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Analysing some QCD Observable at Low Energy Scale, Based on AdS/CFT Correspondence

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Abstract. In the AdS/CFT correspondence, the space-time metric can be modified by a dilaton background with a positive sign. Modifying the metric and action with the dilaton field in the non-Abelian sector of Quantum Field Theory, such as Quantum Chromodynamics (QCD), produces an analytical running coupling constant applicable in the non-perturbative domain of the theory. The computed running coupling constant aligns closely with experimental results at low energy scales. The Burkert-Ioffe model can additionally modify this $\alpha_s^{AdS}(Q^2)$ to more closely align with experimental results at high energy levels. Consequently, utilizing the AdS/CFT correspondence, we analyze specific QCD observables including the Bjorken sum rule, electron-positron annihilation into hadrons, and hadronic tau decay at low energy scales, below the QCD cut-off parameter, and we compare the outcomes with experimental data that match them closely.

Keywords: AdS/CFT Correspondence; Non Perturbative QCD; Modifed Strong Coupling.

Contents

1 Introduction

In QCD, the gauge theory of the strong force, the concept of a running coupling $\alpha_s(Q^2)$ is usually restricted to large Q^2 and the magnitude of this quantity is well defined within perturbative QCD at high energy scales [1,2], but its magnitude grows to infinity at low energy scale. At large distances or low energies scale (IR domain), most of these theoretical results and experimental data predict that QCD coupling constant does not depend on the scale as it does at high momentum domain [3,4]. Hence in Quantum Chromo dynamics (QCD), defining an analytic function for coupling constant at a low energy scale is very essential. This function should be valid over the full space-like and time-like domains.

Recently many experiments have been done at low energies scale [5–15]. Theoretical analysis of these experiments is very important. Using QCD theory to analyze these data directly is not feasible due to the limitation imposed by the QCD cutoff parameter, Λ_{QCD}, on the range of energy scale that can be utilized. Although the numerical value of Λ_{QCD} is specified via the fitting of experimental data but it can not definitely approach very small values. In the other words, α_{QCD} at low energies scale does not have adequate behavior and is not analytical. As a result, it is unsuitable for use as an expansion coefficient in perturbative calculations and cannot be utilized to analyze experimental data at low energy scale. To resolve this difficulty some different solutions have been suggested, including the method based on the lattice QCD [16], technical solution to Dyson-Schwinger equations [17], AdS/CFT duality [18], principle of maximum conformability [19], Freezing technique [20] and etc. Since in AdS/CFT correspondence, there is a relation between the coupling constant of strings and the coupling constant of Yang-Mills theories, we can obtain an analytic behaviour for coupling constant in QCD considerations at low energy scales. In this paper, using AdS/CFT correspondence we study some QCD observables such as the Bjorken sum rule, the $R_{e^+e^-}(s)$ ratio for the annihilation of electron-positron to hadron at center of mass energy $\sqrt{s}$ and finally hadronic tau decays which all show that the results obtained from AdS/CFT duality are in good agreement with the concerned experimental data.

The organization of this paper is as follows. In Sec. 2 a brief review of required explanations of AdS/CFT correspondence is given where more details as AdS space and conformal theory are discussed respectively in Subsecs. 2.1 and 2.2. One of the important results of this correspondence is the Holography princilple is illustrated in Subsec. 2.3. We back to AdS/CFT correspondence in Subsec. 2.4 with the aim of achieving its application in non-abelian gauge theory. For this purpose we discuss in Subsec. 2.5 how the strong coupling constant is modified, considering the AdS/CFT correspondence. In Sec. 3 a further modification on the coupling constant is discussed in which we are able to use it in the whole range from low to high energy scales. Applications of the final version of the modified coupling constant to estimate some QCD observables like Bjorken sum rule, electron-positron annihilation, and Hadronic tau decay are rendered in Sec. 4. Finally we give our conclusion in Sec. 5.

2 Basic concepts in AdS/CFT correspondence

Utilizing the holography principle, Anti-de-Sitter (AdS)/Conformal Field Theory (CFT) correspondence establishes a conformity between different string theories and CFTs. Considering conformal field theories and their relation with Quantum Chromodynamic (QCD), the correspondence between QCD and string theories could be obtained. In this connection, we first argue in the following subsection the AdS space and then we briefly review CFT theories. At the end, considering the holography principle we discuss about some aspects of AdS/CFT correspondence. One of the outstanding aspects of this correspondence provides us with the modified strong coupling constant which involves many phenomenological applications in QCD which is finally discussed in the following subsection.

2.1 AdS space

Here a brief review is given about Anti-de-Sitter (AdS) space time. Einstein in 1917 introduced his general relativity (GR) field equation. AdS space time is a solution of the Einstein equation in the vacuum and has maximum symmetry in 5 dimensions [21,22].

The Anti-de-Sitter space is defined to be quadratic. AdS space time and de-Sitter (dS) space-time are Euclidian spaces with negative and positive cosmological constants respectively. These are changed to the Minkowski space time, if the cosmological constant is zero [23]. General AdS metric is given by:

$$ds^2 = -f\left(r\right)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2. \tag{2.1}$$

This metric can be represented in various coordinates. Unlike dS space, AdS space is temporally circular and open in all spatial directions. Anti de-Siteer and de-Sitter space-time evolved in hyperboloid form.

On the other hand, one can say that Anti-de Sitter space is in fact a 4 dimensional manifold in 5 dimensional (5D) Minkowski space-time where the metric of such space time reads generally as

$$ds^2 = dt^2 + dx^2 - dr^2 - r^2 d\Omega^2. \tag{2.2}$$

On one hand side, Anti-de Sitter metric in 4-dimensional spherical coordinates is as follows [18]:

$$ds^2 = \left(1 + \frac{r^2}{R^2}\right)dt^2 - \left(1 + \frac{r^2}{R^2}\right)^{-1} dr^2 - r^2 d\Omega^2, \tag{2.3}$$

where $-R^2 = \frac{3}{\Lambda^2}$ in which Λ is the cosmological constant. On the other side, one can write AdS space as a hypersurface in a five-dimensional Peseudo-Minkowsky with constant curvature of $1/R^2$ [24]. Finally, the Anti-de Sitter metric in Poincare's coordinate system can be written in the light cone coordinates as follows [25],

$$ds^2 = R^2\left((d\bar{x})^2 + dz^2 - dt^2\right)/z^2, \tag{2.4}$$

where $d\bar{x}$ denotes to the non-transformed coordinates.

2.2 Conformal theory

In order to get the correspondence between AdS and CFT, we first need to deal with the conformal theories. Conformal symmetry is a basic element in considerations of string theory. The conformal algebra is usually taken into account in infinite-dimensional space-time and hence in four dimensions as a finite dimensional space-time, the conformal algebra is less powerful. In conformal theories, one can write in d dimension space-time, the metric tensor as it follows:

$$g(x)_{\mu\nu} \to g\left(x\right)_{\mu\nu} = \Omega\left(x\right)g(x)_{\mu\nu}. \tag{2.5}$$

Generators of conformal theories are:

$$\begin{aligned} P_\mu &= -i\partial_\mu\,,\ L_{\mu\vartheta} = i(x_\mu\partial_\vartheta - x_\vartheta\partial_\mu), \\ D &= ix^\mu\partial_\mu,\ K_\mu = -i\left[x^2\partial_\mu - 2x_\mu x^\vartheta\partial_\vartheta\right]. \end{aligned} \tag{2.6}$$

For the special case of four-dimensional space time, the conformal group is easily constructed from four generators for translations, six generators of the Lorentz group, one generator for scale transformations and four generators for proper conformal transformations with totally of 15 generators [26].

Consequently for finite rather than infinitesimal transformations, the following transformations will be obtained from the above generators:

$$\begin{aligned} x^{\mu} &\rightarrow x^{\mu'} = x^{\mu} + a^{\mu} \\ x^{\mu} &\rightarrow x^{\mu'} = \Lambda^{\mu}_{\upsilon} x^{\upsilon} \\ x^{\mu} &\rightarrow x^{\mu'} = \lambda x^{\mu} \\ x^{\mu} &\rightarrow x^{\mu'} - \frac{x^{\mu} + b^{\mu} x^2}{1 + b.x + b^2 x^2} \end{aligned} \tag{2.7}$$

The total number of all above transformations in d-space is $\frac{(d+1)(d+2)}{2}$ that is identical to the rank of SO(2,d) group in Minkowski space time. This conformal group in four-dimension can be represented such as: $SO(4,2) \sim SU(2,2)$ [27].

2.3 Holography

‘t Hooft showed that two-dimensional space is rich enough to describe all three-dimensional phenomena. Therefore, the world can be described as a Hologram [28]. After he introduced this idea, Maldesena proposed that large N limits of certain CFTs can be described in terms of supergravity (and string theory) in a dimension which is higher by one. Consequently, conformal theories in 4 dimensions can be lived on the AdS space with a compact manifold. Therefore in a corresponding d+1 dimensional space, a compact manifold appears as a sphere. After this suggestion by Maldesena, studies shifted to CFT and field theories which ended finally to an AdS/CFT correspondence. On this base super Yang-Mills theory with N=4 is equivalent to a special type of superstring theory on AdS with 5-dimension. As a result of this Holography, the space geometry of a black hole in its near-horizon is an AdS geometry. Therefore, people usually work on the Anti-de Siter space. Field theory on AdS with d+1 dimensions can be related to an M field theory with d dimension, represented by M_d [29].

After equipping to required information for AdS, conformal theories and Holography, we are at the stage to follow the duality of AdS and conformal theories.

2.4 AdS/CFT correspondence

The AdS/CFT correspondence provides a proper tool for studying the dynamics of strongly coupled quantum field theories. Here strong coupling of Gauge theories, which involve SU(N) symmetry group, is considered [30]. Gauge theory with N = 4 can be a kind of supersymmetry theory. The aim is to obtain a strongly coupling constant in Yang-Mills theories, using the AdS/CFT correspondence. This approach is based on a connection between the large N limit of supergravity theory with a superstring/M-theory. M-theory is constructed on a 10-dimensional space and supergravity is on the AdS space-time. The large N limit of a maximally N = 4 is defined on the AdS boundary which is put in a D dimensional space time. [29,31,32]. The application of this conjecture to QCD is not straightforward because QCD is neither supersymmetry nor conformal. Quantum chromodynamics (QCD) is a gauge theory in 4-dimensional space-time.

The idea of the existence of higher-dimensional space was first introduced by Kaluza and Klein in the 1920s which leads to a duality between electromagnetic theory and gravity [33].

Later on A. Witten presented a procedure to extend duality to QCD and other gauge theories [29]. Klein postulated that the extra dimension should be curled down to a length of around the Plank length in order that the curvature corresponds to the correct magnitude of electric charge [34]. This idea has been generalized in string theory. After then superstring/M-theory is defined on bigger space $S^5 \times (AdS)^5$ that is a ten-dimensional space performed by the direct product of two spaces. The conformal field theory (under the restriction of large N) living on the boundary submanifold of an Anti-de-Sitter space-time is equivalent to the bulk supergravity (or string) theory on the Anti-de-Sitter space-time. According to the string theory, under an appropriate unitary transformation, which involves the required duality, a large wrapping number of p-dimensional branes and a large momentum number can be interchanged. As a result, the relationship among these holographies may exist [35].

Since in the heterotic string, one set of vibrations exists in 26 dimensions and the superstring exists in 10 dimensions, 16 dimensions do not need to curl up like a ball [27]. Thus the metric of superstring theory can be written in 10 dimensions as follows:

$$ds_{10}^2 = -\frac{1}{\sqrt{f_0}}dt^2 + \sqrt{f_0}(dx_1^2 + \cdots + dx_9^2) \; , \tag{2.8}$$

where f_0 is a nine-dimensional harmonic function:

$$f_0 = 1 + g_s l_s^7 \sum_{n=1}^{N} \frac{Q_n}{|\vec{x} - \vec{x}_n|^7} . \tag{2.9}$$

Here Q_n represents the charge or the associated parameter of D-Particles (that is the constituent entities in string theory) at X_n and x_n represents the position of D-Particles. Also, x denotes a nine-dimensional spatial point.

One can simply get the dilaton ϕ (a field in string theory as the primary generator of the length scale) and the R-R one-form gauge field A (as a field associated with the charged currents carried by strings) which are given by [35]:

$$e^{-2\phi} = g_s^{-2} f_0^{-3/2}, \qquad A_t = 1 - \frac{1}{f_0} \; . \tag{2.10}$$

Here the string coupling constant g_s satisfies $g_s = e^{\phi_\infty}$ where the parameter ϕ_∞ denotes the string scale and denotes the value of the dilaton at infinite distance, which can serve as a key parameter in determining the string coupling constant [35].

The dilaton ϕ can also affect the AdS metric and as a result Eq. (2.1) can be modified by introducing a dilaton profile in the AdS action. Furthermore, Polchinski and Strassler show that AdS/CFT duality is correspondingly modified to incorporate a mass scale [36]. In the modified theory the conformal metric of Anti-de Sitter space is modified by introducing an additional warp factor $e^{\pm\kappa^2 z^2}$ [37]. Hence Eq. (2.1) can be rewritten as:

$$ds^2 = R^2 e^{\pm\kappa^2 z^2} \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right) / z^2. \tag{2.11}$$

where z is an extra dimension in the anti-de-Sitter space-time. Using the holographic principle we can see the result of strings moment in space-time with 10 dimensions appearing on the border of four-dimensional AdS space. In Eq. (2.11) R is the Anti-de Sitter radius which is related to the string coupling, number of colors, and string scale such as $R^2 \sim g_s N \alpha^2$ [30].

Considering the AdS/CFT correspondence, it is possible now to derive the strong coupling constant of a Yang-Mills theory which is applicable in nonperturbative region. The context of the next section is devoted to this subject.

2.5 Nonperturbative effective coupling in anti-de sitter space

The group of transformations, belongs to SO(4,2) symmetry group, and leaves the Anti-de Sitter metric invariant. This is due to the fact that under a dilatation of all coordinates, such as

$$x^{\mu} \to \lambda x^{\mu}, \qquad z \to \lambda z, \tag{2.12}$$

the metric in Eq. (2.11) would be invariant. Here the variable z is a scaling variable in Minkowski space.

It should be noted that QCD approaches a conformal theory in both the far ultraviolet and deep infrared regions. When quantum corrections are included, the conformal behavior is preserved at very large Q because of asymptotic freedom and near $Q \to 0$ because the theory develops a fixed point. On the other hand, the renormalization group (RG) equations describe how the coupling constant evolves with energy scale. In AdS/CFT correspondence, these equations reveal that at very low energies, below the Landau pole, the running coupling constant behaves like a fixed point of the RG flow, effectively freezing to a constant value rather than diverging. This behavior is indicative of a quasi-conformal regime where QCD can be treated similarly to a conformal field theory at short distances.

The conformal invariance of Anti-de Sitter space should be broken in order to achieve a confining theory. There are two ways to break the invariance. The first way is "hard-wall". In this way, conformal invariance is broken at $z_0 \sim 1/\Lambda_{QCD}$ [36] where Λ_{QCD} is the QCD cutoff parameter. Since the scale invariant is $x^{\mu} \to x^{\mu'} = \lambda x^{\mu}$ at a finite value $z_0 \sim 1/\Lambda_{QCD}$, the "hard-wall" breaks conformal invariance [36]. It should be noted that different values of z correspond to different energy scales in QCD [38]. Equivalent to modification of the Anti-de Sitter metric in Eq. (2.11), a dilation profile is introduced in the AdS action.

The second way is "soft-wall". In this way, for asymptotical Anti-de Sitter, the geometries warp factor appears as $e^{\pm\kappa^2 z^2}$ and it vanishes at small z [37,39]. According to the assumption of Sonnenscheim in [40] the g_{00} array of the AdS metric is such as

$$\partial_z(g_{00})\,|_{z=z_0} = 0, \qquad g_{00}\,|_{z=z_0} \neq 0. \tag{2.13}$$

The metric which is modified by the warp factor $e^{\pm\kappa^2 z^2}$ satisfies the above conditions with $z_0 = \frac{1}{\sqrt{2\kappa}}$ and is used to derive a confining potential between heavy quarks [37,39].

On the other hand, the gravitational potential energy for an object of mass m in general relativity is given by [41]

$$V = mc^2\sqrt{g_{00}} = mc^2 R \frac{1}{z} e^{\pm\kappa^2 z^2/2}\ . \tag{2.14}$$

Therefore the related action is obtained as it follows [38]:

$$S = -\frac{1}{4}\int d^5x\sqrt{g}e^{\varphi(z)}\frac{1}{g_5^2}F^2. \tag{2.15}$$

Here F is a five-dimensional gauge field which propagating in Anti-de Sitter space, $\varphi(z) = \pm\kappa^2 z^2$ and perfector $e^{\varphi(z)}g_5^{-2}$ can be identified by $g_5^{-2}(z)$ as the effective coupling of the theory in the AdS action at the length scale z and finally $\sqrt{g} = (\frac{R}{z})^5$ [38].

To facilitate the computations, people usually utilize the Light-front (LF) approach [42]. In the LF holography, a correspondence is observed between the Hamiltonian formulations of QCD in 4-dimensional space-time quantized on the light front at a fixed light front time and the equations of motion in AdS space. In this connection the required action in 4 dimensions would be given by

$$S = \int d^4xdz\sqrt{g}e^{\varphi(z)}\mathcal{L}, \tag{2.16}$$

where $\mathcal{L}$ is the QCD lagrangian. We then have a direct relation between hadronic amplitude in Anti-de Sitter space, denoted by $\Phi(z)$, and a function which describes the structure of hadrons in LF, presented by $\phi(\zeta)$, where ζ is the LF invariant variable and is corresponding with coordinate z in AdS. By the spontaneous chiral symmetry breaking, it can be explained how flavor in AdS/CFT is living on the world volume of flavor branes. By matching the UV asymptotic of current-current two-point function between bulk and boundary theories, the gauge coupling g_5 is fixed [43].

Furthermore, the physical states in AdS space are introduced by normalizable modes

$$\Phi_P(x^\mu, z) = e^{-iP.x\Phi(z)}, \tag{2.17}$$

where $\Phi_P(x^\mu, z)$ is a normalizable string mode which is dual to LF hadronic state. On this base, one can write the single-variable light-front relativistic Schrodinger equation as follows [44]:

$$-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta) = M^2\phi(\zeta), \tag{2.18}$$

where L is the relative orbital angular momentum and M is eigenvalue of the corresponding Eigen mode $\phi(\zeta)$. In above equation $U(\zeta)$ is effective potential in the LF equation of motion which is given by [41]:

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L + S - 1). \tag{2.19}$$

Modified metric is done by a positive-sign dilaton background which is related to a non perturbative coupling constant in AdS space where it is a function of $Q^2 = -q^2$ as the transferred momentum. This coupling constant shows behavior of asymptotic freedom at large Q^2 which has a little difference with respect to the reported experimental data. The limit of coupling constant tends to a fixed point as Q tends to 0. The coupling constant in AdS space can be written as $\alpha_s^{AdS}(Q^2) \sim e^{-Q^2/\kappa^2}$. To achieve this behaviour one should note that light-front holography can map the AdS coupling constant $\alpha_s^{AdS}(Q^2)$ into the Yang-Mills (YM) coupling constant $\alpha_{YM}(\zeta)$. Thus

$$\alpha_s^{AdS}(\zeta) = g^2_{YM}(\zeta)/4\pi \propto e^{-\kappa^2/\zeta^2}, \tag{2.20}$$

where ζ as the invariant impact separation variable in $g_{YM}(\zeta)$ which appears in the LF Hamiltonian is identified with z in $g_5(z)$ so as: $g_5(z) \to g_{YM}(\zeta)$. Using the Bessel transformation, the coupling constant is converted to the momentum space and one obtains:

$$\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0)e^{-Q^2/4\kappa^2}, \tag{2.21}$$

where $\alpha_s^{AdS}(0) = \pi$ [38]. The Gaussian form for the running coupling, follows from the requirement that the action remains conformal though a mass scale appears in the confining potential of the light-front Hamiltonian. Eq. (2.21) is valid only at the domain of Q^2 where QCD is a strongly coupled theory and the AdS/CFT correspondence can be applied. Furthermore, quantum effects are neglected in the light-front holographic approach Nonetheless, $\alpha_s^{AdS}(Q^2)$ can be supplemented at large Q^2 by either parameterizing the well-known pQCD effects at the origin of the large Q^2 dependence by matching Eq. (2.21) to the one in the conventional pQCD.

To further elucidate the significance of the modified running coupling constant derived from AdS/CFT correspondence, it is important to recall that the notion of a running coupling $\alpha_s(Q^2)$ in QCD is typically confined to the perturbative regime. Nonetheless, similar to QED, it is advantageous to designate the coupling as an analytic function applicable across both the complete spacelike and timelike domains. Indeed, examining the non-Abelian QCD coupling at low momentum transfer presents a challenging issue due to color confinement. Its conduct in the nonperturbative infrared (IR) region has been extensively investigated as it is a quantity of crucial significance. It has been demonstrated here that the light-front holographic mapping of effective classical gravity in AdS space, adjusted by a positive-sign dilaton background, can serve to identify a nonperturbative effective coupling $\alpha_s^{AdS}(Q^2)$.

Although the modified strong coupling constant, resulting from AdS/CFT correspondence, indicates analytical behaviour at energy scales below the Λ_{QCD} but in order to match it properly to whole experimental data, including low and high energy scales, one needs to do more modifications on the coupling constant which is illustrated in next section.

3 Modified analytical form of the coupling constant

The preliminary form of the running coupling constant, resulting from AdS/CFT correspondence, is given by Eq. (2.21) where the normalization factor $\alpha^{AdS}(0) = \pi$ can be obtained from Lattice computations [16]. The value of $\kappa = 0.54\ GeV$ has been determined from the principal Regge trajectory of the vector meson [41]. Erlich and his collaborations have obtained the value of the five-dimensional coupling for a SU(2) flavor gauge theory. Their computed value is $\left(g_2^5\right)_{SU(2)} = 12\pi^2\ \frac{R}{N_C}$, with $N_C = 3$ and $R = 1$ leads to $\left(\frac{g_2^5}{4\pi}\right)_{SU(2)} = \pi$ [45] that is corresponded to the normalized coupling $\alpha^{AdS}(0)$.

The coupling constant in Eq. (2.21) at $Q \geq 1$ GeV differs with respect to the reported experimental data. Therefore the coupling constant $\alpha^{AdS}(Q^2)$ requires to be modified. The smooth behavior of the holographic strong coupling constant allows for extrapolation of its form in the perturbative region and this fact also allows to extend the functional dependence of the coupling to large distances. At these distances or low energy scales, the role of the Landau pole is dominating. At the Landau pole, a divergence occurs in the coupling constant which indicates that the perturbative expansion used to describe the QCD breaks down. In this case, the thoery may not be valid at any energy scale, as physical predictions become unreliable. While QCD has an infrared Landau pole, the non-perturbative behavior can be analyzed using AdS/CFT techniques.

Since perturbative confining effects do not vanish exponentially at large Q^2, it is required to add in AdS coupling a fitted term denoted by $g_\pm$ such as

$$g_\pm\left(Q^2\right) = \frac{1}{\left(1 + e^{\pm\frac{Q^2 - Q_0^2}{\tau^2}}\right)}, \tag{3.1}$$

with the values $Q_0^2 = 0.8\text{GeV}^2$ and $\tau = 0.3\ GeV^2$. Regarding the above consideration, one would obtain [38]:

$$\alpha^{AdS}_{Modified}\left(Q^2\right) = \alpha^{AdS}\left(Q^2\right) g_+\left(Q^2\right) + \alpha^{fit}\left(Q^2\right) g_-\left(Q^2\right). \tag{3.2}$$

In the above equation one needs to employ a smeared step function to connect smoothly two contributions from different regions. This is due to the fact that the initiated coupling constant does not indicate a smooth transition without using an analytical expression like $g_{\pm}$. In other words, one encounters with difficulty to do perturbative calculations near or below the transition region and consequently it is inevitable to use the $g_{\pm}\left(Q^2\right)$ function [38].

Deur et al. have presented a new extraction of the fitted coupling constant, α^{fit}, in Eq. (3.2) [46]. This coupling constant agrees with experimental data in the low and high Q^2. This modified form is required to apply in AdS/CFT correspondence and later on in QCD calculations. In this connection, the available data have been used to do a fit for the modified part of the coupling constant to resemble the pQCD evolution equation for α_s. The new version of α_{fit} is such as

$$\alpha_{fit} = \frac{\gamma n\left(Q\right)}{\log\left(\frac{Q^2+m_g^2(Q)}{\Lambda^2}\right)}, \tag{3.3}$$

where $n(Q)$ forces the modified coupling constant to be π when $Q^2 \to 0$ and an additional term, m_g^2 , causes this coupling not to be diverted when $Q^2 \to \Lambda^2$. In this equation $\gamma = 4/\beta_0 = 12/\left(33 - n_f\right)$ where n_f is the number of active quark flavor,

$$n\left(Q\right) = \pi\left(1 + \left[\frac{\gamma}{log\left(m^2/\Lambda^2\right)\left(1+Q/\Lambda\right)^{-\gamma}} + \left(b\,Q\right)^c\right]^{-1}\right),$$

and $m_g\left(Q\right) = 4\left(m/\left(1 + \left(a\,Q\right)^{d}\right)\right)$ where [46]

$$\begin{array}{lll} \Lambda = 0.349 \pm 0.009\, GeV, & a = 3.008 \pm 0.081\, GeV^{-1}, & b = 1.425 \pm 0.032\, GeV^{-1}, \\ c = 0.908 \pm 0.025, & m = 1.204 \pm 0.018 GeV, & d = 0.840 \pm 0.051. \end{array}$$

Now that we have access to the updated coupling constant derived from the AdS/CFT correspondence, we can use it for various phenomenological purposes outlined in the next section.

4 QCD observable in light-front holography

In this section, we study some QCD observables at low energy scales using AdS/CFT correspondence and then compare the theoretical results with the available experimental data.

4.1 Bjorken sum rule

In order to indicate the advantage of utilizing the modified coupling constant, based on AdS/CFT correspondence, we investigate some QCD observables which contain experimental data at low energy scale. The first one is the Bjorken sum rule (BSR) that is of particular importance and is relating to the spin dependence of quark densities to the axial charge. The study of nucleon spin structure is important in quantum chromodynamics. The concerned studies have been actively pursued over the past recent decades from the experimental point of view at CERN, SLAC, DESY and Jefferson laboratory [47–53]. They are well tested at high energy but at low energy, characterizing the domain of quark confinement, their examination is challenging [54]. BSR is important to understand the nucleon spin structure that is confirmed quantum chromodynamics (QCD) can describe well

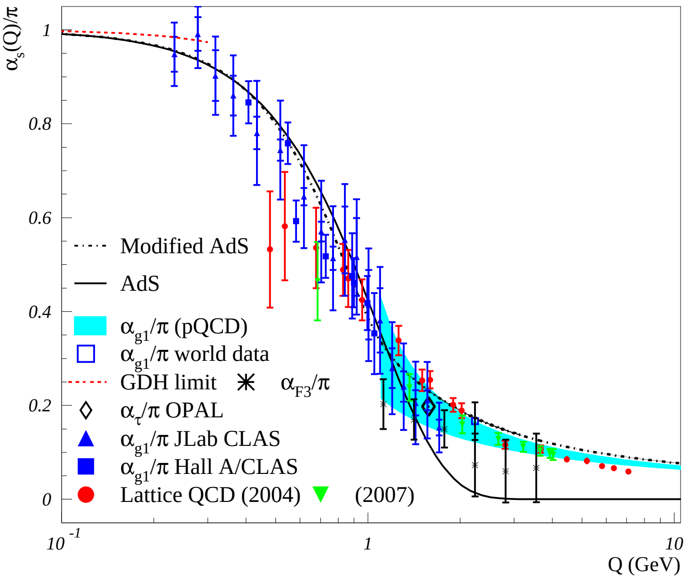

Figure 1: Comparing the effective coupling from LF holographic mapping for $\kappa = 0.54\ GeV$ with effective QCD couplings extracted from different observable and lattice results. This figure has been quoted from Ref. [38].

the strong force in polarized case. It is related to the first moment of polarized nucleon structure function, g_1, which takes the form

$$\Gamma_1^{p(n)}(Q^2) \equiv \int_0^1 g_1^{p(n)}(x, Q^2)dx = \frac{C_1^{NS}(Q^2)}{12}\left[\frac{a_8}{3} \pm a_3\right] + \frac{C_1^S(Q^2)}{9} a_0(Q^2), \tag{4.1}$$

where $\pm$ in the square brackets refer to proton and neutron respectively and $C_1^{NS,S}$ are the SU(3) nonsinglet/singlet coefficient functions that are known up to $O(a_s^3)$. The a_3 and a_8 terms denote the non-singlet combinations of the first moment of polarized quark densities which are related to the weak matrix elements that are measured in neutron and hyperon β decay [55]. The β decay is controlling by nucleon axial charge, presenting hereinafter by g_A [56,57]. Note that only a_0 as singlet axial current depends on Q^2 because a_3 and a_8 are matrix elements of conserved currents in the limit of massless quarks. Conservation of the related axial current operators is a physical statement independent of scale. The reason why this is not the case for a_0 is related to existence of the axial anomaly [58]. From Eq. (4.1) one would easily get

$$\Gamma_1^p(Q^2) - \Gamma_1^n(Q^2) = \frac{C_1^{NS}(Q^2)}{6} a_3, \tag{4.2}$$

where presented Γ_1^p and Γ_1^n refer the the Ellis- Jaffe sum rule and the combination appears in Eq. (4.2) is known as Bjorken sum rule.

On the other hand, going beyond the operator product expansion (OPE) at the leading twist, a new format of the Bjorken sum rule, related to polarized nucleon structure function is obtained as it follows [59]:

$$\Gamma_1^{p-n}(Q^2) \equiv \Gamma_1^p - \Gamma_1^n = \int_0^1 [g_1^p(x) - g_1^n(x)]dx = \frac{g_A}{6} C^{Bjp} + \sum_{i=2}^{\infty} \frac{\mu_{2i}^{p-n}(Q^2)}{Q^{2i-2}}, \tag{4.3}$$

where [60]

$$\begin{aligned} C^{Bjp} =& 1 - \alpha_s + (-4.583 + 0.3333n_f)a_s^2 + (-41.44 + 7.607n_f - 0.1775n_f^2)a_s^3 \\ &+ (-479.4 + 123.4n_f - 7.697n_f^2 + 0.1037n_f^3)a_s^4, \end{aligned} \tag{4.4}$$

and $|g_A| = 1.2670 \pm 0.0035$ [57]. Here n_f denotes to number of quark active flavour. At leading twist order two Eq. (4.2) and Eq. (4.3) are equivalent to each other. The sum term

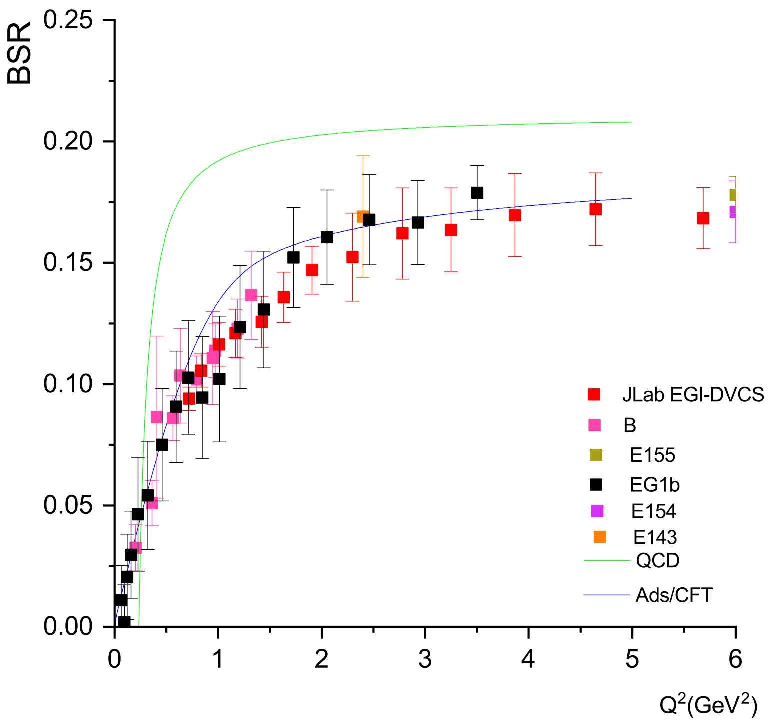

Figure 2: Bjorken sum rule in conventional pQCD which are compared with the results from modified AdS/CFT and experimental dates.

in Eq. (4.3) refers to a higher twist effect. The term with $D = 2$ dimension, i.e., μ_4^{p-n}/Q^2, has the following coefficient [56]:

$$\mu_4 = \frac{M_N^2}{9}(a_2^{p-n} + 4d_2^{p-n} + 4f_2^{p-n}(Q^2)). \tag{4.5}$$

In this equation, the nucleon mass is $M_N \approx 0.94$ GeV. The coefficient a_2^{p-n} represents the twist-2 target mass correction and d_2^{p-n} is related to the twist-3 matrix element. The f_2^{p-n} function is in fact a well-defined operator with a specific physical meaning. More details about these coefficients can be found in [56,61]. Computing result of underlying QCD is done here up to $D = 2$ dimension.

To indicate the advantage of amended coupling constant, based on the improvement which is done by AdS/CFT duality, we plot in Fig. 2 the result for the Bjorken sum rule using two approaches. First, we employ the improved coupling constant in AdS/CFT duality and as a second approach, we resort to conventional pQCD, using Eq. (4.3) while C^{Bjp} is given by Eq. (4.4). In the conventional QCD approach the utilized energy range is $(0.24\,GeV^2, 6\,GeV^2)$ while in another approach, based on AdS/CFT correspondence, the energy range can be started from zero that is below the QCD cutoff parameter which is chosen $\Lambda^2 = 0.12\,GeV^2$ ($\Lambda = 0.349\ GeV$). It should be noted that the result of the first approach is obtained while we employ the following relation:

$$\Gamma_1^{p-n}(Q^2) = \int_0^1 [g_1^p(x) - g_1^n(x)]dx = \frac{g_A}{6}(1 - \alpha_{Modified}^{AdS}(Q^2))\ , \tag{4.6}$$

which is involving a perturbative series up to the first order. Therefore it seems that the coupling constant in AdS/CFT duality plays a role as an effective coupling which is containing all the corrections of higher orders. This coupling incorporates confinement and agrees well with effective charge observables and lattice simulations. It also exhibits an infrared fixed point at small Q^2 and asymptotic freedom at large Q^2. The equation indicated above represents an alternative version of Eq. (4.3), in which the contribution of higher twist effects has been ignored.

Examining Fig. 2 reveals that the outcome of the initial method, utilizing the AdS/CFT correlation, aligns well with the available data, whereas the calculated analytical outcome competently encompasses the data at low energy scale, below the QCD cutoff parameter.

As a second observable where its behaviour particularly at a low energy scale is noteworthy, we consider below the ratio of electron-positron annihilation to hadrons and muon-antimuon.

4.2 Electron-positron annihilation to hadrons, $R_{e^+e^-}$ ratio

In this section, we consider another QCD observable where its behaviour at low energy infrared regime is important for us. We then follow to see whether AdS/CFT correspondence is working well or not. We focus on studying the $R_{e^+e^-}$ ratio for electron-positron annihilation to hadrons with respect to the same annihilation to muon-antimuon at the center of mass (c.m) energy $\sqrt{s}$. In the conventional pQCD, this ratio involves a part that can be written as a power series in terms of the renormalized QCD coupling $a_s(s) = \frac{\alpha_s(s)}{\pi}$. The ratio at c.m energy $\sqrt{s}$ is given by:

$$R_{e^+e^-}(s) \equiv \frac{\sigma_{tot}(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)} = 3\sum_f Q_f^2(1 + R(s)). \tag{4.7}$$

Here Q_f is the electric charge of the different quark flavors and $R(s)$ is perturbative corrections to the parton model result for the concerned ratio. The $R(s)$ has the following perturbation series:

$$R(s) = a + \sum_{n>0} r_n a^{n+1} \; , \tag{4.8}$$

where r_1 and r_2 have been computed in the $\overline{MS}$ scheme with renormalization scale $\mu^2 = s$ [20,62,63]. Substituting the numerical results for r_1 and r_2 in above equation, one will arrive at [64]:

$$\begin{aligned} R(s) =& 3\sum_f Q_f^2 \left\{ \begin{array}{l} 1 + a_s + a_s^2(1.98571 - 0.115295 n_f - 0.345886 n_{\tilde{g}}) \\ +a_s^3(-6.63694 - 1.20013 n_f - 0.00518 n_f^2 \\ -2.85053 n_{\tilde{g}} - 0.03107 n_f n_{\tilde{g}} - 0.04661 n_{\tilde{g}}^2) \end{array} \right\} \\ & - (\sum_f Q_f)^2 \alpha_s^3 \; (1.2395). \end{aligned} \tag{4.9}$$

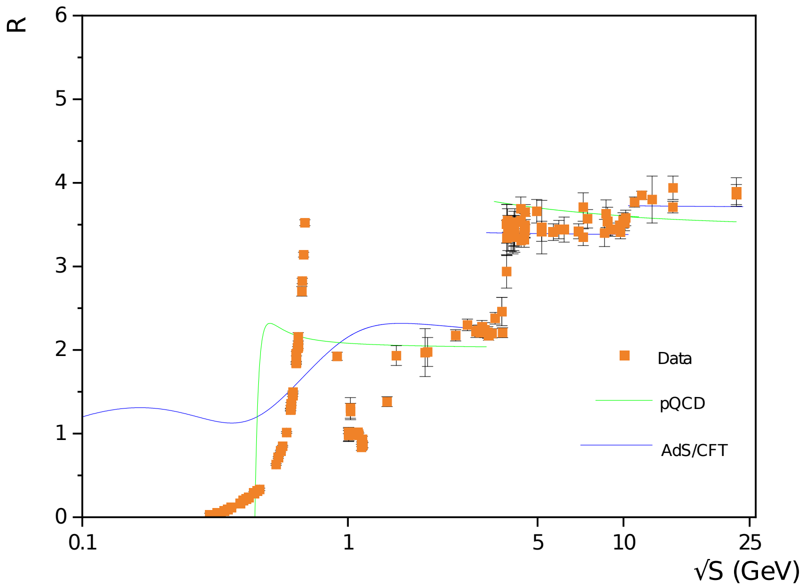

Figure 3: $R_{e^+e^-}$ ratio, resulted from AdS/CFT correspondence and conventional pQCD which are compared with the available experimental data [65].

Here there is no light by light corrections which enter at $O(\alpha_s^3)$ since summing over u, d, and s quarks leads to $(\sum_f Q_f)^2 = 0$. In Fig. 3 we plot $R_{e^+e^-}$ ratio, employing two different approaches at three different energy ranges ($0.45\ GeV$ to $3.2\ GeV$), ($3.2\ GeV$ to $10\ GeV$) and finally ($10\ GeV$ to $25\ GeV$) where three, four and five quark flavours are activated respectively. The first approach is based on the conventional pQCD and the in the second approach we take into account the improved coupling constant, resulted from AdS/CFT duality. As it is seen and expected the underlying pQCD does not have proper behaviour at low energy scale, below the QCD cutoff parameter, Λ_{QCD}. The result of the second approach indicates adequate behaviour at low energy scales toward a range below the QCD cutoff parameter while it is also able to cover properly the experimental data at this range which can be taken as the advantage of this approach.

Below, we discuss the third observable for which there are experimental data available at a low energy scale.

4.3 Hadronic tau decay

The τ particle is a member of the third lepton generation. Tau decays into different particles via the W propagation. The decay of this lepton has five equal contributions. Two of them are $\tau^- \to \upsilon_\tau e^- \bar{\upsilon}_e$ and $\tau^- \to \upsilon_\tau \mu^- \bar{\upsilon}_\mu$. Other three decay modes are typically presenting by $\tau^- \to \upsilon_\tau d_\theta \bar{u}$ where $d_\theta \equiv \cos\theta_C d + \sin\theta_C s$.

Ratio for τ decay to hadron with respect to the first channel is given by [66]:

$$R_\tau \equiv \frac{\Gamma(\tau^- \to \upsilon_\tau + hadrons)}{\Gamma(\tau^- \to \upsilon_\tau e^- \bar{\upsilon}_e)} \,. \tag{4.10}$$

Like to what we had for $R_{e^+e^-}$ (see Eq. (4.9)) the perterbative part for ratio of τ decay has the following expansion:

$$\tilde{R}_\tau = a + r_1 a^2 + r_2 a^3 + \cdots + r_k a^{k+1} + \cdots \tag{4.11}$$

Considering the above expansion in the $\overline{MS}$ scheme (with $\mu^2 = s$) the completed ratio form of lepton decay width is as follows [67]:

$$R_\tau = N(|V_{ud}|^2 + |V_{us}|^2) S_{EW}[1 + \frac{5}{12}\frac{\alpha(m_\tau^2)}{\pi} + \tilde{R}_\tau] \,, \tag{4.12}$$

where $|V_{ud}|^2 + |V_{us}|^2 \approx 1$ and $m_\tau^2 = s$. Here $\alpha(m_\tau^2)$ is electromagnetic coupling [68,69] and $S_{SW} \simeq 1.0194$ [15]. This observable has been calculated analytically in pQCD. One can find for this observable numerical value $3.660^{+0.12}_{-0.12}$ that is based on the principle of maximum conformability [70]. Considering the completed renormalization group improvement (CORGI) approach the obtained numerical value is $3.652^{+0.23}_{-0.22}$ [19]. The value obtained for R_τ via AdS/CFT duality is $3.620^{+0.002}_{-0.003}$, consistent with other approaches and in close agreement with the experimental result $3.593^{+0.008}_{-0.008}$ [71]. In the context of AdS/CFT correspondence, it is important to note that in the perturbative aspect of the calculation described in Eq. (4.12), the coupling constant a defined in Eq. (3.2) is employed, and the perturbative coefficients r_i up to the third order are analogous to those for $R_{e^+e^-}$ as shown in Eq. (4.9). The AdS/CFT modified coupling constant could give us physical value for R_τ below the QCD cutoff parameter that can be matched with ALEPH collaboration data [72] while we are not able to get reasonable values in conventional pQCD for R_τ at low energy scale. As mentioned earlier, the modified coupling yields more adequate numerical results for R_τ at $Q = m_\tau = 1.777\ GeV$ compared to conventional pQCD, showing the benefit of utilizing the AdS/CFT modified coupling even at moderate and high energy scale.

5 Conclusion

In this paper, we discussed how by writing a proper space-time metric, given by Eq. (2.3), for an anti-de sitter space and employing the holography principle one could reach a modified coupling constant which indicates appropriate behaviour at low energy scales. The soft-wall approach in AdS/CFT correspondence can give us a nonperturbative coupling constant at low energy scales. Due to the usefulness of the AdS coupling constant, considering the Burkert-Ioffe model, the obtained $\alpha_s^{AdS}(Q^2)$ is modified. The coupling constant which is used in this paper is obtained using AdS/CFT correspondence. In order to get a proper coupling constant, the AdS metric is modified using a positive-sign dilaton background. This modification changes the $\alpha_s^{AdS}(Q^2)$ amount at high energies while it leaves its amount approximately unchanged at low energies scales.

As a consequence of the above considerations, taking into account the modified $\alpha_s^{AdS}(Q^2)$, we studied some QCD observables such as the Bjorken sum rule, the ratio of electron-positron annihilation to hadrons, and muon-antimuon, and finally hadronic tau decay. Computation of pQCD calculations and extended to a region below the QCD cutoff parameter that is based on employing the modified coupling constant, indicate adequate analytical behaviour for these observables. As can be seen from Fig. 2 and Fig. 3 and the outcome of previous subsection, the computed results for these observables, based on AdS/CFT correspondence, are in good agreement with the available experimental data at any energy scales, specially at the low ones.

For additional research, other QCD observables like Higgs decay width, Compton scattering amplitude, and meson form factors can be investigated, incorporating AdS/CFT considerations. We look forward to reporting on these topics in the future.

Authors' Contributions

All authors have the same contribution.

Data Availability

The manuscript has no associated data or the data will not be deposited.

Conflicts of Interest

The authors declare that there is no conflict of interest.

Ethical Considerations

The authors have diligently addressed ethical concerns, such as informed consent, plagiarism, data fabrication, misconduct, falsification, double publication, redundancy, submission, and other related matters.

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