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Regular article

The Modified Chaplygin Gas and Dark Degeneracy with Phantom Model

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Abstract. In this paper, we consider two known models of dark energy and make dark degeneracy. The first one is a modified generalized Chaplygin gas, and the second one is the Phantom model. The dark degeneracy leads us to obtain the explicit form of the creation rate. By using the matter density with respect to the rate of dark matter, we define the rate of dark matter creation. In that case, we consider the modified generalized Chaplygin gas (MGCG), and calculate the creation rate and adiabatic sound speed. Also, we introduce the Phantom model and split the corresponding field into components. We write the Klein-Gordon equation and obtain the potential and H in terms of the creation rate.

Keywords: Dark Energy; Modified Generalized Chaplygin Gas; Phantom Model; Klein-Gordon Equation.

Contents

1 Introduction

Over the last decades, cosmology has seen high-quality data become available. The most surprising conclusion from these data is the existence of a dark contribution to the energy density in the universe, which seems to make up 95 percent of the total energy density today, on the other hand, the vacuum energy in expanding space-time faces a problem in theoretical cosmology and quantum field theories. This problem creates some differences between the vacuum energy density and the observed value in cosmology. Also, we note that the vacuum energy density is regularized by imposing an ultraviolet cutoff on the order of the Planck mass. So, the results of vacuum energy are theoretical with a cutoff of 122 orders of magnitude the observed value. Also, if we use cutoff by the QCD vacuum transition the result is still 40 orders above the observed. On the other hand in flat space-time, the vacuum energy-momentum tensor is zero. So, in curve space-time the vacuum density is non zero and must be derived by an appropriate renormalization procedure. In de Sitter space-time the vacuum density will be as $\Lambda \simeq H^2$ (H is the expansion rate) [1–4]. As we know the result $\Lambda \simeq H^2$ creates a radiation phase and the vacuum density decays producing relativistic matter [5]. Any model with matter creation from a decaying vacuum has phenomenological status, because vacuum energy-momentum conservation is one of the conditions behind renormalization techniques [6]. The scaling of the vacuum density with H^2 from the inflation period can explain the back reaction of relativistic particle creation. Also, several arguments will be considered in de Sitter space-time $P_\Lambda = -\Lambda$. In the present FLRW space-time, observations show that the universe is in accelerated expansion. This means that the dark energy equation of state parameter ω, defined by

$$P = \omega\rho. \tag{1.1}$$

We note here the background observations can not fix this function because the effects of dark energy and dark matter are degenerated [7–9]. Such degeneracy can be broken at the perturbation level. In that case, it defined dark matter as the clustering component observed in galaxies and clusters. Also, dark matter is assumed to be cold, that is non relativistic. So, the degeneracy will be reduced to two distinct classes of dark energy models with $\omega = -1$ namely ΛCDM model, with constant Λ, and interacting models with an energy flux from dark energy to dark matter.

A holographic perspective in dark energy models suggests that the universe may be like a giant hologram, in which the observed three-dimensional reality is encoded on a distant two-dimensional boundary, and the energy driving the accelerating expansion of the universe (dark energy) is related to the information contained in a space that is essentially lower than the space it is supposed to be. This concept is based on the holographic principle in theoretical physics, which states that the information content of a volume is limited by the area of its boundary. The holographic dark energy model, in which, according to the Hubble scale, the density of dark energy is inversely proportional to the square of the size of the universe, is the most notable model. This approach provides a solution to the problem of cosmic coincidence and ties dark energy to the thermodynamics of space-time. The holographic dark energy principle expresses that the quantum zero-point energy in the given volume of space cannot exceed the mass of a black hole of the same size, which, by definition, acts as a limit on the energy density of the vacuum, which implies that all information within a volume can be described by its boundary surface. It means the information content of a region is limited by its size, preventing it from collapsing into a black hole [21].

In this paper we are going to consider two known models of dark energy and make dark degeneracy. The first one is a modified generalized Chaplygin gas and the second one is the Phantom model. In section 2 we review the dark degeneracy and obtain the explicit form of the creation rate. By writing the ρ_m with respect to Γ we show that Γ defines the rate of dark matter creation. In section 3 we consider the modified generalized Chalygin gas (MGCG), we calculate the creation rate and adiabatic sound speed. In section 4 we introduce the Phantom model and splite the corresponding field into components. We write the Klein-Gordon equation and obtain the potential and H in terms of creation rate. The Holographic Dark Energy and the creation rates and limitations of holographic principles are also discussed. In this section, we have some figures for the different models as various of it $H(t)$. In that case, we draw the rate of dark matter with respect to the field and for the MGCG and GCCG gases. Different figures completely agree with any data and information from the literature. Finally, in section five, we have some conclusions and suggestions.

2 Review Dark Degeneracy

As we know in the present letter dark degeneracy is formulated in the following manner. We assume dark fluid that is following,

$$\rho = \Lambda + \rho_m, \tag{2.1}$$
$$P_\Lambda = -\Lambda, \tag{2.2}$$
$$P_m = \omega_m \rho_m, \tag{2.3}$$

where $\omega_m \geq 0$ and $\Lambda > 0$. By using (1.1-2.3), we have the following equation,

$$\rho_m = \frac{\omega + 1}{\omega_m - \omega}\Lambda. \tag{2.4}$$

In case of $-1 \leq \omega < 0$ we have $\rho_m \geq 0$ and this component can be interpreted as dark matter. By using the equations (2.1-2.3) and $\omega_m = 0$, the Friedmann and continuity equations will be as,

$$\rho_m + \Lambda = 3H^2, \tag{2.5}$$
$$\dot{\rho}_m + 3H\rho_m = -\dot{\Lambda}, \tag{2.6}$$

where the dot means derivative with respect to cosmological time. Equation (2.6) expresses the total energy conservation. In this case, two component may interact together [10–13]. Differentiating (2.5) and using (2.6) we will arrive at,

$$\rho_m = -2\dot{H}. \tag{2.7}$$

We put (2.4) in (2.5) in case of $\omega_m = 0$, we have following equation,

$$\Lambda = -3\omega H^2. \tag{2.8}$$

We take derivative Λ with respect to cosmological time in equation (2.8), one can obtain,

$$\dot{\Lambda} = -3\dot{\omega}H^2 - 6\omega\dot{H}H. \tag{2.9}$$

By using equations (2.7), (2.5) and (2.4) in equation (2.9) we drive,

$$\dot{\Lambda} = \left(3\omega H - \frac{\dot{\omega}}{\omega + 1}\right)\rho_m, \tag{2.10}$$

so, equation (2.6) can be rewritten as,

$$\dot{\rho}_m + 3H\rho_m = \Gamma\rho_m, \tag{2.11}$$

where Γ is the creation rate will be as,

$$\Gamma = \frac{\dot{\omega}}{\omega + 1} - 3\omega H. \tag{2.12}$$

By using $\rho_m = Mn$ and $H = \frac{\dot{a}}{a}$, from equation (2.11), we obtain the following expression,

$$\frac{1}{a^3}\frac{d}{dt}(a^3 n) = \Gamma n, \tag{2.13}$$

where M, n and a are defined dark matter particle mass, number density and scale factor respectively. Here, we note the Γ defines the rate of dark matter creation.

3 The Modified Generalized Chaplygin Gas and Degeneracy

In order to discuss the rate of dark matter creation we are going to consider several models with different H. So, in this case, the equation of state for the modified generalized Chaplygin gas (MGCG) is given by [14–19].

$$P = A\rho - \frac{B}{\rho^\alpha}, \tag{3.1}$$

also, the modified generalized Chaplygin gas is characterized by an adiabatic sound speed [14–19],

$${c_s}^2 = \frac{\dot{P}}{\dot{\rho}} = A(1 + \alpha) - \alpha\omega, \tag{3.2}$$

when $A = 0$, we will obtain the generalized gas sound speed as ${c_s}^2 = \alpha\omega$ (α is constant). If we look at the relation (3.2), in order to have the stability of the system one has to consider $A(\alpha + 1) - \alpha\omega > 0$. So the corresponding stability lead us to have $\frac{A(\alpha+1)}{\alpha} > \omega$ Hence, by differentiating $P = \omega\rho$ as $\dot{P} = \dot{\rho}\omega + \dot{\omega}\rho$ and put$\dot{P}$ in equation (3.2) one can obtain,

$$\dot{\omega}\rho = \dot{\rho}(A - \omega)(\alpha + 1), \tag{3.3}$$

By using the following conservation equation

$$\dot{\rho} + 3H(\rho + P) = 0, \tag{3.4}$$

in expression (3.3) we have following,

$$\dot{\omega} = 3H(\alpha + 1)(1 + \omega)(\omega - A), \tag{3.5}$$

Now we have to apply the Chaplygin gas equation of state in the equation (2.12) one can obtain the creation rate as,

$$\Gamma = 3H\alpha(\omega - A) - 3HA, \tag{3.6}$$

where in generalized Chaplygin gas the creation rate is $\Gamma = 3H\omega\alpha$. In equation (3.6) the creation rate is positive if α is negative, so in case of $A = 0$, we have $C_s^2 = \alpha\omega$ The modified Chaplygin gas equation of state parameter is given by,

$$\omega = \frac{P}{\rho} = A - \frac{B}{\rho^{\alpha+1}}, \tag{3.7}$$

where A and B are positive constant. Since $\rho = 3H^2$, from equation (3.7) one can obtain following,

$$\omega = A - B\frac{H^{-2(\alpha+1)}}{3^{\alpha+1}}, \tag{3.8}$$

We put equation (3.8) in (3.6) and achieve the following equation,

$$\Gamma = -\alpha B\frac{H^{-(2\alpha+1)}}{3^{\alpha}} - 3HA, \tag{3.9}$$

when $\alpha < 0$ we have energy flux from dark energy to dark matter, since $\Gamma > 0$. For $\alpha = 0$ and $A = 0$, we arrive at the ΛCDM model with $\Gamma = 0$ and $C_s^2 = 0$. For $\alpha = -\frac{1}{2}$ we have $\Gamma = \frac{\sqrt{3}B}{2} - 3HA$ where

$$H = \frac{\frac{\sqrt{3}B}{2} - \Gamma}{3A}. \tag{3.10}$$

Here from equation (3.9), we see that $\Gamma \sim 3H_{dS}$ where H_{dS} is the expansion rate in the de Sitter limit. This result shows that a universe dominated by matter evolves from an Einstein-de Sitter phase to an asymptotically de Sitter era.

4 Phantom model and Degeneracy

In the FLRW space-time the energy density and pressure of a minimally coupled scalar field Φ Phantom model are given by,

$$\begin{aligned} \rho_\Phi &= -\frac{1}{2}\dot{\Phi}^2 + V(\Phi) \\ P_\Phi &= -\frac{1}{2}\dot{\Phi}^2 - V(\Phi) = -\Lambda. \end{aligned} \tag{4.1}$$

So, Λ will be as,

$$\Lambda = \frac{1}{2}\dot{\Phi}^2 + V(\Phi). \tag{4.2}$$

Now, we are going to use the equation (4.1) in equation (2), one can obtain ρ_m as,

$$\rho_m = -\dot{\Phi}^2. \tag{4.3}$$

The equations (4.2), (2.5) and (2.7) help us to arrive at following equation,

$$3H^2 = V(\Phi) - 2H'^2, \tag{4.4}$$

where

$$\dot{\Phi} = 2H', \tag{4.5}$$

and prime means derivative with respect to Φ. We use (4.1) and (4.2) into the conservation equation (2.6), we derive the following Klein- Gordon equation,

$$\ddot{\Phi} + 3H\dot{\Phi} - V'(\phi) = 0. \tag{4.6}$$

All the above information for cosmological be general, we shall now consider the special case of Γ, which is $\Lambda = 2\Gamma H$. From (4.1), (4.2), (4.4) and (4.5) one can obtain following,

$$V(\Phi) = \frac{3}{2}H^2 + \Gamma H. \tag{4.7}$$

We substitute this equation in (4.4) and (4.7) we will arrive at following,

$$4H'^2 - 2\Gamma H + 3H^2 = 0. \tag{4.8}$$

So, in that case, one can write the field Φ as,

$$\Phi = \int \frac{dH}{\sqrt{\frac{\Gamma H}{2} - \frac{3}{4}H^2}}. \tag{4.9}$$

The corresponding Hubble parameter in terms of the field will be as,

$$H = \frac{\Gamma}{3}(1 - \cos\frac{\sqrt{3}}{2}\Phi). \tag{4.10}$$

We see in equations (4.7) and (4.10) the potential and H for the corresponding Phantom model are written in terms of field and creation rate. This result is important to describe the degeneracy in generalized Chaplygin and Phantom models.

In the following, we take several examples with different scale factors. Using some scale factors $a(t)$, for GCG we can discuss the time dependence of the creation rate as well as the evolution on ϕ in a given time. So, in the first example, we take the scale factor as,

$$a(t) = a_0(B + e^{kt})^m, \tag{4.11}$$

where

$$H(t) = \frac{mke^{kt}}{B + e^{kt}},$$

so, the creation rate is,

$$\Gamma(\phi, t) = \frac{3mke^{kt}}{(B + e^{kt})(1 - \cos\frac{\sqrt{3}}{2}\Phi)}.$$

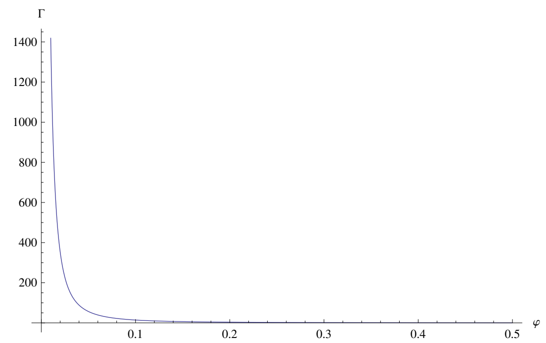

Figure 1: The evolution of Γ on ϕ for $m = 1.1$, $k = 0.03$, $B = 1$, $t = 5$.

Figure 1 shows the evolution of the creation rate Γ as a function of the scalar field ϕ. The sharp increase in Γ at higher values of ϕ indicates that dark matter creation accelerates

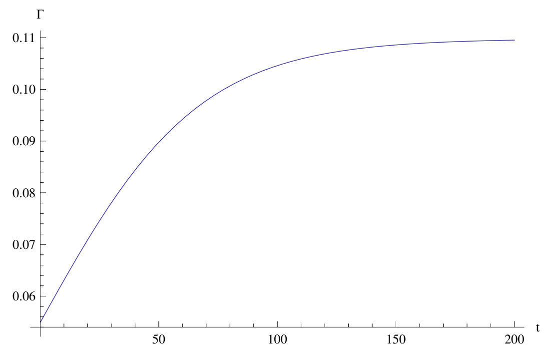

Figure 2: The evolution of Γ on t for $m = 1.1$, $k = 0.03$, $B = 1$, $\phi = 1.7$.

as the scalar field evolves, which may have important consequences for the evolution of the universe.

In these figures, we took $a_0 > 0$, $k > 0$, $> B > 0$, $m > 1$ and $a(t) = a_0(B + e^{kt})^m$. In the second example, the scale factor is given by

$$a(t) = e^{\lambda t^{\beta}}, \tag{4.12}$$

where

$$H(t) = \lambda\beta t^{\beta-1}.$$

So, the creation rate is achieved by the following equation,

$$\Gamma(\phi, t) = \frac{3\lambda\beta t^{\beta-1}}{(1 - \cos\frac{\sqrt{3}}{2}\Phi)}.$$

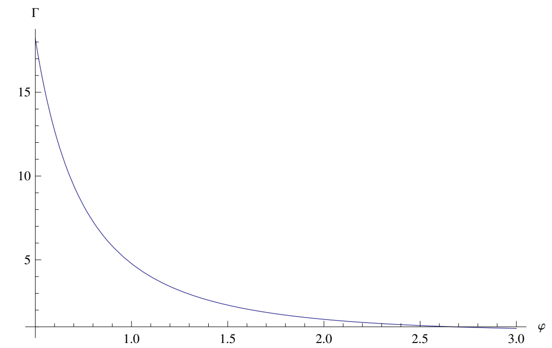

Figure 3: Evolution of Γ on ϕ for $\lambda = 0.7$, $\beta = 0.8$, $t = 1$.

In those cases, we took $\lambda > 0$, $0 < \beta < 1$ and $a(t) = e^{\lambda t^{\beta}}$.
Now we take the third example. In that case, by considering a class of possible cosmological solutions with indefinite expansion, the scale factor showing the accelerating expansion of the universe which is given by,

$$a(t) = e^{x(\ln t)^{\beta}}, \tag{4.13}$$

where

$$H(t) = \frac{\beta x}{t}(\ln t)^{\beta-1},$$

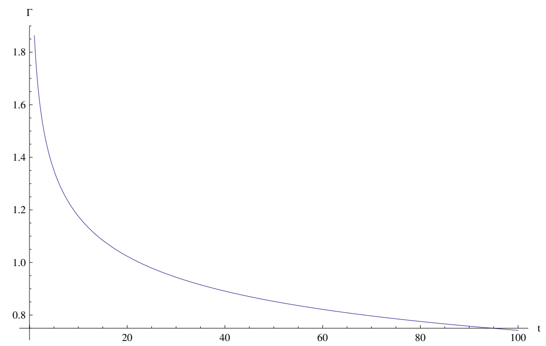

Figure 4: Evolution of Γ on t for $\lambda = 0.7$, $\beta = 0.8$, $\phi = 1.7$.

so, the creation rate is

$$\Gamma(\phi, t) = \frac{3\beta x(\ln t)^{\beta-1}}{t(1 - \cos\frac{\sqrt{3}}{2}\Phi)}.$$

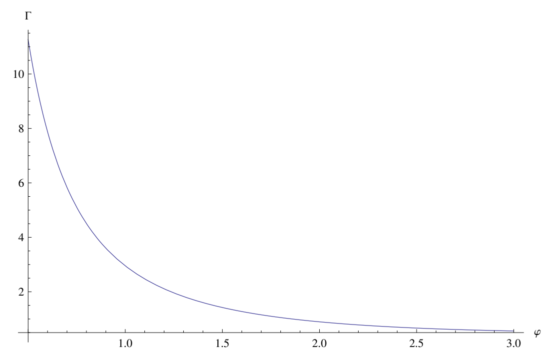

Figure 5: Evolution of Γ on ϕ for $x = 0.5$, $\beta = 2$, $t = 2$.

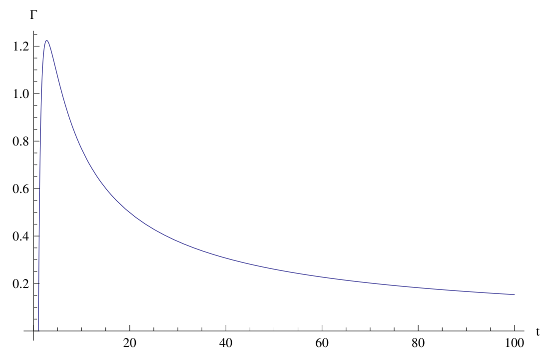

Figure 6: Evolution of Γ on t for $x = 0.5$, $\beta = 2$, $\phi = 1.7$.

In these cases we took $x > 0$, $\beta > 1$ and $a(t) = e^{x(\ln t)^{\beta}}$. Here we take advantage of equation (3.2) and investigate the stability of the system for the three models. In order to discuss the stability of the system one can write the speed of sound, the condition of $C_s^2 \geq 0$ gives us such stability. For this reason, we calculate the C_s^2 in GCG for the three examples. And also, we draw the C_s^2 in terms of t which are shown by figures 7, 8 and 9. Finally, the information of the paper gives us to work with generalized cosmic Chaplygin gas (GCCG) and also discuss the stability of the system with three examples. In those cases, we have figures 10, 11 and 12.

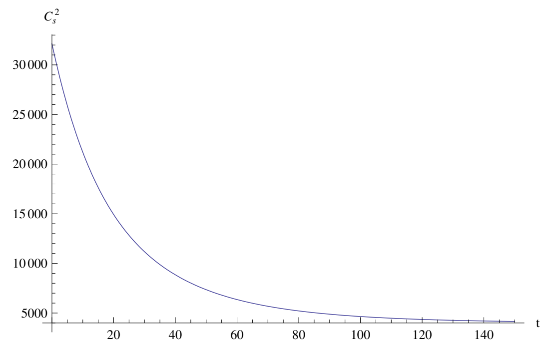

Figure 7: Square of sound speed in terms of time for $H(t) = \frac{mke^{kt}}{B+e^{kt}}$ whith $B = 1.5$, $\alpha = 0.5$and $A = 0.5$.

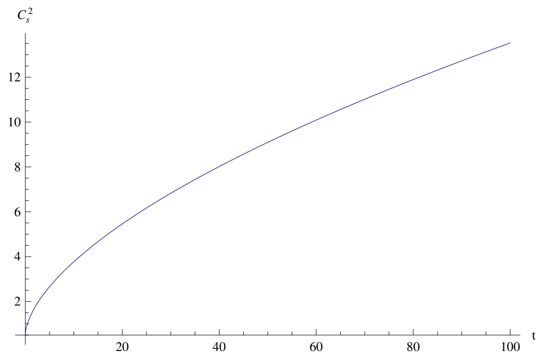

Figure 8: Square of sound speed in terms of time for $H(t) = \lambda\beta t^{\beta-1}$ whith $B = 1.5$, $\alpha = 0.5$and $A = 0.5$.

Figures 7 through 12 show the square of the adiabatic sound speed C_s^2 as a function of time for different models of the scale factor a(t). These figures show the stability of the system by analyzing the behavior of the sound speed squared, C_s^2, for each model. The condition $C_s^2 \geq 0$ ensures the stability of the system. A negative sound speed squared would indicate instability, which is critical for determining the viability of each cosmological model. By evaluating C_s^2, these figures provide a way to test the stability of different dark energy models under various conditions. Stability is essential for understanding whether these models can describe a physically realistic universe. The results shown in these figures help assess whether the equations of state lead to viable cosmologies or if they predict unphysical behavior.

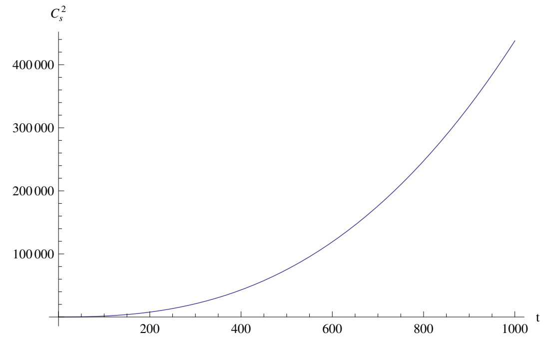

Figure 9: Square of sound speed in terms of time for $H(t) = \frac{\beta x}{t}(\ln t)^{\beta-1}$ whith $B = 1.5$, $\alpha = 0.5$and $A = 0.5$.

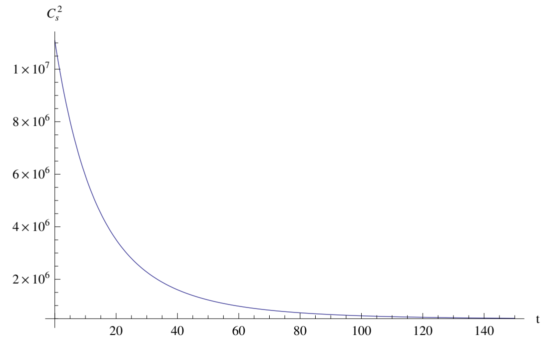

Figure 10: Square of sound speed in terms of time for $H(t) = \frac{mke^{kt}}{B+e^{kt}}$ whith $B = 1.5$, $\alpha = 0.5$and $\omega = 0.5$.

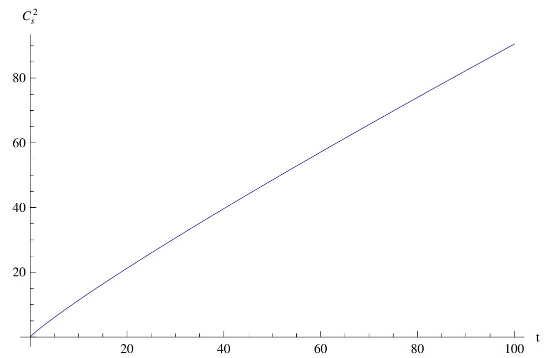

Figure 11: Square of sound speed in terms of time for $H(t) = \lambda\beta t^{\beta-1}$ whith $B = 1.5$, $\alpha = 0.5$and $\omega = 0.5$.

4.1 The Holographic Dark Energy and the Phantom Field Theory

Using the future event horizon as an infrared (IR) cutoff in holographic dark energy models, it can be directly related to the behavior of the phantom field, since the resulting dark energy density can become negative, leading to a rapid expansion of the universe that violates the standard energy condition, which is an important characteristic of the phantom field. In

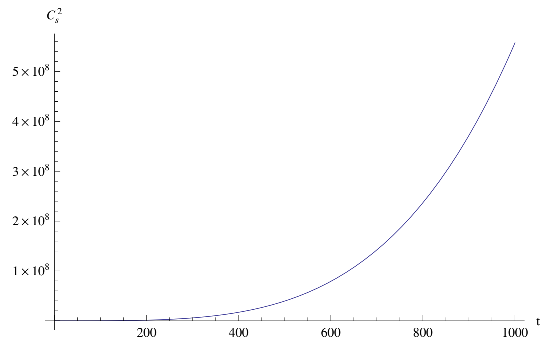

Figure 12: Square of sound speed in terms of time for $H(t) = \frac{\beta x}{t}(\ln t)^{\beta-1}$ whith $B = 1.5$, $\alpha = 0.5$and $\omega = 0.5$.

holographic dark energy models, the energy density is related to the IR cutoff, which in this case is chosen as the future event horizon. This allows us to relate the properties of the phantom model to holographic principles [22].

The holographic dark energy density, ρ_D, can be expressed as:

$$\rho_D = \frac{3c^2 m_p^2}{L^2}, \tag{4.14}$$

where c is a constant, m_p is the Planck mass, and L is the IR cutoff. In the context of the Phantom model, we take the IR cutoff L to be the future event horizon:

$$L(t) = a(t)r(t), \tag{4.15}$$

$r(t)$ represents the future event horizon at time t: $r(t) = \frac{1}{H(t)}$, here H(t) is the Hubble parameter at time t.
The relationship between the Hubble parameter and the future event horizon plays an important role in linking the holographic principles to the creation rate Γ in the phantom model. We can now write the equation for the creation rate Γ, taking into account the holographic nature of dark energy:

$$\Gamma = 3H(\frac{c^2 M_p^2}{r(t)^2} - \frac{3}{a(t)^2}). \tag{4.16}$$

4.2 Creation rates and limitations of holographic principles

The rate of creation of Γ in holographic dark energy models can be bounded by the holographic principle, which requires that the energy density be bounded by the degrees of freedom present at the boundary of the universe. This can impose a relationship between Γ, $H(t)$, and the future event horizon. The holographic principle states that information cannot exceed the limits set by the boundary conditions, and therefore we expect that there is a natural limit to the rate of creation of dark matter, ensuring that it does not exceed the energy available in the system at any given time [22]. This limit can be expressed as:

$$\Gamma \leq \frac{3c^2 M_p^2}{r(t)^2}. \tag{4.17}$$

This ensures that the creation rate Γ is consistent with the holographic nature of dark energy, where the amount of dark energy in the universe should be limited by the IR cut-off.

5 Conclusion

In this paper, we introduced two models of dark energy and investigated their degeneracy. The two models considered here were the generalized Chaplygin gas and the Phantom model. We discussed dark degeneracy, which led us to arrange the creation rate. Additionally, for the selected models, we calculated the creation rate and the adiabatic sound speed. Finally, we introduced the Klein-Gordon equation for the Phantom model and obtained the potential and H in terms of the creation rate, which plays an important role in dark energy. We extended the Phantom model by relating it to holographic dark energy, where the IR cutoff is taken as the future event horizon. By considering this relationship, we derived equations linking the creation rate Γ to the holographic density ρ_D, which adds a layer of physical insight into the evolution of dark energy and dark matter.

Our results show that the creation rate plays a crucial role in determining the dynamics of the universe. Future work could involve testing these models against observational data from large-scale surveys, such as the upcoming Euclid mission, to determine their compatibility with the latest cosmological observations.

Authors' Contributions

All authors have the same contribution.

Data Availability

The manuscript has no associated data or the data will not be deposited.

Conflicts of Interest

The authors declare that there is no conflict of interest.

Ethical Considerations

The authors have diligently addressed ethical concerns, such as informed consent, plagiarism, data fabrication, misconduct, falsification, double publication, redundancy, submission, and other related matters.

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References

[1] L. H. Ford,"Quantum vacuum energy in general relativity", Phys. Rev. D **11**, 3370 (1975). DOI:10.1103/PhysRevD.11.3370.

[2] J. S. Dowkler and R. Critchley, "Effective Lagrangian and energy-momentum tensor in de Sitter space", Phys. Rev. D **13**, 3224 (1976). DOI: 10.1103/PhysRevD.13.3224.

[3] B. K. Shukla, R.K. Tiwari, A. Beesham, D. Sofuoglu, "Modified Chaplygin gas solutions of f(Q) theory of gravity", Elsevier, New Astronomy, **117**, 102355, (2025). DOI: 10.1016/j.newast.2025.102355

[4] A. A. Starobinsky, "A new type of isotropic cosmological models without singularity", Phys. Lett. B **91**, 99 (1980). DOI: 10.1016/0370-2693(80)90670.

[5] S. Carneiro and R. Tavakol, "Thermodynamical properties of dark energy in loop Quantum cosmology", Gen. Rel. Grav. **41**, 2287 (2009). DOI: 10.1142/S0218271811018731.

[6] S. Carneiro, AIP Conf. Proc. "Inflation driven by particle creation", **61**, 1471 (2012). DOI: 10.1142/S2010194512008173.

[7] M. Kunz, "Degeneracy between the dark components resulting from the fact that gravity only measures the total energy-momentum tensor", Phys. Rev. D **80**, 123001 (2009). DOI: 10.1103/PhysRevD.80.123001.

[8] I. Wasserman, "On the Degeneracy Inherent in Observational Determination of the Dark Energy Equation of State", Phys. Rev. D **66**, 123511 (2002). DOI: 10.1103/PhysRevD.66.123511

[9] C. Rubano and P. Scudellaro, "On some exponential potentials for a cosmological scalar field as quintessence", Gen. Rel. Grav. **34**, 1931 (2002). DOI: 10.1023/A

[10] W. Zimdahl, J. Schwarz, A. B. Balakin and D. Pavon, "Cosmic antifriction and accelerated expansion", Phys. Rev. D **64**, 063501 (2001). DOI: 10.1103/PhysRevD.64.063501.

[11] S. Del Campo, R. Herrerg and D. Pavon, "Interacting models may be key to solve the cosmic coincidence problem", JCAP **0901**, 020 (2009). DOI: 10.1088/1475-7516/2009/01/020.

[12] L. P. Chimento, "Linear and nonlinear interactions in the dark sector", Phys. Rev. D **81**, 043525 (2010). DOI: 10.1103/PhysRevD.81.043525.

[13] J. H. He, B. Wang and E. Abdalla, "Testing the interaction between dark energy and dark matter via latest observations", Phys. Rev. D **83**, 063515 (2011). DOI: 10.1103/PhysRevD.83.063515.

[14] A. Kamenshchik, U. Moschella and V. Pasquier, "Chaplygin-like gas and branes in black hole bulks", Phys. Lett. B **7**, 487 (2000). DOI: 10.1016/S0370-269328002900805-4.

[15] A. Kamenshchik, U. Moschella and V. Pasquier, "An alternative to quintessence", Phys. Lett. B **511**, 265 (2001). DOI: 10.1016/S0370-2693

[16] N. Bilic, G. B. Tupper and R. D. Viollier, "Unification of dark matter and dark energy: the inhomogeneous Chaplygin gas", Phys. Lett. B **535**, 17 (2002). DOI: 10.1016/S0370-2693

[17] Y. Wang, D. Wands, L. Xu, J. De- Santiago and A. Hojjati, "Cosmological constraints on a decomposed Chaplygin gas", Phys. Rev. D **87**, 083503 (2013). DOI: 10.1103/PhysRevD.87.083503

[18] D. Wands, J. De- Santiago and y. Wang, "Inhomogeneous vacuum energy", Class. Quant. Grav. **29**, 145017 (2012). DOI: 10.1088/0264-9381/29/14/145017.

[19] J. Zheng, Sh. Cao, Y. Lian, T. Liu, Y. Liu, Z-H Zhu, “Revisiting Chaplygin gas cosmologies with the recent observations of high-redshfit quasars”, The European Physical Journal C **82**, 582, (2022). DOI: 10.1140/epjc/s10052-022-10517-4

[20] M. C. Bento, O. Bertolami and A. A. Sen, “Generalized Chaplygin gas, accelerated expansion, and dark-energy-matter unification”, Phys. Rev. D **66**, 043507 (2002). DOI: 10.1103/PhysRevD.66.043507.

[21] A. Tita, B. Gumjudpai, P. Srisawad, “Dynamics of holographic dark energy with apparent-horizon cutoff and non-minimal derivative coupling gravity in non-flat FLRW universe”, Elsevier, Physics of the Dark Universe, **45**, 101542, (2024). DOI: 10.1016/j.dark.2024.101542.

[22] S. D. Campo, J. C. Fabris, R. Herrera, W. Zimdahl, “On holographic dark-energy models”, Phys. Rev. D **83**, 123006 (2011). DOI: 10.1103/PhysRevD.83.123006.