Journal of Holography Applications in Physics
Volume 4, Issue 2, Spring 2024, 63–65
©Available online at http://jhap.du.ac.ir
DOI: 10.22128/jhap.2024.838.1084
Online ISSN: 2783–3518

Letter

Non-Perturbative Correction to the Black Holes Distribution Function

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Received: June 4, 2024; **Accepted:** June 29, 2024

Abstract. This letter investigates non-perturbative corrections to the entropy of black holes and their impact on the associated statistical mechanics and thermodynamics. An exponential correction term to the entropy is proposed, modifying the standard Bekenstein-Hawking formula. The corrected partition function is derived, enabling calculations of thermodynamic quantities like the Helmholtz free energy. For the Schwarzschild black hole case, the free energy reduces to the standard result when the correction term vanishes, providing a consistency check. The implications of these non-perturbative entropy corrections for the statistical mechanics and thermodynamic descriptions of black holes are discussed.

Keywords: Quantum correction; Thermodynamics; Black hole.

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Black holes have provided a remarkable window into connections between gravity, quantum mechanics, thermodynamics and information theory. However, our understanding of their microscopic statistical origin remains incomplete. Recent work has suggested that non-perturbative effects could lead to exponentially small corrections to the entropy of black holes beyond the celebrated Bekenstein-Hawking area law. Such corrections may offer insights into the underlying quantum gravity theory.

In this letter, we explore the consequences of an exponential correction term to the entropy on the statistical mechanics and thermodynamic description of black holes. We derive the corrected partition function and use it to calculate thermodynamic potentials like the Helmholtz free energy. Applying this to the Schwarzschild case recovers the standard results when the correction vanishes, providing a consistency check. We discuss how these entropy corrections could shed light on the quantum nature of black holes.

It has been recently argued that the non-perturbative corrected entropy of black hole could expressed as [1],

$$S = S_0 + \eta e^{-S_0}, \tag{1}$$

where S_0 is the original entropy of a given black hole, and η is the correction coefficient.

The entropy of a physical system is related to the partition function via the following relation (in unit of the Boltzmann constant),

$$S = \ln Z + T\left(\frac{\partial \ln Z}{\partial T}\right), \tag{2}$$

where T is the black hole temperature. For the Schwarzschild black hole with mass M, we know that

$$S_0 = 4\pi M^2, \tag{3}$$

and

$$T = \frac{1}{8\pi M}. \tag{4}$$

So, combining the equations (1) and (2), using the relations (3) and (4) one can obtain the following partition function,

$$Z = \exp\left(\eta[2\pi M \mathrm{erf}(2\sqrt{\pi}M) + e^{-4\pi M^2}] - 4\pi M^2\right), \tag{5}$$

where

$$\mathrm{erf}(2\sqrt{\pi}M) = \frac{2}{\sqrt{\pi}}\int_0^{2\sqrt{\pi}M} e^{-t^2}dt, \tag{6}$$

is the Gauss error function. Using the partition function, one can obtain all thermodynamic quantities. For example, Helmholtz free energy is given by,

$$F = -T\ln Z, \tag{7}$$

which is identical to

$$F = -\int S dT. \tag{8}$$

Both equations (7) and (8) yield the following result,

$$F = \frac{\eta[2\pi M \mathrm{erf}(2\sqrt{\pi}M) + e^{-4\pi M^2}] + 4\pi M^2}{8\pi M}. \tag{9}$$

It is clear that in the case of $\eta = 0$ we have $F = \frac{M}{2}$ which is well-known for the Schwarzschild black hole [2].

We have investigated the impacts of a proposed exponential correction term to the entropy of black holes on their statistical mechanics and thermodynamics. By deriving the corrected partition function, we calculated the Helmholtz free energy and other thermodynamic quantities. The formulas reduce to the standard cases when the correction vanishes, verifying self-consistency.

While just a first step, incorporating non-perturbative effects in the entropy may ultimately lead to a deeper understanding of the quantum gravitational degrees of freedom responsible for the entropy of black holes. The thermodynamic derivations here provide a framework for exploring such corrections and their physical implications [3]. Further work is needed to determine if the ansatz for the entropy correction is justified from a fundamental theory of quantum gravity. Nevertheless, this analysis shows how even tiny corrections can profoundly impact the thermal properties of black holes.

Data Availability

The manuscript has no associated data or the data will not be deposited.

Conflicts of Interest

The author declares that there is no conflict of interest.

Ethical Considerations

The author has diligently addressed ethical concerns, such as informed consent, plagiarism, data fabrication, misconduct, falsification, double publication, redundancy, submission, and other related matters.

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