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Letter

Development of a General Functional Form of the Gravitational Potential: Addressing the Flatness Problem in Galactic Rotation Curves

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Abstract. This letter presents a mathematical model that may help address the long-standing problem of galaxy rotation curves. The strategy involves exploring the variation of circular velocity 'v' with respect to radial distance 'r' as a result of which a Bessel differential equation is developed, yielding a general functional form of the potential as a Bessel function. Employing this potential in the case of galaxies, we find the velocity profile in terms of Bessel functions. This approach establishes Bessel functions as a powerful choice for investigating galactic rotation and deepening our understanding of the universe.

Keywords: Galactic Rotation Curves; Flatness Problem; Functional Form.

1 A General Functional Form of the Gravitational Potential

The motion of the stars and gas within a galaxy is governed by the circular velocity model [1] given as;

$$v^2(r) = r\frac{d\phi(r)}{dr} \tag{1}$$

where $v(r)$ is the velocity, $\phi(r)$ is the gravitational potential and r is the radial distance. This equation has limitations in explaining the observed rotational speed of stars and gas in galaxies. The flatness issue, a significant discrepancy between the expected and observed distribution of rotational velocities, necessitates a deeper insight into the circular orbit model [1, 2, 3], therefore, in this work, we explore the variation of velocity v with respect to the radial distance r;

$$r\frac{d(v^2(r))}{dr} = r^2\frac{d^2\phi(r)}{dr^2} + r\frac{d\phi(r)}{dr} \tag{2}$$

Comparing this equation with the standard Bessel differential equation [4], we arrive at;

$$r^2\frac{d^2\phi(r)}{dr^2} + r\frac{d\phi(r)}{dr} + (r^2 - n^2)\phi(r) = 0 \tag{3}$$

The solution of equation (3) can be expressed as;

$$\phi(r) = -CJ_n(r) \tag{4}$$

where

$$J_n(r) = \sum_{s=0}^{s=\infty}(-1)^s\frac{(r/2)^{n+2s}}{s!(n+s)!}$$

represents the Bessel function of first kind [5, 6] and C is the proportionality constant depending upon the physical model. When considering gravitational effects, a reasonable estimation for the value of C is approximately $\simeq -GM$, where G represents the Gravitational Constant and M corresponds to the mass of the galaxy. The obtained solution provides the general functional form of the gravitational potential $\phi(r)$ in terms of Bessel functions.

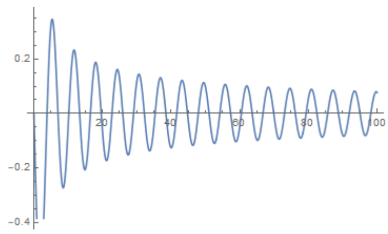

Figure 1: Behavior of $\phi(r)$ as a Bessel function.

2 Analysis into the Velocity Profile

Further analysis into equations (1) and (4) gives the velocity profile of the form;

$$v^2(r) = -CrJ_n'(r) \tag{5}$$

Using the Bessel recurrence relation;

$$J_n'(r) = J_{(n-1)}(r) - n\frac{J_n(r)}{r}$$

The general velocity profile can be written as;

$$v^2(r) = C\left(nJ_n(r) - rJ_{n-1}(r)\right) \tag{6}$$

This explicitly demonstrates the dependence of velocity on the radial distance r, through Bessel functions $J_n(r)$ and $J_{n-1}(r)$, and reveals the overall pattern of velocity distribution in the circular orbit.

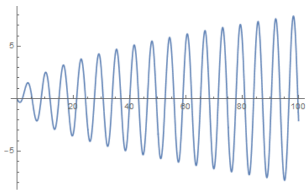

Figure 2: Behavior of $v^2(r)$ as a Bessel function.

From Fig. 1 and Fig. 2, it can be seen that the oscillatory nature of the Bessel functions provides an interesting insight into the behavior of the velocity of galaxies. The velocity curve Fig.2 suggests that the velocity is quantized (the negative part of v^2 has no physical sense and is therefore prohibited). While examining the observational data of galaxy rotation curves [1, 7], we see the data points remain confined to a particular range and interestingly, the oscillations of the velocity align in the same range. This co-occurrence between the theoretical Bessel function-based plot and the observed rotation curve suggests that there could be a holographic relationship between the observed rotational curve and the oscillations of the Bessel function which reinforces the wider applicability of Bessel functions.

3 Conclusion

We attempted to explore the variation of circular velocity with respect to radial distance as a result of which a Bessel differential equation emerged offering a general functional form of the potential characterized by Bessel functions and a general oscillatory velocity profile. Furthermore, analysis reveals employing this potential, the density too has an oscillatory nature. Upon careful analysis of the observation data, we could find the data points within the galactic rotational curve consistently fall within a specific range [1, 7]. Intriguingly, our investigation reveals that the oscillatory behavior of the Bessel function exactly aligns with the observed data. This correlation between the observational curve [1, 7] and the oscillations of the Bessel function holds immense possibility in explaining the study of galactic rotation. In conclusion, our study has successfully developed a general functional form of gravitational potential in terms of Bessel function that presents a compelling avenue for understanding the dynamics of galaxies.

Furthermore, this gravitational potential can also be employed to studying the thermodynamics of galaxy clusters. We reserve further investigation and application of this Bessel function based potential for the future courses of study.

Authors' contributions

All authors have the same contribution.

Data Availability

No data are available.

Conflicts of Interest

The authors declare that there is no conflict of interest.

Ethical Considerations

The authors have diligently addressed ethical concerns, such as informed consent, plagiarism, data fabrication, misconduct, falsification, double publication, redundancy, submission, and other related matters.

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