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Regular article

Hawking radiation as tunnelling from dilatonic BTZ black hole

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Abstract. In this paper, we study Hawking radiation for a dilatonic BTZ black hole solution and derive the transmission probability of tunnelling through the barrier of the event horizon. Furthermore, we discuss the black hole chemistry of the black hole solution under the effect of thermal fluctuation and find that the thermal fluctuation plays a significant role for the black hole with a small horizon radius.

Keywords: Hawking Radiation; Dilatonic BTZ Black Hole; Thermodynamics.

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1 Introduction

Gravitational force is the main force to governs the Universe at a large scale [1]. The dilaton gravity is found interesting because of the emergence of a scalar field in the low energy limit of string theory. The interactions of dilaton with the gauge fields have interesting features [2, 3, 4, 5]. For instance, the dilaton field can modify the asymptotic nature of the geometry. In the presence of Liouville-type dilaton potentials, black hole spacetimes are neither asymptotically flat nor (A)dS [6, 7, 8, 9] because the dilaton field does not vanish for asymptotic horizon radius.

The black hole in three dimensions is one of the interesting solutions to the field equations. Banados-Teitelboim-Zanelli (BTZ) were the first who found three dimensional black hole solution [10]. The BTZ black holes were generalized to introduce the dilaton field in Refs. [11, 12].

It has been found that thermal fluctuations due to small statistical fluctuation lead to logarithmic correction to the entropy of the black hole which plays a crucial role in the thermodynamics of small black holes [13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23].

The Hawking radiation as tunnelling and black hole chemistry under thermal fluctuation for dilatonic BTZ black holes are not studied yet. Therefore, it is worth studying these here. In the present work, we consider a constrained solution of a dilatonic BTZ black hole. This black hole solution has a coordinate singularity. In order to avoid this, we use the Painleve coordinate. Moreover, we calculate the transmission probability of tunnelling through the barrier of the event horizon by assuming a pair production of particles inside the horizon. In addition to that, we study the black hole chemistry of the given solution under the influence of small statistical thermal fluctuation around the equilibrium. The thermal fluctuation perturbs the equilibrium entropy of the system which is attributed to other thermal properties of the solution. Here, we find that Gibbs free energy and Helmholtz free energy of the system reduces to their equilibrium values under the influence of thermal fluctuation. In fact, the effect of thermal fluctuation is more significant for the smaller sized dilatonic BTZ black holes. We derive a particular condition for mass for which the given black hole satisfies the first law of thermodynamics.

The paper is organized as follows. In Sec. 2, we emphasize the tunnelling for dilatonic BTZ black hole. However, in Sec. 3, we discuss the black hole thermodynamics under the influence of thermal fluctuation. We summarize the results in the last section.

2 Tunnelling for dilatonic BTZ black hole

The line element for the dilatonic BTZ black hole is given by [24],

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\phi^2, \tag{1}$$

where $f(r)$ has the following form:

$$f(r) \quad = 4r - m\sqrt{r} + (0.0691)r^{-1}. \tag{2}$$

Here, the constants in Ref. [24] are constrained to have the following values: strength of coupling of the scalar and electro-magnetic field $\beta = 1$, arbitrary constants $b = 1$ and $\gamma = 0.5$, cosmological constant $\Lambda = -1$, power of nonlinearity $s = 0.75$. The last term of the metric function will be very small for $r > 0$ and, therefore, can be ignored. This leads to

$$f(r) \quad = 4r - m\sqrt{r}. \tag{3}$$

This confirms that the event horizon exists at $r_0 = \frac{m^2}{16}$. Therefore, the metric function in terms of the event horizon is given by

$$f(r) \simeq \frac{m}{\sqrt{r_0}}(r - \sqrt{rr_0}). \tag{4}$$

Thus, the black hole solution becomes

$$ds^2 = -\frac{m}{\sqrt{r_0}}(r - \sqrt{rr_0})dt^2 + (\frac{m}{\sqrt{r_0}}(r - \sqrt{rr_0})^{-1}dr^2 + r^2 d\phi^2. \tag{5}$$

At r_0, there is a coordinate singularity. So, to study across-horizon physics, we need to change the coordinate system so that the metric is well behaved at the horizon. For this, we use Painleve coordinate

$$t_s = t - \mathbb{S}(r), \tag{6}$$

where $\mathbb{S}(r)$ refers to an arbitrary function of r. This gives

$$dt = dt_s + \mathbb{S}'(r)dr, \tag{7}$$

$$dt^2 = dt_s^2 + \mathbb{S}'^2(r)dr^2 + 2\mathbb{S}'(r)drdt_s. \tag{8}$$

Therefore, the solution in Painleve coordinates is expressed as

$$ds^2 = -adt_s^2 + 2\sqrt{1-a}drdt_s + dr^2 + r^2 d\phi^2, \tag{9}$$

where $a = \frac{m}{\sqrt{r_0}}(r - \sqrt{rr_0})$.

The radial null geodesic is given by

$$-adt_s^2 + 2\sqrt{1-a}drdt_s + dr^2 = 0, \tag{10}$$

$$-a + 2\sqrt{1-a}\left(\frac{dr}{dt_s}\right) + \left(\frac{dr}{dt_s}\right)^2 = 0. \tag{11}$$

This gives

$$\dot{r} = \pm 1 - \sqrt{1-a}. \tag{12}$$

Now, we consider pair production just inside the horizon at $r_{in} \simeq r_0$. Let ω be the energy of the created particles. So, when a particle is emitted from the black hole its mass decreases $m - \omega$ and, hence, the horizon contracts to $r_{out} = \frac{(m-\omega)^2}{16}$ from $r_{in} = \frac{m^2}{16}$. The difference between r_{out} and r_{in} acts as a barrier for the particle tunnelling. In this region, ω is less than the potential, so action is imaginary. Therefore, the imaginary part of the action given as

$$\mathrm{Im}\, S = \mathrm{Im} \int_{r_{in}}^{r_{out}} p_r dr, \tag{13}$$

$$\mathrm{Im}\, S = \mathrm{Im} \int_{r_{in}}^{r_{out}} \int_0^{p_r} dp'_r dr. \tag{14}$$

Hamilton's equation relates the momentum and Hamiltonian as $dp_r = \frac{dH}{\dot{r}}$. Therefore,

$$\begin{aligned} \mathrm{Im}\, S &= \mathrm{Im} \int_{r_{in}}^{r_{out}} \int_m^{m-\omega} \frac{dH}{\dot{r}} dr, \\ &= \mathrm{Im} \int_{r_{in}}^{r_{out}} \int_0^{\omega} \frac{-d\omega}{1 - \sqrt{1-a}} dr, \\ &= \mathrm{Im} \int_{r_{in}}^{r_{out}} \int_0^{\omega} \frac{-d\omega}{1 - \sqrt{1 - \frac{m}{\sqrt{r_0}}(r - \sqrt{rr_0})}} dr. \end{aligned} \tag{15}$$

This implies that

$$\text{Im } S = \text{Im} \int_{r_{in}}^{r_{out}} \int_0^{\omega} \frac{-d\omega}{1 - \sqrt{1 - 4r + (m-\omega)\sqrt{r}}} dr. \quad (16)$$

By defining $u = 1 - 4r + (m - \omega)\sqrt{r}$, we have

$$du = -d\omega\sqrt{r}. \quad (17)$$

Therefore, relation (16) becomes

$$\text{Im } S = \text{Im} \int_{r_{in}}^{r_{out}} \int_{u(0)}^{u(\omega)} \frac{du}{(1-\sqrt{u})\sqrt{r}} dr. \quad (18)$$

This equation has a simple pole at $u = 1$. So residue at $u = 1$ is

$$\lim_{u \to 1} (1 - \sqrt{u}) \frac{1}{1 - \sqrt{u}} = 1. \quad (19)$$

Therefore, integral over u gives $2\pi i$. Now (18) becomes

$$\text{Im } S = \text{Im} \int_{r_{in}}^{r_{out}} 2\pi \frac{dr}{\sqrt{r}}, \quad (20)$$

$$= 4\pi(\sqrt{r_{out}} - \sqrt{r_{in}}). \quad (21)$$

The transmission probability is given by

$$\Gamma(\omega) \simeq e^{2\text{Im } S} = e^{8\pi(\sqrt{r_{out}} - \sqrt{r_{in}})} \quad (22)$$

To see the Boltzmann factor, we need to calculate $(\sqrt{r_{out}} - \sqrt{r_{in}})$ which reads

$$\sqrt{r_{out}} - \sqrt{r_{in}} = \frac{m-\omega}{4} - \frac{\omega}{4} = -\frac{\omega}{4}. \quad (23)$$

Consequently, the transmission probability becomes

$$\Gamma(\omega) \simeq e^{-2\pi\omega}. \quad (24)$$

The term in the exponential is identified to Boltzmann factor $\exp[-\frac{\omega}{T_H}]$. Therefore, the Hawking temperature identified to

$$T_H = \frac{1}{2\pi}. \quad (25)$$

3 Black hole chemistry under thermal fluctuation

In this section, we calculate the thermal properties of the diltonic BTZ black hole under thermal fluctuation. To describe these, we calculate the temperature, entropy, pressure, mass and volume respectively

$$T_H = \frac{f'(r_0)}{2\pi} = \frac{m}{2\pi\sqrt{r_0}} = \frac{1}{2\pi}, \quad (26)$$

$$S_0 = \frac{1}{2}\pi r_0, \quad (27)$$

$$P = \frac{1}{8\pi l^2}, \quad (28)$$

$$M = \sqrt{r_0} = 4l\sqrt{PS_0}, \quad (29)$$

$$V = \left.\frac{\partial M}{\partial P}\right|_{(S_0)} = 16 l^3 \pi \sqrt{\frac{\pi r_0}{2}}. \quad (30)$$

Here, we are interested in studying black hole chemistry under the influence of thermal fluctuation. Under the effect of thermal fluctuation, entropy gets the following correction:

$$S = S_0 - \frac{\alpha}{2} \ln S_0 T_H^2, \tag{31}$$

where α is a correction parameter which measures the strength of fluctuation [25] and has two possibilities either 0 or 1. The vanishing value of the correction parameter describes the equilibrium state. The Gibbs free energy for the $\alpha = 0$ is given by,

$$G = M - T_H S_0, \tag{32}$$

$$= 4l\sqrt{PS} - \frac{m}{4}\sqrt{r_0}, \tag{33}$$

$$= (1 - \frac{m}{4})\sqrt{r_0}, \tag{34}$$

$$= \left(1 - \frac{m}{4}\right) \frac{m}{4}. \tag{35}$$

Moreover, Gibbs free energy in the presence of thermal fluctuation is given by

$$G = \left(1 - \frac{m}{4}\right) \frac{m}{4} + \frac{\alpha}{4\pi} \ln \frac{m^2}{128\pi}. \tag{36}$$

To study the behaviour of Gibbs free energy we plot the Fig. 1. Here, we find that the thermal fluctuation decreases the value of it, and it is important for the small m. However, for the large m, the effects of thermal fluctuation are not significant.

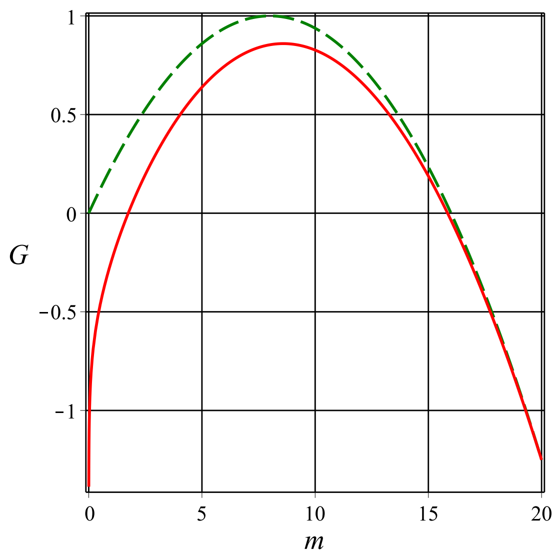

Figure 1: Gibbs free energy in terms of m for $\alpha = 0$ (green dash) and $\alpha = 1$ (red solid).

The Helmholtz free energy can be calculated from the standard relation

$$F = M - PV - T_H S. \tag{37}$$

This gives

$$F = m\left(\frac{1}{4} - \frac{m}{64} - \frac{l\sqrt{2\pi}}{4}\right) - \frac{\alpha}{4\pi}\left[7\ln 2 + \ln\pi - 2\ln m\right]. \tag{38}$$

The behaviour of Helmholtz free energy is depicted in the Fig. 2. Here, we can see that Helmholtz free energy decreases with thermal fluctuation. In the presence of correction due to fluctuation, we can see an extremum for the Helmholtz free energy.

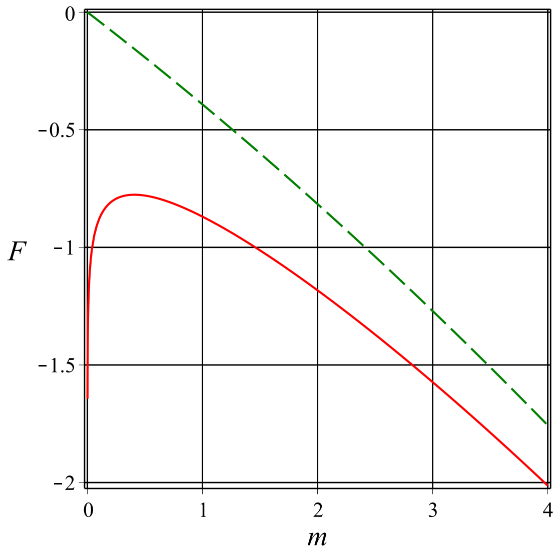

Figure 2: Helmholtz free energy in terms of m for $\alpha = 0$ (green dash) and $\alpha = 1$ (red solid).

We can also check conditions to satisfy the first law of thermodynamics

$$dM = TdS. \tag{39}$$

Here, we find that the first law holds only if

$$m = \frac{4(\pi + \sqrt{\pi(1+\alpha)})}{\pi}. \tag{40}$$

It is indeed corresponding to the maximum in the Fig. 1. This means that the black hole chemistry of the considered system suggests that the give black hole does not satisfy the first law of thermodynamics for all masses rather this holds for a particular mass for which Gibbs free energy is maximum. This is an interesting result.

4 Conclusion

In this paper, we have considered a particular solution of dilatonic BTZ black hole and studied the derived transmission probability of tunnelling through the barrier of the event horizon. Furthermore, we have studied the black hole chemistry of the given solution under the influence of small statistical thermal fluctuation. The thermal fluctuation modified the equilibrium entropy of the system. Here, we have found that Gibbs free energy and Helmholtz free energy of the system decrease under the influence of thermal fluctuation. However, the effect of thermal fluctuation is more significant for the smaller dilatonic BTZ black holes. We have also obtained a condition for mass for which the given black hole satisfies the first law of thermodynamics.

Authors' contributions

All authors have the same contribution.

Data Availability

No data are available.

Conflicts of Interest

The authors declare that there is no conflict of interest.

Ethical Considerations

The authors have diligently addressed ethical concerns, such as informed consent, plagiarism, data fabrication, misconduct, falsification, double publication, redundancy, submission, and other related matters.

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