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Regular article

Fast Electric Quenches and Scaling Behavior at Finite Coupling

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Abstract. We investigate the dynamical response of a gauge theory with a holographic dual at both finite- and infinite-coupling regimes to time-dependent electric field quenches of various profiles. Using the AdS/CFT correspondence, we analyze the resulting electric current as a function of the quench profile and system parameters, including temperature and coupling strength. Our study reveals a universal scaling behavior in the early-time response to fast quenches. Specifically, we find that for tanh-like quenches, the rescaled first peak of the current scales as $(1/\delta t)^0$, while for pulse-like quenches, it scales as $(1/\delta t)^{-1}$, where δt is the transition time of the electric field. This scaling persists across different theories, including infinite coupling, finite coupling, and finite temperature, demonstrating its independence from the underlying theory and quench details.

Keywords: AdS/CFT Correspondence; Electric Field Quench; Universal Scaling.

1 Introduction

The study of quantum quenches, which involve deforming a quantum field theory by a time-dependent change in the coupling of a relevant operator, $\mathcal{L} \to \mathcal{L} + \lambda(t)\mathcal{O}$, is of significant interest from both theoretical and experimental perspectives. Despite the extensive research dedicated to this problem, many questions remain open regarding the out-of-equilibrium behavior induced by the quenched coupling and the subsequent evolution of the system. Understanding the dynamics of far-from-equilibrium systems presents a challenging problem across various areas of physics, particularly in the strong coupling regime. This challenge underscores the importance of further investigating quantum quenches.

Another important aspect that makes the study of quantum quenches fascinating is the observation of universal behavior in the system's response under certain special conditions. One such universal feature was identified in a series of papers [1–4]. Some of these works [1–3] explored the response of a strongly coupled holographic conformal field theory to a deformation induced by a time-dependent coupling to a relevant operator. Specifically, the coupling $\lambda(t)$ starts from zero and increases to a finite value $\delta\lambda$ over a finite duration, referred to as the transition time δt. These studies considered bosonic and fermionic mass operators, examining the transition from a (thermal) conformal field theory to a mass-deformed theory, as well as the reverse transition from the massive theory back to the conformal theory.

Through these investigations, they discovered a remarkable scaling property for strongly coupled holographic systems in the limit of fast but smooth quenches. The response of such systems, i.e., the expectation value of the quenched operator rescaled by $\delta\lambda$, scales as $\delta t^{d-2\Delta}$, where d is the spacetime dimension and Δ is the conformal dimension of the operator.

They then extended this study to investigate the scaling behavior of free field theories using non-holographic considerations [4]. Surprisingly, they found the same scaling property in the case of scalar and fermionic field theories. This result indicates that the scaling behavior observed under fast quenches is universal and does not require strong coupling as a necessary condition.

The scaling property observed in these studies is remarkable because it suggests a universal response that transcends specific details of the system, such as the nature of the operator or the dynamics of the underlying theory. This universality is crucial for understanding far-from-equilibrium phenomena in quantum field theories, as it provides a predictive framework that applies across diverse systems. Testing this scaling at finite coupling is particularly important because most physical systems, such as the quark-gluon plasma (QGP) produced in heavy-ion collisions, exist in regimes where neither weak nor infinitely strong coupling fully applies. Demonstrating that the scaling persists at finite coupling would strengthen the case for its universality and deepen our understanding of the dynamics of real-world strongly coupled systems.

A remarkable approach to studying field theories in the strong coupling limit is the AdS/CFT correspondence, which relates a strongly coupled theory at infinite 't Hooft coupling and an infinite number of colors to a weakly coupled supergravity theory [5]. To explore the regime where the coupling lies between the extremes of infinitely strong and weak, one can incorporate finite coupling corrections by including additional contributions of order $\mathcal{O}(\alpha'^3)$ to the gravity theory [6,7]. Here, α' is related to the string length.

It is intriguing to investigate whether the aforementioned scaling behavior arises in the response of holographic theories with finite coupling under fast quenches. To this end, we examine the response of a holographic model with massless quarks to electric field quenches, considering both finite and infinite coupling regimes. To introduce an external electric field on the field theory side, we embed a probe D7-brane with a gauge field into the gravity bulk. By solving the DBI equation, we analyze the universal properties of the system, focusing in particular on the scaling behavior discussed earlier.

In the next section, we introduce the model describing a finite coupling field theory. In Section 3, we derive the equations of motion from the DBI action of the probe D7-brane with the gauge field. These equations provide the electric current produced in response to the quenched electric field. Section 4 presents the results through various graphs and investigates the presence of universal scaling. Finally, a brief summary and discussion are provided in Section 5.

2 Gravity background with $\mathcal{O}(\alpha'^3)$ corrections

Our aim is to examine the response of the following backgrounds to a time-dependent electric field of different profiles.

The ten-dimensional $\mathrm{AdS}_5 - \mathrm{BH} \times S^5$ metric which is an exact solution of type IIB supergravity, can be expressed as

$$ds_{10}^2 = \frac{R^2}{z^2}\left[-f(z)dt^2 + d\vec{x}^2 + \frac{1}{f(z)}dz^2\right] + R^2 d\Omega_5^2, \tag{2.1}$$

where R is the radius of AdS_5 and S^5. The blackening function is $f(z) = 1 - \frac{z^4}{z_h^4}$, where z is the radial coordinate, ranging from $z = 0$ at the boundary to $z = z_h$ at the event horizon. This metric is dual to the planar limit of the strong coupling $\mathrm{SU}(N_c)$ $\mathcal{N} = 4$ SYM theory at finite temperature, given by $T = \frac{1}{\pi z_h R^2}$.

By incorporating $\mathcal{O}(\alpha'^3)$ corrections to the $\mathrm{AdS}_5 - \mathrm{BH} \times S^5$ metric, the modified metric becomes [6,7]

$$ds_{10}^2 = \frac{R^2}{z^2}\left[-f(z)K^2(z)dt^2 + d\vec{x}^2 + \frac{P^2(z)}{f(z)}dz^2\right] + R^2 L^2(z) d\Omega_5^2, \tag{2.2}$$

where

$$f(z) = 1 - \left(\frac{z}{z_h}\right)^4, \quad K(z) = e^{\gamma[a(z)+4b(z)]}, \quad P(z) = e^{\gamma\, b(z)}, \quad L(z) = e^{\gamma\, c(z)}. \tag{2.3}$$

Here $\gamma = \frac{\zeta(3)}{8}\lambda^{-\frac{3}{2}}$, with the 't Hooft coupling $\lambda \propto \alpha'^{-\frac{1}{2}}$. The functions $a(z)$, $b(z)$ and $c(z)$ represent the corrections to the metric components, defined as

$$a(z) = -\frac{1625}{8}\left(\frac{z}{z_h}\right)^4 - 175\left(\frac{z}{z_h}\right)^8 + \frac{10005}{16}\left(\frac{z}{z_h}\right)^{12}, \tag{2.4}$$

$$b(z) = \frac{325}{8}\left(\frac{z}{z_h}\right)^4 + \frac{1075}{32}\left(\frac{z}{z_h}\right)^8 - \frac{4835}{32}\left(\frac{z}{z_h}\right)^{12}, \tag{2.5}$$

$$c(z) = \frac{15}{32}\left[1 + \left(\frac{z}{z_h}\right)^4\right]\left(\frac{z}{z_h}\right)^8. \tag{2.6}$$

The dilaton field also receives corrections at order $\mathcal{O}(\gamma)$ expanded as

$$\phi(z) = \phi_0(z) + \gamma\phi_1(z) + \mathcal{O}(\gamma),$$

where

$$\phi_0(z) = -\log(g_s), \tag{2.7}$$

$$\phi_1(z) = -\frac{45}{8}\left(\frac{z}{z_h}\right)^4 - \frac{45}{16}\left(\frac{z}{z_h}\right)^8 - \frac{45}{24}\left(\frac{z}{z_h}\right)^{12}. \tag{2.8}$$

$(g_s N)^{1/2} \propto \alpha'$. Moreover, The corrected temperature, accounting for finite coupling effects, is given by

$$T = \frac{1}{\pi z_h R^2}\left(1 + \frac{265}{16}\gamma\right). \tag{2.9}$$

As expected, in the limit $\gamma \to 0$ all relations revert to their forms at infinite 't Hooft coupling, consistent with the standard results in AdS/CFT.

2.1 D3/D7 brane configuration

To introduce fundamental quarks into the field theory, we add a D7-brane in the probe limit. In this approximation, the background geometry remains unchanged by the presence of the D7-brane, simplifying the analysis.

To implement a time-dependent electric field $E(t)$ in the x direction (one of the spatial directions in the field theory), we adopt the following ansatz for the gauge field on the brane:

$$A_x = -\int^t E(s)ds + A(t,z). \tag{2.10}$$

The D7-brane is chosen to span the coordinates $(t, \vec{x}, z, \Omega_3)$ while the perpendicular directions are fixed to $\theta = 0$ and $\phi = 0$. This choice is guided by symmetry and the massless nature of the quarks. For massless quarks, the case we are interested in, the brane embedding is trivial, and the dynamics are determined solely by the gauge field $A(t,z)$. In contrast, for massive quarks, the brane embedding becomes non-trivial and is described by a function $\phi(t,z)$ that must be solved alongside $A(t,z)$ [8–10].

Substituting the gauge field $A_x(t,z)$ and the induced metric into the DBI action, we obtain

$$S_{D7} = -\tau_7 \int d^8\xi \frac{R^2 L^3(z)}{z^5}\sqrt{\frac{z^4 P^2(z)\dot{A_x}^2(t,z) - R^2 f(z)K^2(z)P^2(z) - z^4 f^2(z)K^2(z)A_x'^2(t,z)}{f(z)}}, \tag{2.11}$$

where the tension of the D7 brane is given by $\tau_7 = 1/[g_s\alpha'^4(2\pi)^7]$ and ξ^a denote the brane coordinates. Variation of the action with respect to the gauge field, yields a partial differential equation, which must be solved subject to appropriate boundary conditions on $A(t,z)$. These conditions are $A(t,z_0) = \partial_z A(t,z)|_{z_0} = 0$ and $A(t_0,z) = \partial_t A(t,z)|_{t_0} = 0$, where z_0 is a small cutoff near the boundary and t_0 is an initial time before which the electric field is zero. Then, using the AdS/CFT dictionary, the time-dependent electric current, which is the response to the applied electric field, is given by $j(t) \propto \partial_z^2 A(t,z)|_{z_0}$.

To investigate the system's response, we consider four forms of time-dependent electric fields, referred to as $M1$, $M2$, $M3$ and $M4$:

$$E(t) = \frac{E_0}{2}\left[1 + \tanh\left(\frac{4t}{\delta t}\right)\right], \tag{2.12}$$

$$E(t) = E_0 \begin{cases} 0, & t < 0, \\ \cos^2\left(\frac{\pi t}{2\delta t} + \frac{\pi}{2}\right), & 0 \leqslant t \leqslant \delta t, \\ 1, & t > \delta t, \end{cases} \tag{2.13}$$

$$E(t) = \frac{E_0}{\cosh^2\left(\frac{4t}{\delta t}\right)}, \tag{2.14}$$

$$E(t) = E_0 \begin{cases} 0, & t < 0, \\ \cos^2\left(\frac{\pi t}{2\delta t} + \frac{\pi}{2}\right), & 0 \leqslant t \leqslant 2\delta t, \\ 0, & t > 2\delta t. \end{cases} \tag{2.15}$$

The profiles $M1$ and $M2$ are *tanh-like*, starting from zero and asymptotically reaching a finite value E_0. In contrast, $M3$ and $M4$ *pulse-like*, starting from zero, reaching a peak value E_0 and eventually returning to zero. In these expressions, δt represents the transition time, characterizing how quickly the electric field evolves from zero to its peak value E_0. In the profiles $M2$ and $M4$, the changes in the electric field occur over an exact, finite duration of time. In both cases, the electric field is turned on precisely at $t = 0$, reaches its peak value at $t = \delta t$, and remains constant thereafter in $M2$. In $M4$, however, the electric field returns to zero at $t = 2\delta t$. By contrast, the general trends in $M1$ and $M3$ are similar to $M2$ and $M4$, respectively, but the transitions in these profiles are not confined to an exact duration. In $M1$, the electric field starts increasing from zero at asymptotically distant past ($t \to -\infty$) and reaches its final value only at asymptotically distant future ($t \to +\infty$). Similarly, in $M3$, the electric field returns to zero at $t \to +\infty$.

In the next section, we explore the system's response to these electric field quenches, focusing on the universal scaling behavior.

3 Universal scaling in various coupling constants

Our aim is to explore the dynamical response of the system to time-dependent electric fields in Eqs. (2.12-2.15). We specifically focus on searching for a scaling property similar to that discovered in [1–4]. We are interested in considering the effect of temperature along with the finite coupling on this scaling. Finding universal behavior in the response of the system to quenched operators is significant, as it highlights circumstances under which the behavior of the system is predictable irrespective of details such as the theory and the quench.

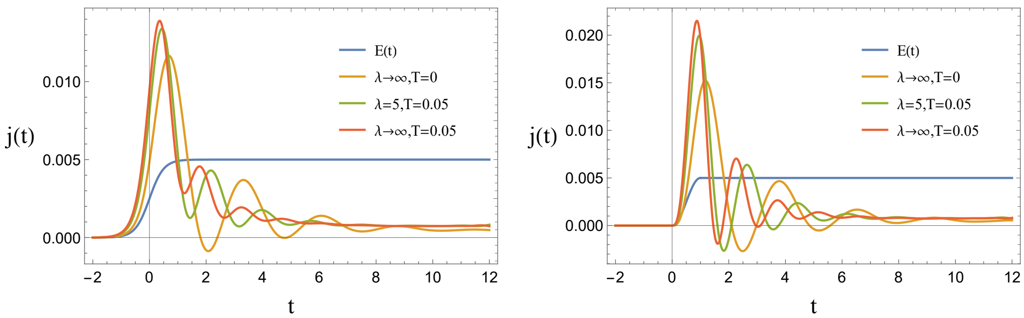

Figure 1: The left and right graphs show the time evolution of the electric current $j(t)$ in response to the quenches $M1$ and $M2$, respectively, both with $E_0 = 0.005$ and $\delta t = 2$.

For the field theory we are working with, the application of any minute electric field leads to an electric current due to the Schwinger effect [11], since this theory is deconfined and the quarks are massless, and as a consequence, its critical electric field is zero [12–19]. We first display the electric current for various parameters of the theory and the electric field quench. Figure 1 depicts the dynamical evolution of $j(t)$ when quenches $M1$ and $M2$ with parameters $E_0 = 0.005$ and $\delta t = 2$ are turned on. Each figure compares three cases: zero temperature and infinite coupling, finite temperature and finite coupling, and finite temperature and infinite coupling. The electric field profile has also been drawn in each case. Notice that the response is similar in both $M1$ and $M2$. Since in $M2$ the transitions in the electric field occur over an exact finite duration of time, the oscillations in the response of the system are more intense, with higher amplitudes, compared to $M1$, where the evolution of the electric field follows a smoother trend.

In all these cases, the electric current tends to its corresponding value for a static $E = E_0$ as the system approaches its final steady state. The final fate of the response is independent of the details of the quench profile and its transition time. Figure 2 shows a similar display of the response of the system to models $M3$ and $M4$. In these cases, the final fate of the electric current is approaching zero regardless of the system's and quench's parameters.

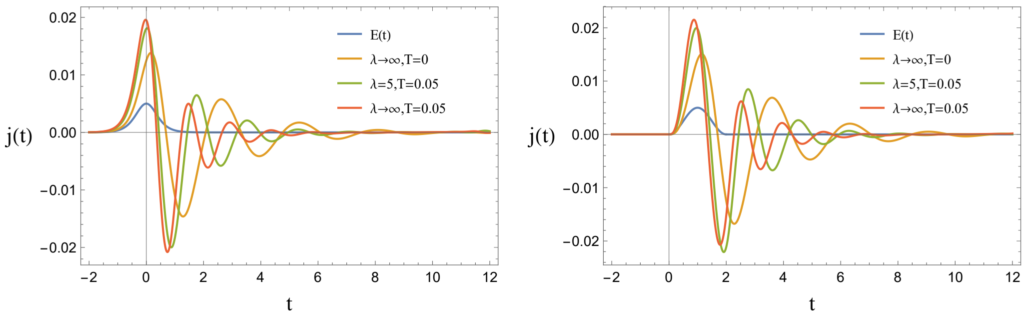

Figure 2: Left and right graphs show the time evolution of the electric current $j(t)$ in response to the quenches $M3$ and $M4$, respectively, both with $E_0 = 0.005$ and $\delta t = 2$.

Now, we move to our original goal: searching for universal scaling in the early-time response of the system. To this end, we compute the value of the first peak of $j(t)$, i.e., $j_{\max}$, rescaled by the maximum value of the electric field E_0 as a function of $1/\delta t$ for small values of δt.

The graphical outcomes of this computation are represented in Fig. 3. The left graph illustrates the results for all four quench models in the finite-coupling background with $\lambda = 5$ and $T = 0.05$. Notice that both axes are logarithmic, although the tick marks indicate the original values. As can be seen, a scaling behavior is observed for fast enough quenches, i.e., small δt, consistent with prior studies. The results in this limit are independent of the exact form of $E(t)$, as they are exactly the same for tanh-like quenches $M1$ and $M2$, and also for pulse-like quenches $M3$ and $M4$. Straight lines in log-log plots confirm that $j_{\max}/\delta t$ has a power-law dependence on $1/\delta t$ at the fast-quench limit. The power is zero and -1 for tanh-like and pulse-like quenches, respectively. Note that by increasing δt, the response differs for different quench models.

Another important point is that we have similar graphs with exactly the same powers for all the theories with zero temperature and infinite coupling, finite temperature and finite coupling, and finite temperature and infinite coupling. Here we have only shown one of these results. Therefore, this scaling demonstrates that, in the limit of fast quenches, the response is not sensitive to the theory, its coupling, its temperature, or the exact functionality of the electric field quench. The only determining factor is whether the electric field eventually returns to zero (pulse-like quenches) or results in a nonzero electric current in the final steady state of the system (tanh-like quenches).

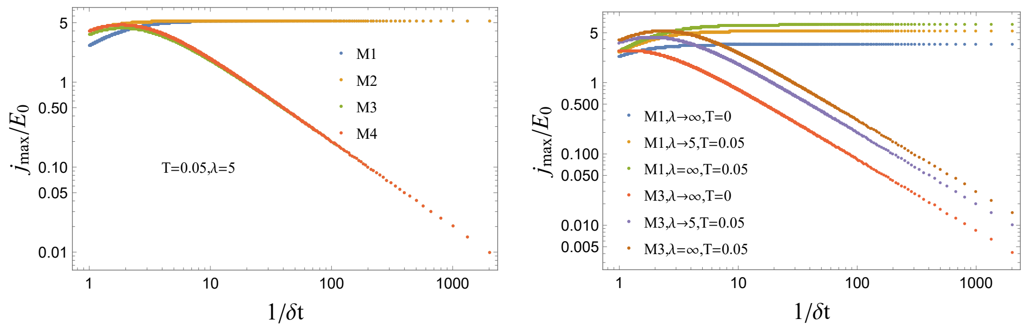

Figure 3: Left graph: $j_{\max}/\delta t$ against $1/\delta t$ for all four models in the finite-coupling background with $\lambda = 5$ and $T = 0.05$. Right graph: $j_{\max}/\delta t$ against $1/\delta t$ for $M1$ and $M3$, and for various backgrounds.

In the right graph of Fig. 3, we show $j_{\max}/\delta t$ for the quench models $M1$ and $M3$ for zero temperature and infinite coupling, finite temperature and finite coupling, and finite temperature and infinite coupling. This comparison reveals the existence of the universal scaling for all three theories. We see that the power is 0 and -1 in all theories, as stated before. The differences arise only in the slope of the power-law functions.

4 Conclusion

In this paper, we have applied holographic techniques to study the dynamical response of a gauge theory with a gravity dual to electric field quenches of various profiles. Gauge/gravity duality provides a powerful framework for studying quantum field theories in the strong (infinite) coupling regime. It is particularly effective for investigating out-of-equilibrium phenomena, where other techniques often face significant challenges.

A key motivation for this study is the discovery of universal scaling behaviors in the response of quantum systems to quenches. Such scaling properties are remarkable because they reveal predictable behavior in systems that are otherwise highly sensitive to their specific details, such as coupling strength, temperature, or the precise form of the quench. Previous studies on fermionic and bosonic mass quenches have demonstrated that, in both holographic and non-holographic settings, the rescaled first peak of the quenched operator exhibits a universal scaling in the fast-quench regime. Specifically, this scaling follows the relation $\delta t^{d-2\Delta}$, where d is the spacetime dimension, and Δ is the conformal dimension of the operator. These studies incorporated the backreaction of the quench operator in the background, providing a comprehensive picture of the scaling behavior. However, these studies solved the equations perturbatively in the amplitude of the bulk scalar, leaving room for exploration of non-perturbative approaches and different types of quenches.

In our study, we instead turn on an external, time-dependent electric field while neglecting its backreaction in the background. To achieve this, we introduced the corresponding gauge field on a D7-brane in the probe limit within a bulk $\mathrm{AdS}_5 \times S^5$ metric. Additionally, recognizing that most physically interesting systems operate in regimes of intermediate coupling, we extended our analysis to include the $\mathcal{O}(\alpha'^3)$ corrected $\mathrm{AdS}_5 \times S^5$ black hole metric. This allowed us to investigate whether the observed scaling behavior persists in holographic theories with finite coupling and at finite temperature.

We have employed four distinct electric field quench profiles. Two of these quenches rise from zero to a final finite value and eventually return to zero (pulse-like quenches), while the other two remain at their maximum value indefinitely after rising from zero (tanh-like quenches). In the absence of a mass gap, the critical electric field for deconfined field theories is zero, which means an electric current is immediately induced when the electric field increases from zero. This is indeed the case for our system, where the current arises instantly upon activating the electric field.

We have analyzed the system's response to different quench profiles by plotting the electric current under various conditions to capture the general behavior. Additionally, we produced logarithmic plots of the maximum value of the first peak of the current, $j_{\max}$, rescaled by the maximum electric field, E_0, as a function of the inverse transition time, $1/\delta t$. For tanh-like quenches, we observed a horizontal straight line, indicating that $j_{\max}/E_0$ is independent of δt in the fast-quench limit. In contrast, for pulse-like quenches, we found a straight line with a fixed negative slope, suggesting a power-law dependence. Curve fitting confirmed that $j_{\max}/E_0 \sim (1/\delta t)^0$ for tanh-like profiles and $j_{\max}/E_0 \sim (1/\delta t)^{-1}$ for pulse-like quenches. Note that for bosonic and fermionic mass operators, the conformal mass dimensions are $\Delta = d - 2$ and $\Delta = d - 1$, respectively, in d spacetime dimensions, corresponding to 2 and 3 in $d = 4$. Consequently, the scaling exponents for these operators are 0 and 2, respectively. These results were derived for tanh-like quenches. In contrast, in our study, the quenched operator, coupled to the gauge field, is the current operator, with $\Delta = 3$ in $d = 4$. Our calculations show a scaling exponent of 0 for tanh-like quenches, differing from the findings of previous studies for an operator of the same mass dimension. While the specific exponents differ from previous results, the scaling behavior itself aligns with the earlier studies.

In summary, our results demonstrate that the scaling behavior in the early-time response of the system to fast quenches persists in the context of electric field quenches, even in the fully non-perturbative regime and without backreaction on the background. Moreover, this universality holds for both finite and infinite coupling cases.

The inconsistency of the scaling exponents found in our case with previous results prompts curiosity about whether other scenarios should be explored to gain a deeper understanding of the nature of this universality. Scaling exponents serve as measurable signatures that can validate theoretical predictions. From an experimental perspective, such as in heavy-ion collisions, the complexity of quark-gluon plasma (QGP) dynamics can be simplified by focusing on universal properties, which are more accessible to identify and analyze.

For future research, it would be intriguing to investigate the response of holographic theories incorporating confinement and/or a mass gap. This exploration could reveal the existence and characteristics of similar scaling behaviors in these more realistic setups, particularly near the critical point of the system. Such studies hold significant value as they better reflect the strongly coupled theories like QCD, potentially bridging the gap between theoretical models and experimental observations.

Data Availability

The manuscript has no associated data.

Conflicts of Interest

The author declares that there is no conflict of interest.

Ethical Considerations

The author has diligently addressed ethical concerns, such as informed consent, plagiarism, data fabrication, misconduct, falsification, double publication, redundancy, submission, and other related matters.

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