Journal of Holography Applications in Physics
Volume 4, Issue 2, Spring 2024, 43–54
©Available online at http://jhap.du.ac.ir
DOI: 10.22128/jhap.2024.818.1080
Online ISSN: 2783–3518

Regular article

The Nucleus Rotating Effect on the Electron Energy Levels via the Seiberg-Witten Map: A Semi-Classical Approach

Abolfazl Jafari
Department of Physics, Faculty of Science, Shahrekord, Iran;
E-mail: jafari-ab@sku.ac.ir

Received: March 31, 2024; **Revised:** May 3, 2024; **Accepted:** May 14, 2024

Abstract. In a semi-classical approach, we relate the spin and rotation of the nucleus. We find the effect of the rotation attributed to the nucleus on the energy of the electron layers. The aim of the present work is to investigate the correction of the magnetic moment and electron vertex function in the lowest order of approximation in the presence of field correction by Seiberg-Witten's method. We will also exploit Seiberg-Witten's relations in the official method, reconstruct the sources and solve modified Maxwell's equations. Furthermore, we will show that in the first approximation of non-commutativity, Seiberg-Witten's and Bopp's shift methods are unequal. The present work is based on non-relativistic quantum mechanics; therefore, the results of this research are expected to change by applying the principles of holography and using the Schrodinger equation compatible with gravitational effects.

Keywords: Non-commutative coordinates; Bopp's shift; Seiberg-Witten map; θ deformed electrodynamics.

1 Introduction

The Seiberg-Witten map is one of the methods for introducing non-commutative geometry in physics. The principles of this map have been explained in many texts [1–3]. The most common instances of the studies of physicists in the field of non-commutative geometry are the studies on the effects of importing non-commutative coordinates, which are mainly limited to the first order of the non-commutative parameters. One of the conventional methods for rewriting the physics of non-commutative geometry is Moyal-Weyl mapping [4–8]. However, Moyal-Weyl mapping is also used up to the first order of non-commutativity. The non-commutative geometry is a space of sufficiently smooth functions on R^4 established with the Moyal-Weyl product:

$$[\hat{y}^{\mu}, \hat{y}^{\nu}]_{\star} = \imath\theta^{\mu\nu}. \tag{1}$$

Here, θ as a real constant and antisymmetric tensor has a square length dimension. The sign of the star product, is a method for the formulation of physics in non-commutative geometry. A simple rule explains how we should apply it. Indeed, replacing the ordinary products between quantities with a $\star$-product [9–13],

$$\breve{f}(\hat{\mathbf{y}})\breve{g}(\hat{\mathbf{y}}) = \breve{f}(\mathbf{y}) \star \breve{g}(\mathbf{y}) = \breve{f}(\mathbf{y}) \mathrm{e}^{\frac{\imath}{2}\overleftarrow{\partial}_{\mu}\theta^{\mu\nu}\overrightarrow{\partial}_{\nu}} \breve{g}(\mathbf{y}).$$

In most cases, because of the causality, time coordinates do not contribute to the non-commutativity. According to the literature, the mean value of the non-commutativity parameter is in the order of $\sqrt{\theta} \sim 10^{-15}$m [14,15].
After accepting the existence of a volume for the nucleus instead of a point nucleus, in the semi-classical concept of the nucleus, the stationary state of the nucleus volume is a hard idea to believe. Therefore, the nucleus can include more degrees of freedom, and this motivation strengthens the assumption of nucleus rotation. However, the main reason for the rotation of the nucleus is a quasi-classical relationship between the orbital momentum and the spin of the nucleus in determining the dipole moment of the nucleus [16–18].
Assuming the existence of nuclei rotation, we extract the effects of rotation and the correction of non-commutativity coordinates. This assumption requires the assumption of a radius for the nucleus. The rotation of the nucleus requires the assumption of a nucleus radius. Therefore, this work is based on the nucleus' radius. We compare the effects of non-commutative geometry in the nucleus, which is studied by Bopp's shift method and the Seiberg-Witten map. Due to the corrections of the electron energy levels, we demonstrate that the two approaches are not equivalent. We show that the electric and magnetic dipole moments, as well as one-loop vertex correction, change due to the mapping selection. Furthermore, we provide a way to communicate the results of the two methods.
We take the direction of nucleus rotation as the normal of the preferred plane, and for convenience, we limit the non-commutativity coordinates to the preferred plane as the case study. So, we have the following assumption about non-commutativity:

$$\boldsymbol{\theta} = \begin{pmatrix} 0 & \theta & 0 \\ -\theta & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{2}$$

That is, we assume that the direction of the nucleus, $\boldsymbol{\omega}$, is normal to the xoy-plane. We consider the coincidence of the $\hat{z}$ with the direction of nuclear rotation as a reason for the difference of the third components of $\boldsymbol{\theta}$ in the above relationship.
We work in the Heaviside-Lorentz unit, [19–21]. Based on equation (1), at the first order of

θ, the variation of electromagnetic fields can be explained by the Seiberg-Witten (SW) map [1–3]

$$\breve{A}_\mu = A_\mu - \frac{1}{4}\theta^{\alpha\beta}\{A_\alpha, \partial_\beta A_\mu + F_{\beta\mu}\}_\star, \tag{3}$$

where

$$\breve{F}_{\mu\nu} = \partial_\mu \breve{A}_\nu - \partial_\nu \breve{A}_\mu - \imath[\breve{A}_\mu, \breve{A}_\nu]_\star.$$

The Symbol,$\breve{}$, is used for is used for the non-commutative coordinate functions. Substituting equation (1) and defining $f_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, the changes of the field strength tensor, read

$$\breve{F}_{\mu\nu} = F_{\mu\nu} + \frac{1}{4}\theta^{\alpha\beta}(2\{F_{\mu\alpha}, F_{\nu\beta}\}_\star - \{A_\alpha, \mathfrak{D}_\beta \star F_{\mu\nu} + \partial_\beta F_{\mu\nu}\}_\star, \tag{4}$$

where the equation of motion for the strength tensor is as follows:

$$\breve{\mathfrak{D}}_\mu \star \breve{F}^{\mu\nu}(\hat{\mathbf{y}}) = \frac{1}{c}\breve{j}^\nu(\hat{\mathbf{y}}). \tag{5}$$

Moreover, the covariant derivative is defined by

$$\breve{\mathfrak{D}}_\mu \star = \partial_\mu - \imath[\breve{A}_\mu, \star].$$

The four-vector of the current is also changed in the SW map. The current term transforms by

$$\breve{\breve{j}}^\mu = u_\star(\breve{\Lambda}) \star \breve{j}^\mu \star u_\star^{-1}(\breve{\Lambda}),$$

in which Λ should be changed:

$$\breve{\Lambda} = \Lambda + \frac{1}{4}\theta^{\alpha\beta}\{\partial_\alpha \Lambda, A_\beta\}_\star.$$

So, the variation rule for the four-vector current is written as [3,22,23]

$$\breve{j}^\mu = j^\mu - \frac{1}{2}\theta^{\alpha\beta}\{A_\alpha, \partial_\beta j^\mu\}_\star + \psi(\boldsymbol{\theta}.\mathbf{F}.\mathbf{j})^\mu, \tag{6}$$

where

$$(\boldsymbol{\theta}.\mathbf{F}.\mathbf{j})^\mu := \frac{1}{2}\theta^{\mu\alpha}F^{(0)}_{\alpha\beta}j^{(0)\ \beta} + \frac{1}{2}\theta^{\alpha\beta}F^{(0)}_{\alpha\beta}j^{(0)\ \mu},$$

and $\psi = 1$. In this work, we admit the non-commutative geometry and restrict ourselves to the first order of θ. We know that the SW map is only valid when we have $\theta^{\mu\nu}\theta_{\mu\nu} > 0$ and this condition is realized in this paper. We also calculate the corrections to the Maxwell equations and make deformed electrodynamics based on the SW map. The main aim of this paper is to compare the effects of nucleus rotation on the energy levels of the electron layers and emission lines and the absorption of atoms.

Although the basis of this article is a non-relativistic regime, we can refer to its holographic version. The dynamics of the nucleus are expressed by a theory that is not conformal in itself. Basically, without relativistic approximations, the correspondence between the theories of Ads and QCD cannot be established. This topic contains many deviations from the basic order, such as that QCD is not supersymmetric while SYM is maximally supersymmetric, e.t.c. Thus, the adaptation of the holographic principle to the QCD requires special considerations and conditions. Of course, it must be acknowledged that, recently, many efforts have been made to achieve this goal, and results have been obtained [24–28]. But we use

the nuclear spin in the semi-classical approximation. Therefore, the way to achieve nuclear spin values has no effect on its use.
The energy level of electron layers is certainly dependent on the status of the Schrodinger equation. For the Schrodinger equation and its solutions, we have used the usual method and matrix model, which otherwise required more approximation. Schrodinger's equation, whose results are used in this article, is a non-relativistic equation, while due to the correspondence of Ads and CFT, the equation must be Lorentz invariant [29,30]. So, any change in the structure of the Schrodinger equation can be a motivation for the correction research in the results of this article. By applying changes and using the holographic versions of the QED and QCD, corrections will certainly occur in the energy layers of the electrons.

2 Basic notations

We work in the Heaviside-Lorentz unit. By setting $A \to gA$, with the help of the coupling constant g, SW's correction equations, equations (3)–(6) will be modified. In a semi-classical interpretation, the center of atoms has a charge distribution function that lies within the nucleus position, where the electron cannot be seen. Approximately, the shape of a nucleus can be assumed to be a sphere with a radius R. As we mentioned in the previous section, the radius of the nuclear is at least larger than $\sqrt{\theta}$. We denote the region $r < R$ and its related quantities with the index of "$_<$", and in the same style, we denote the region $r > R$ with "$_>$". We can split the electromagnetic fields into two separate parts,

$$\breve{A}_\mu = A^{(0)}_\mu + A^{(1)}_\mu + 0(\theta)^2,$$

which means the second term is small. Likewise, we consider the nucleus radius, $R_{_{nuc}} = R \sim 10^{-15}m$, so R, which has a reference to the proton diameter. Based on the nucleus radius, space is divided into two parts, $r < R$ and $r > R$. By appending $<$ or $>$ indices, we distinguish the quantities related to these areas. For the nucleus case, the distribution function is

$$j^{(0)}_{0\ <} = \frac{\mathbb{Z}e}{\frac{4\pi}{3}R^3}, \qquad j^{(0)}_{0\ >} = 0.$$

This implies that the current density

$$\mathbf{j}^{(0)}_{(<)>} = 0,$$

and it is obvious that,

$$\lim_{R\to 0} j^{(0)}_{0\ <} \to\ \mathbb{Z}e\delta(\mathbf{r}).$$

$A^{(0)}_{0\ <}$ and $A^{(0)}_{0\ >}$ are solutions to Maxwell's equations of motion:

$$\Box(A^{(0)}_\mu + A^{(1)}_\mu + \cdots) = -\frac{1}{c}(j^{(0)}_\mu + j^{(1)}_\mu + \cdots).$$

Which provides

$$\begin{cases} \Box A^{(0)}_{0\ (>)<} = -j^{(0)}{}_{0\ (>)<} \\ \\ \Box \mathbf{A}^{(0)}_{(>)<} = -\frac{1}{c}\mathbf{j}^{(0)}_{(>)<} \end{cases}$$

with the conditions $A^{(0)}_{<} = A^{(0)}_{>}|_R$ and $\partial_r A^{(0)}_{<} = \partial_r A^{(0)}_{>}|_R$. Due to the linearity of the dependence on non-commutativity,

$$\begin{cases} \Box A^{(1)}_{0\ (>)<} = -j^{(1)}{}_{0\ (>)<} \\ \Box \mathbf{A}^{(1)}_{(>)<} = -\frac{1}{c}\mathbf{j}^{(1)}_{(>)<} \end{cases} \tag{7}$$

Without non-commutative effects,

$$A^{(0)}_{0\ <} = -\frac{\mathbb{Z}e}{8\pi R^3} r^2 + \frac{3\mathbb{Z}e}{8\pi R},$$

and

$$A^{(0)}_{0\ >} = \frac{\mathbb{Z}e}{4\pi r}. \tag{8}$$

The nucleus, as a sphere of radius R carries a uniform surface charge distribution Q based on volume density j_0, is rotated about a diameter with constant angular velocity $\boldsymbol{\omega} = \omega\hat{z}$, which is proportional to nucleus spin. This is a semi-classical analogy and can be defended by assuming a lump nucleus. One finds:

$$\mathbf{J}^{(0)}_{<} = \sigma\boldsymbol{\omega} \wedge \mathbf{r}, \qquad \mathbf{J}^{(0)}_{>} = 0.$$

According to the Maxwell's equation,

$$\mathbf{A}^{(0)}_{>} = \frac{\sigma R^4}{3cr^3}\boldsymbol{\omega} \wedge \mathbf{r}, \qquad \mathbf{A}^{(0)}_{<} = \frac{\sigma R}{3c}\boldsymbol{\omega} \wedge \mathbf{r}. \tag{9}$$

The compatibility of equation (7) is the basis of the calculations. In this work, first we obtain the modified currents according to equation (6) of SW's approach, and then we try to find the solutions of the generalized Maxwell's equations based on equation (7). By finding the modified fields using Bopp's shift method, we show the inconsistency of the results in the first order of the non-commutative parameter.
When $\theta^{0\alpha} = 0$, [31,32], equations (2) and (6) give

$$\begin{aligned} \breve{j}_{\mathbf{1}\ <} &= j^{(0)}_{1\ <} - g\theta^{12} A^{(0)}_{1\ <} \partial_2 j^{(0)}_{1\ <} - g\theta^{21} A^{(0)}_{2\ <} \partial_1 j^{(0)}_{1\ <} + g\theta^{12} F^{(0)}_{21\ <} j^{(0)}_{1\ <} + \frac{1}{2} g\theta^{12} F^{(0)}_{12\ <} j^{(0)}_{1\ <} \\ &\quad + \frac{1}{2} g\theta^{21} F^{(0)}_{21\ <} j^{(0)}_{1\ <} \\ &= -\sigma\omega(1 + \frac{g}{3c}\theta^{12}\sigma\omega R) y_2, \end{aligned} \tag{10}$$

and repeating the same calculations for $\breve{j}_{\mathbf{2}\ <} = \sigma\omega(1 + \frac{g}{3c}\theta^{12}\sigma\omega R) y_1$, we have

$$\breve{\mathbf{J}}_{<} = \sigma(1 + \frac{g}{3c}\theta^{12}\sigma\omega R)\boldsymbol{\omega} \wedge \mathbf{r}. \tag{11}$$

So, without any change in $\mathbf{J}^{(0)}_{<}$ and initial conditions, also if $\beta_1 := (1 + \frac{g}{3c}\theta^{12}\sigma\omega R)$, thus we reach to

$$\breve{\mathbf{J}}_{<} = \beta_1 \sigma\boldsymbol{\omega} \wedge \mathbf{r}. \tag{12}$$

Clearly, $\mathbf{J}_{>}$ does not change; $\mathbf{J}^{(1)}_{>} = 0$.

$$\breve{\mathbf{J}}_{>} = \mathbf{J}_{>}. \tag{13}$$

Equations (12) and (13) mean there is no change in the general form of currents $\mathbf{J}_{(<)>}$, compared to equation (11). Substituting $\beta_1\boldsymbol{\omega} =: \boldsymbol{\Omega}$ instead of $\boldsymbol{\omega}$, the solution of the relevant regions' equations of motion is predicted. Thus, equation (9) becomes:

$$\breve{\mathbf{A}}^{SW}_{>} = \frac{\sigma R^4}{3cr^3}\boldsymbol{\Omega}\wedge\mathbf{r}, \qquad\qquad \breve{\mathbf{A}}^{SW}_{<} = \frac{\sigma R}{3c}\boldsymbol{\Omega}\wedge\mathbf{r}. \tag{14}$$

Now, similarly, we have

$$\begin{aligned}\breve{j}_{0\,<} &= j^{(0)}_{0\,<} - g\theta^{12}A^{(0)}_{1\,<}\partial_2 j^{(0)}_{0\,<} - g\theta^{21}A^{(0)}_{2\,<}\partial_1 j^{(0)}_{0\,<} + \frac{1}{2}g\theta^{12}F^{(0)}_{12\,<}\, j^{(0)}_{0\,<} + \frac{1}{2}g\theta^{21}F^{(0)}_{21\,<}\, j^{(0)}_{0\,<}\\ &= \frac{3\mathbb{Z}e}{4\pi R^3}(1+\frac{2g}{3c}\theta^{12}\sigma\omega R),\end{aligned} \tag{15}$$

likewise

$$\begin{aligned}\breve{j}_{0\,>} &= j^{(0)}_{0\,>} - g\theta^{12}A^{(0)}_{1\,>}\partial_2 j^{(0)}_{0\,>} - g\theta^{21}A^{(0)}_{2\,>}\partial_1 j^{(0)}_{0\,>} + \frac{1}{2}g\theta^{12}F^{(0)}_{12\,>}\, j^{(0)}_{0\,>} + \frac{1}{2}g\theta^{21}F^{(0)}_{21\,>}\, j^{(0)}_{0\,>}\\ &= 0.\end{aligned} \tag{16}$$

By defining $\beta_2 = (1+\frac{2g}{3c}\theta^{12}\sigma\omega R)$ and absorbing it in $\mathbb{Z}$, we will have

$$\breve{A}^{SW}_{0\,<} = -\frac{\breve{\mathbb{Z}}e}{8\pi R^3}r^2 + \frac{3\breve{\mathbb{Z}}e}{8\pi R}, \qquad\qquad \breve{A}^{SW}_{0\,>} = \frac{\breve{\mathbb{Z}}e}{4\pi r}. \tag{17}$$

There is a point to be mentioned, and that is, according to the relationship of the difference of the coefficient of change in relations (10) and (15), it seems that ψ in relation (6) has a value of $\frac{1}{2}$. In this case, with the same change weight, we should be justified in applying the effect of geometric non-commutation to the current vector. In this situation, β_1 and β_2 will be the same, and their attraction will be determined.

2.1 Comparison of Bopp's shift and SW's methods

In the following, the potential is re-obtained by applying Seiberg-Witten's approximation, without solving the equation of motion. With the following calculations in the Seiberg-Witten's approach, it is possible to understand the difference in the order of disorder due to the non-commutativity geometry. If $\gamma := \frac{\sigma\omega R^4}{3c}$, then from equation (3),

$$\mathbf{A}^{(1)}_{>} = -\frac{g}{2}\theta^{\alpha\beta}A^{(0)}_{\alpha\,>}(2\partial_\beta\mathbf{A}^{(0)}_{>} - \nabla A^{(0)}_{\beta\,>}),$$

one finds

$$A^{(1)SW}_{i\,>} = -\frac{3g}{2}\theta_{ij}\gamma^2 r^{-6}y^j + \frac{6g}{2}\theta_{ij}\gamma^2 r^{-8}y^j y^j y^j + \frac{6g}{2}\theta_{ij}\gamma^2 r^{-8}y^i y^j y^i + a_i(r,\theta),$$

as well as

$$A^{(1)SW}_{3\,>} = -g\theta^{12}\frac{\gamma^2}{r^8}y_1y_2y_3 + a_3(r,\theta), \tag{18}$$

also,

$$A^{(1)SW}_{0\,>} = -\frac{\mathbb{Z}eg\sigma\omega R^4}{12\pi cr^6}\theta^{12}(y_2^2+y_1^2) + a_0(r,\theta), \tag{19}$$

where $\nabla^2 a_\mu = 0$, is the general solution, and $a_{_\mu}(r,\theta) = \kappa_{_\mu}\theta^{k\mu}y_{_k} + \cdots$. In an ambiguous approach but practical way, the modified potential fields may be obtained from equation (3) [3]. But, these results cannot be more correct than the results we obtained from solving the modified Maxwell's equations (14) and (17). Because our answers are the solutions of transformed motion equations, while results 3 and 4 should apply to the conditions of connection, etc. In the first approximation of the non-commutativity, the difference is significant. The potentials obtained by solving the equations of motion are different from the solutions that we will get from Seiberg-Witten's correction method. We expect that, due to the linearity of the dependence on the non-commutative parameter, we can use the linearity superposition rule and obtain the same results. But it doesn't seem that way. Equation (8) does not reach equation (19) using Bopp's shift method:

$$\hat{y}^i := y^i - \frac{\theta^{ij}}{2\hbar}\hat{p}_j. \tag{20}$$

In the following, we set $\boldsymbol{\theta} = (0,0,\theta)$, and because of the simplicity of the calculations. A simple estimate shows: $\hat{r}^{-n} = r^{-n} + \frac{n\boldsymbol{\theta}\cdot\mathfrak{L}}{2\hbar}r^{-n-2} + 0(\theta)^2$ which results in the following relationship:

$$\breve{A}^{BS}_{_{0\ >}} = \frac{\mathbb{Z}e}{4\pi r} + \frac{\mathbb{Z}e}{16\pi\hbar r^3}\boldsymbol{\theta}\cdot\mathfrak{L}. \tag{21}$$

For the vector potential equation (9), by setting $\hat{\boldsymbol{\omega}} = \frac{\boldsymbol{\omega}}{\omega}$, we will have:

$$\begin{aligned}\breve{\mathbf{A}}^{BS}_{_{>}} =& \frac{\gamma}{\breve{r}^3}\hat{\boldsymbol{\omega}}\wedge\breve{\mathbf{r}} = \frac{\gamma}{\breve{r}^2}\hat{\boldsymbol{\omega}}\wedge\hat{r}\\ =& \frac{\gamma}{r^2}\hat{\boldsymbol{\omega}}\wedge\hat{r} + \frac{1}{\hbar}\frac{\gamma}{r^4}\hat{\boldsymbol{\omega}}\wedge\hat{r}\ (\boldsymbol{\theta}\cdot\mathfrak{L})\\ =& \frac{\gamma}{r^3}\hat{\boldsymbol{\omega}}\wedge\mathbf{r} + \frac{\gamma}{\hbar r^5}\hat{\boldsymbol{\omega}}\wedge\mathbf{r}\ (\boldsymbol{\theta}\cdot\mathfrak{L}).\end{aligned} \tag{22}$$

Equations (14) and (17) are not equal to (21) and (22) in any way, although, the expansion of functions in Bopp's shift method should be extracted from their direct multiplication. In terms of quantity, the existence of $\mathfrak{L}_z$ is not a problem; the main problem after dependence on the coefficients is the behavior of r^n in the denominator. According to the above, the understandable result is the correctness of all these approaches: Seiberg-Witten's relations, solving the equation of motion, or exploiting Bopp's shift method. However, the present work is based on Seiberg-Witten's approach.

3 Energy modification

In the following, our calculations are only in the region $r > R$. Thus, we can drop the separator indicator from this area. In addition, all the quantities that come after will belong to the area $r > R$. Here, we estimate the order of γ, numerically and estimated, without imagining their dimensions, and according to the Heaviside units values. The results were obtained without considering the effects of the parameters of the non-commutativity geometric and the nucleus rotating frequencies. Therefore, the deletion of some terms can be referred to as the above estimate. β_1 and β_2 show that the coupling constant with its coefficient is equal to the dimensionless character, that is, $[g^H]\frac{[\theta][\sigma][R][\omega]}{[c]} = 1$, and finally, $[g^H] = C^{-1} \equiv \frac{1}{e}$. In this way, the coefficients of variations, b_1 and β_2, are also equal to $\frac{g\sigma R}{3c}\theta\omega \propto \theta\omega$. In an approximate calculation, the nucleus's rotating frequency can be of

order $10^5 Hz$, so the value of θ is a determining factor. However, the rotation of the nucleus alone, without the contribution of the non-commutative parameter, will be effective. For the electrons from the region $r > R$, the covariant momentum $\hat{\mathbf{\Pi}}$ is given by

$$\hat{\mathbf{\Pi}} = \hat{\mathbf{p}} - g\breve{\mathbf{A}}_{>}$$

In the first order of θ, the operator of momentum becomes:

$$\hat{\mathbf{\Pi}}^{SW} = \hat{\mathbf{p}} - g\frac{\sigma R^4}{3cr^3}\mathbf{\Omega} \wedge \hat{\mathbf{r}}.$$

Comparing the order of the terms in equations (14) and (18), we can ignore terms with a higher order of $(\mathbf{A}_{>}^{(1)})^2$ or, in the stronger case, we ignore $(\mathbf{A}_{>})^2$. The Hamiltonian is summarized as

$$\hat{H}^{SW} = \frac{\hat{\mathbf{\Pi}}^2}{2\mu} + V_{eff}(r;\theta,R),$$

where

$$V_{\text{eff}}^{SW}(r;\theta,a) = -\frac{\mathbb{Z}e^2}{4\pi r} - \frac{\mathbb{Z}e^2 g\sigma\omega R}{6c\pi r}\theta^{12}. \tag{23}$$

It is evident that the effective potential contains the nucleus rotation frequency and is different from the potential offered in [3,33]. Finally, the perturbation potential is:

$$\hat{H}(r;\theta;\boldsymbol{\gamma}) = \frac{1}{2\mu}\hat{\mathbf{P}}\cdot\hat{\mathbf{P}} - \frac{\mathbb{Z}e^2}{4\pi r} - \frac{\mathbb{Z}e^2 g\sigma\omega R}{6c\pi r}\theta^{12} - g\frac{\sigma R^4}{6c\mu r^3}\boldsymbol{\omega}\cdot\mathfrak{L} - g\frac{g\sigma^2\omega^2 R^5}{18c^2\mu r^3}\boldsymbol{\theta}\cdot\mathfrak{L}. \tag{24}$$

Equation (24) yields the energy for first-order perturbation:

$$\Delta E_{NC}^{\text{Rotating Nuc}} = < nljm|\hat{H}_p|nljm > . \tag{25}$$

It can be seen that in calculating the $< \acute{n}\acute{l}\acute{m}|\frac{\mathfrak{L}_z}{r^k}|nlm >$ expression, the result will be proportional to $\delta_{\acute{n}n}\delta_{\acute{l}l}\delta_{\acute{m}m}$. We will include the recent impression in the final result. For the chosen case of non-commutativity, by substituting the second part of equation (23) in equation (25), equation (25) becomes,

$$\Delta E_{NC}^{\text{Rotating Nuc}} = -\frac{\mathbb{Z}e\sigma R}{6c\pi}\omega\theta < r^{-1} > -\frac{\sigma R^4}{6ce\mu}\omega(1+\frac{\sigma R}{3ec}\omega\theta) < \mathfrak{L}_z r^{-3} > .$$

We also know that

$$< r^{-1} > = \frac{1}{n^2 a_0},$$
$$< \mathfrak{L}_z r^{-3} > = \frac{2\mathbb{Z}^3 m\hbar}{a_0^3 n^3 l(l+1)(2l+1)} := \frac{u^3(\mathbb{Z},l)m\hbar}{a_0^3 n^3}.$$

So we have,

$$\Delta E_{NC}^{\text{Rotating Nuc}} = -\frac{\mathbb{Z}e\sigma R}{6c\pi}\omega\theta\frac{1}{n^2 a_0} - \frac{\sigma R^4}{6ce\mu}\omega(1+\frac{\sigma R}{3ec}\omega\theta)\frac{u^3(\mathbb{Z},l)m\hbar}{a_0^3 n^3}. \tag{26}$$

By calculating the order of appearing terms in equation (26), one can obtain the first-order correction to the energy in each layer and sub-layer. Now, we choose the comparison of total energy correction with Bopp's shift and Seiberg-Witten's method as the main goal. In this way and based on the perturbed theory, relations (17) and (21) give:

$$\Delta E^{Bop}_{NC(0)} = -\frac{\mathbb{Z}e^2}{16\pi\hbar} < \frac{\boldsymbol{\theta}\cdot\mathfrak{L}}{r^3} >,$$
$$\Delta E^{SW}_{NC(0)} = -\frac{\mathbb{Z}e\theta\sigma\omega R}{6c\pi} < \frac{1}{r} > .$$

Corresponding to the scalar potential only,

$$\frac{\Delta E^{Bop}_{NC(0)}}{\Delta E^{SW}_{NC(0)}} \approx \frac{cR}{\omega a_0^2}.$$

The difference in the order of the results is very significant. This ratio shows that the magnitude of correction due to Bopp's shift is 10^6 times the one caused by SW mapping. Therefore, the correction of the electron energy levels in a hydrogen-like atom calculated by the SW map and Bopp's shift methods, are not equivalent to one another.
Of course, with a change in $\breve{A}^{SW}_{0\ >}$, there is a change in the vertex function, $\breve{\mathsf{V}} = \mathsf{V}^{0} + \mathsf{V}^{1}$. Up to the first order in θ, the vertex function is appropriated with

$$\mathsf{V}^{SW} = \mathsf{V}^{(0)} + eF_1(0)A^{(1)\ SW}_{0\ >},$$

in this regard, $F_1(0)$ is the structure factor. In the one-loop order, the vertex correction is,

$$\mathsf{V}^{(1)\ SW}_{>\ Correc} = F_1(0)\frac{\beta_2 - 1}{4\pi r}\mathbb{Z}e^2.$$

The final relationship is different from the result obtained in [15], which was based on the method of Bopp's shift. Furthermore, with a change in $\mathbf{A}_{>}$, a change in $\mathbf{B}_{>}$ and then a change in torque μ is created, $\breve{\mu} = \mu^{(0)} + \mu^{(1)}$. In this case,

$$\mu^{(1)\ SW}_{Correc} = \frac{e}{2m}(F_1(0) + F_2(0))\sigma\tilde{\mathbf{B}}^{(1)\ SW}_{>},$$

where $\tilde{B}^{(1)\ SW}_{k\ >}(q) \propto \epsilon_{ijk}q_i\tilde{A}^{(1)\ SW}_{j\ >}(q)$, is the Fourier transformation of the relevant magnetic fields. The changes are quite evident from Bopp's shift method.

4 Discussion

At the level of quantum mechanics, we found a way to determine the contribution of the rotating nucleus to the electron energy levels. We extracted the Hamiltonian of a system at the classical mechanics level by exploiting the Seiber-Witten map based on the θ deformed electrodynamics. We showed that the Hamiltonian changes from the official version by entering the size of the nucleus in the electrodynamics equations. It was determined that the dependence of the energy corrections depends on the orbital angular momentum of the electron and the frequency of the rotating nucleus. We also proved that the energy corrections from the SW maps and Bopp's shift method are different, and their ratio is in the order of 10^6. Based on this paper, the magnetic dipole moments of the electron as well as the vertex function in the order of one-loop, have many variations that come from the amount of their values given in Ref. [15].

Acknowledgments

The author thanks Shahrekord University for supporting this work with a research grant.

Data Availability

The manuscript has no associated data or the data will not be deposited.

Conflicts of Interest

The author declares that there is no conflict of interest.

Ethical Considerations

The author has diligently addressed ethical concerns, such as informed consent, plagiarism, data fabrication, misconduct, falsification, double publication, redundancy, submission, and other related matters.

References

[1] N. Seiberg and E. Witten, "String Theory and Noncommutative Geometry", JHEP **9909**, 032 (1999) DOI: 10.1088/1126-6708/1999/09/032

[2] M.R. Douglas and N. Nekrasov, "Noncommutative Field Theory", Rev. Mod. Phys. **73**, 977 (2001) DOI: 10.1103/RevModPhys.73.977

[3] T. C. Adorno, D. M. Gitman, A. E. Shabad, and D. V. Vassilevich, "Noncommutative magnetic moment of charged particles", Phys. Rev. **D84**, 085031 (2011) DOI: 10.1103/PhysRevD.84.085031

[4] D. J. Gross and N. A. Nekrasov, "Solitons in Noncommutative Gauge Theory", JHEP **0103**, 044 (2001) DOI: 10.1088/1126-6708/2001/03/044

[5] R. J. Szabo, "Quantum field theory on noncommutative spaces", Phys. Rep. **378**, 207 (2003) DOI: 10.1016/S0370-1573(03)00059-0

[6] A. Fischer and R. J. Szabo, "Duality covariant quantum field theory on noncommutative Minkowski space", JHEP **0902**, 031 (2009) DOI: 10.1088/1126-6708/2009/02/031

[7] M. Chaichian, P. Presnajder, M. M. Sheikh-Jabbari, and A. Tureanu, "Noncommutative Standard Model: Model Building", Eur. Phys. J. **C29**, 413 (2003) DOI: 10.1140/epjc/s2003-01204-7

[8] M. M. Sheikh-Jabbari, "One Loop Renormalizability of Supersymmetric Yang-Mills Theories on Noncommutative Two-Torus", JHEP **9906**, 015 (1999) DOI: 10.1088/1126-6708/1999/06/015

[9] A. Connes and M. Marcolli, "Non-Commutative Geometry, Quantum Fields and Motives", London: Academic Press (1994).

[10] P. Aschieri, M. Dimitrijevic, P. Kulish, and et al, "Notes in Phys, Non-Commutative Space-times: Symmetries in Non-Commutative Geometry and Field Theory", Berlin Heidelberg: Springer **774**, (2009).

[11] R. J. Szabo, "Quantum Field Theory on Noncommutative Spaces", Phys. Rep. **378**, 207 (2003) DOI: 10.1016/S0370-1573%2803%2900059-0

[12] O. Bertolami and C. A. D. Zarro, "Towards a Noncommutative Astrophysics", Phys. Let. **B673**, 83 (2009) DOI: 10.1103/PhysRevD.81.025005

[13] A. Jafari, "Photon dynamic in temporal noncommutative electrodynamics", Eur. Phys. J. C **73**, 2271 (2013) DOI: 10.1140/epjc/s10052-012-2271-z

[14] A. Saha, "Quantum entanglement in analogue Hawking radiation: When is the final state nonseparable?", Phys. Rev. D **89**, 025010 (2014) DOI: 10.1103/PhysRevD.89.105024

[15] M. Chaichian, M. M. Sheikh-Jabbari, and A. Tureanu, "Hydrogen Atom Spectrum and the Lamb Shift in Noncommutative QED", Phys. Rev. Lett. **86**, 2716 (2001) DOI: 10.1103/PhysRevLett.86.2716

[16] J. David Jackson, "Classical Electrodynamics", John Wiley and Sons Inc. (1998)

[17] A. Zangwill, "Modern Electrodynamics", Oxford University Press Inc. (2012)

[18] R. Wald, "Advanced Classical Eletromagnetism", Princeton University Press (2022)

[19] B. Leory, "Collision between two balls accompanied by deformation: A qualitative approach to Hertz's theory", American Journal of Physics **53**, (1985) DOI: 10.1119/1.14164

[20] A. Grag, "The two cultures: SI and Gaussian units in electromagnetism", Eur. J. Phys. **39**, 045205 (2008) DOI: 10.1088/1361-6404/aac233

[21] D. Petrascheck, "Unit system independent formulation of electrodynamics", Eur. J. Phys. **42**, 045201 (2021) DOI: 10.1088/1361-6404/abe8c6

[22] R. Banerjee, C. Lee, and H. Seok Yang, "Seiberg-Witten-type maps for currents and energy-momentum tensors in noncommutative gauge theories", Physical Rev. **D70**, 065015 (2004) DOI: 10.1103/PhysRevD.70.065015

[23] R. Banerjee and K. Kumar, "Maps for currents and anomalies in noncommutative gauge theories", Physical Rev. **D71**, 045013 (2005) DOI: 10.1103/PhysRevD.71.045013

[24] H. Brain JR. and M. L. Michelsohn, "Spin Geometry", Princeton University Press, New Jersey (1989)

[25] M. R. Pahlavani and R. Morad, "Application of AdS/CFT in Nuclear Physics", Advances in High Energy Physics **2014**, 863268 (2014) DOI: 10.1155/2014/863268

[26] S. J. Brodsky and G. F. de Teramond, "Nonperturbative QCD coupling and its β function from light-front holography", Phys. Rev. **D78**, 025032 (2008) DOI: 10.1103/PhysRevD.81.096010

[27] J. A. de Wet, "The nuclear hologram", International Math. Forum **1**, 1841 (2006)

[28] K. Hashimoto, Y. Matsuo, and T. Morita, "Nuclear states and spectra in holographic QCD", JHEP **12**, 001 (2019) DOI: 10.1007/JHEP12(2019)001

[29] H. Maxfield and Z. Zahraee, "Holographic solar systems and hydrogen atoms: non-relativistic physics in AdS and its CFT dual", JHEP **11**, 093 (2022) DOI: 10.1007/JHEP11(2022)093

[30] R. Sandapen, "A Virtual Tribute to Quark Confinement and the Hadron Spectrum (vConf21)", EPJ. Web of Conferences **258**, 10009 (2022) DOI: 10.1051/epjconf/202225810009

[31] J. Gomis and T. Mehen, "Space-Time Noncommutative Field Theories And Unitarity", Nucl. Phys. **B591**, 265 (2000) DOI: 10.1016/S0550-3213%2800%2900525-3

[32] A. Jahan, "Energy shift of interacting non-relativistic fermions in noncommutative space", Iranian J. Phys. **5**, 112 (2005)

[33] K. S. Krane, "Introductory Nuclear Physics", New York, John Wiley and Sons Inc. (1988)