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Regular article

Non-perturbative corrections of shear viscosity to entropy ratio

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Abstract. Shear viscosity to entropy ratio has a universal lower bound which may be violated under some effects. This paper would like to consider non-perturbative quantum corrections on the mentioned ratio in an STU black hole background. The STU model is a gravitational background that is the holographic dual of $N = 4$ super Yang-Mills quark-gluon plasma with the chemical potential. Non-perturbative corrections to the black hole entropy emerge as an exponential term which may affect the shear viscosity to entropy ratio. All possibilities will be studied in this paper to extract the shear viscosity of a quark-gluon plasma. We find that the universal lower bound of the shear viscosity to entropy ratio may be violated due to the non-perturbative corrections.

Keywords: Quark-Gluon Plasma; Black Hole; Holography; Non-perturbative Corrections.

1 Introduction

The early stage of the universe can be modeled by a quark-gluon plasma (QGP) which may be produced at RHIC or LHC. Now, AdS/CFT correspondence [1, 2, 3] is a powerful tool to study quark-gluon plasma. In Ref. [2], the relation between an anti-de Sitter space and holography was considered, while de Sitter space and holography is also an interesting issue [4, 5, 6]. Already there are several works like [7, 8, 9] where a quark-gluon plasma is studied using AdS/CFT correspondence. A quark-gluon plasma has some properties like viscosity which can be investigated using the AdS/CFT correspondence [10, 11, 12, 13, 14, 15] through studying the shear viscosity to entropy ratio. Although there are other ratios [14] including drag force and jet-quenching parameter. Considering a moving quark in plasma, one can calculate a drag force due to the plasma viscosity [16, 17, 18, 19, 20, 21, 22].
On the other hand, the shear viscosity to entropy ratio may have a universal lower bound [23, 24, 25, 26]. This universal lower bound may be violated due to the higher derivative or quantum corrections [27, 28, 29]. Already, we have studied the behavior of the ratio due to the thermal fluctuations [27] which make a logarithmic correction on the black hole entropy [30, 31]. Now, it is interesting to study the ratio under the exponential correction [32] on the black hole entropy. The exponential corrected entropy comes from the non-perturbative corrections which were recently considered to study the thermodynamics of some black objects [33, 34, 35, 36, 37]. As a black hole size decreases due to Hawking radiation, perturbative corrections become important. Decreasing more to the quantum scales, non-perturbative corrections become dominant so one can consider exponential corrections on the black hole entropy. Hence, the shear viscosity to entropy ratio is affected by non-perturbative corrections. It is also interesting to see non-perturbative correction effects on the shear viscosity and other QGP properties like drag force and jet-quenching parameter [38]. We would like to examine the mentioned effect on the shear viscosity to entropy ratio of some black hole backgrounds like the STU model [39, 40]. We explain the method in detail for the STU model and then represent the results for some famous black holes. The STU model includes a charged non-rotating black hole which may be used to study quark-gluon plasma using AdS/CFT correspondence. Black hole electric charges of the STU model correspond to the chemical potential on the CFT side. Effects of the STU black hole variables on the QGP already discussed in the literature [18]. Hence, black hole thermodynamics and hydrodynamics are important from a holography point of view. However, black hole thermodynamics and hydrodynamics may be corrected in quantum scales. So, in this regime, the holographic dual QGP may also affected. In this paper, we would like to study the non-perturbative correction on the shear viscosity, drag force, and jet-quenching parameter in various black hole backgrounds.
So, the rest of the paper is organized as follows. In the next section, we review the STU model and represent some basic properties. In section 3 we introduce the exponential corrected entropy of the black hole. The shear viscosity to entropy ratio is discussed in section 4. Finally, in section 5 we give a conclusion.

2 STU Model

We begin with the one-charged STU black hole in a closed universe which is given by the following metric [20],

$$ds^2 = -\frac{f(r)}{H^{\frac{2}{3}}}dt^2 + H^{\frac{1}{3}}(\frac{dr^2}{f(r)} + \frac{r^2}{R^2}d\Omega_3^2), \tag{1}$$

where,

$$f(r) = 1 - \frac{\mu}{r^2} + \frac{r^2}{R^2}H, \tag{2}$$

and

$$H = 1 + \frac{q}{r^2}, \tag{3}$$

where R is the constant AdS radius, q is the black hole electric charge, and μ denotes the non-extremality parameter which is related to the black hole mass. Also,

$$d\Omega_3^2 = R^2(d\rho^2 + \sin^2\rho d\theta^2 + \sin^2\rho \sin^2\theta d\phi^2). \tag{4}$$

The black hole horizon is specified by $r = r_+$ which is obtained from $f(r) = 0$. It gives us,

$$r_+^2 = \frac{\sqrt{R^4 + 2R^2(q + 2\mu) + q^2} - q - R^2}{2}. \tag{5}$$

It is the only real solution of $f(r) = 0$. In Fig. 1 we can see the position of the horizon for several values of μ and q.

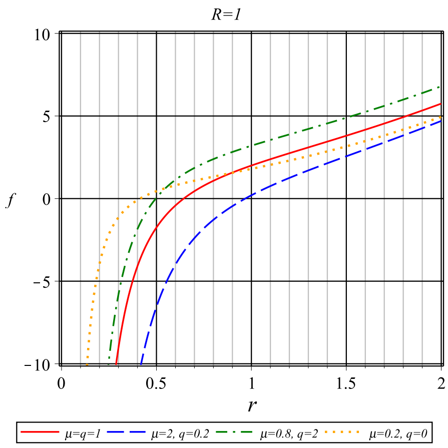

Figure 1: Horizon structure of one charged STU black hole with $R = 1$.

The non-extremality parameter can expressed in terms of the horizon radius as follows,

$$\mu = \frac{(r_+^2 + R^2 + q)r_+^2}{R^2}, \tag{6}$$

where the equation (3) is used.
The black hole temperature is given by [14],

$$T = \frac{2r_+^2 + R^2 + q}{2\pi r_+ R^2\sqrt{1 + \frac{q}{r_+^2}}}, \tag{7}$$

In Fig. 2 we can see the behavior of the black hole temperature with respect to the horizon radius. It is illustrated that the black hole temperature decreased by increasing the horizon

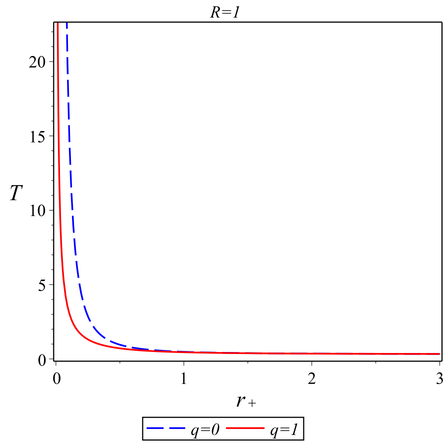

Figure 2: Temperature of one charged STU black hole versus r_+ with $R = 1$.

radius. It means that by decreasing the horizon radius due to Hawking radiation, the black hole temperature becomes infinitely large at small radii.

In that case, the black hole entropy and shear viscosity are given by the following expression respectively [10],

$$S_0 = \frac{r_+^3}{4GR^3}\sqrt{1 + \frac{q}{r_+^2}}, \tag{8}$$

and

$$\eta_0 = \frac{r_+^3}{16\pi GR^3}\sqrt{1 + \frac{q}{r_+^2}}, \tag{9}$$

where G is Newton's constant and subscript "0" denotes uncorrected states. For the charged black hole, the first law of thermodynamics is given by,

$$dM = TdS_0 + \Phi dq, \tag{10}$$

where Φ is the electrostatic potential that is conjugate variable to the physical charge.
In the case of the uncharged black hole ($q = 0$), one can obtain,

$$M = \frac{3}{16\pi GR^3}\mu. \tag{11}$$

Hence, the first law of the black hole thermodynamics, $dM = TdS_0$, satisfied.
In the general case,

$$M = \frac{1}{8\pi GR^3}\left(\frac{3}{2}\mu + q\right), \tag{12}$$

the thermodynamics equations are as follows,

$$T = \left(\frac{\partial M}{\partial S_0}\right)_q, \tag{13}$$

and

$$\Phi = \left(\frac{\partial M}{\partial q}\right)_{S_0}. \tag{14}$$

So, the first law of the black hole thermodynamics extended to $dM = TdS_0 + \Phi dq$. Therefore, it is clear from the equations (8) and (9) that

$$\frac{\eta_0}{S_0} = \frac{1}{4\pi}. \tag{15}$$

It has been argued that there is a universal lower bound for every system. However, some effects like perturbative corrections on the black hole entropy may violate it [27]. Now, we would like to investigate the effect of the recently proposed exponential corrected entropy which was obtained using non-perturbative corrections [32].

3 Exponential Corrected Thermodynamics

Recently, it has been found that non-perturbative quantum correction on the black hole entropy is exponential [32]. So, one can write,

$$S = S_0 + e^{-S_0}. \tag{16}$$

Equation (8) suggests that the uncorrected black hole entropy S_0 is proportional to the horizon radius. So at large radius, the correction term is negligible while at infinitesimal radii the exponential term is dominant.
It yields to the following specific heat,

$$C = T\frac{dS}{dT} = C_0\left(1 - e^{-S_0}\right), \tag{17}$$

where

$$C_0 = T\frac{dS_0}{dT} = -\frac{(3r_+^2 + 2q)r_+^2(2r_+^2 + q + R^2)\sqrt{r_+^2 + q}}{4GR^3(2R^2r_+^2 + qR^2 + q^2)}, \tag{18}$$

is uncorrected specific heat and S_0 is given by the equation (8). In Fig. 3 we can see the effect of the exponential correction on the specific heat. It shows that the exponential correction has no effect on the black hole phase transition and stability. Both corrected and uncorrected STU black holes are unstable.

At the scale where the exponential correction is dominant, the first law of black hole thermodynamics extended to,

$$dM = TdS + \Phi dq + \Theta dX, \tag{19}$$

where $\Theta = e^{-S_0}$ is a new thermodynamics variable responsible for the non-perturbative correction, while $dX = C_0 dT = TdS_0$. It may affect the nature of the quark-gluon plasma and we can investigate it by studying the shear viscosity to entropy ratio. It is easy to find that,

$$X = \frac{(q^2 - qR^2)\ln(r_+^2 + q) + 3r_+^2(r_+^2 + R^2 + \frac{q}{3})}{16\pi GR^5}, \tag{20}$$

4 Shear Viscosity to Entropy Ratio

Now, we are in the place to investigate the shear viscosity to entropy ratio. We consider three different scenarios.
Assuming that non-perturbative corrections are negligible for the shear viscosity.

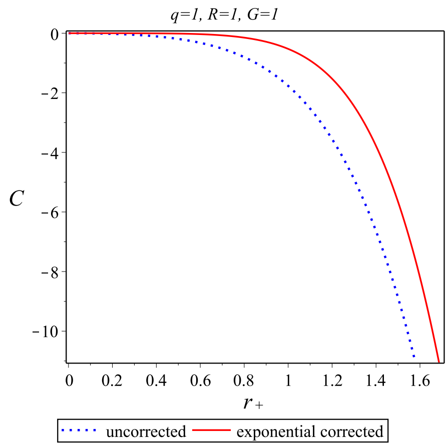

Figure 3: Specific heat of one charged STU black hole versus r_+ with $R = 1$, $q = 1$, and $G = 1$.

Assuming that the shear viscosity is corrected by non-perturbative corrections effectively (after calculating using the Kobu formula) and corrects the the shear viscosity to entropy ratio.
Assuming that the lower bound of the shear viscosity to entropy ratio holds, use it to calculate the non-perturbative corrected shear viscosity.

4.1 Conserved viscosity

First of all, we consider the situation that the shear viscosity is not affected effectively by quantum corrections. It means that $\eta \approx \eta_0$ and one can obtain,

$$\frac{\eta}{S} = \frac{\eta_0}{S_0 + e^{-S_0}}, \tag{21}$$

where η_0 and S_0 are given by the equations (9) and (8) respectively. In Fig. 4 we can see that the lower bound is violated at small radii where the non-perturbative correction is dominant. However, at the large radius where the non-perturbative corrections are negligible, the lower bound $\eta/S = 1/4\pi$ is satisfied.

4.2 Corrected viscosity

In Ref. [14] it has been suggested that

$$\dot{P} = \mathcal{A}\frac{S}{T} = \dot{P}_0 + \frac{\mathcal{A}}{T}e^{-\frac{\dot{P}_0 T}{\mathcal{A}}}, \tag{22}$$

where $\mathcal{A}$ is a constant and $\dot{P}$ denotes the corrected drag force, since

$$\dot{P}_0 = \mathcal{A}\frac{S_0}{T}, \tag{23}$$

is an uncorrected one. It means that non-perturbative corrections may increase the value of drag force.

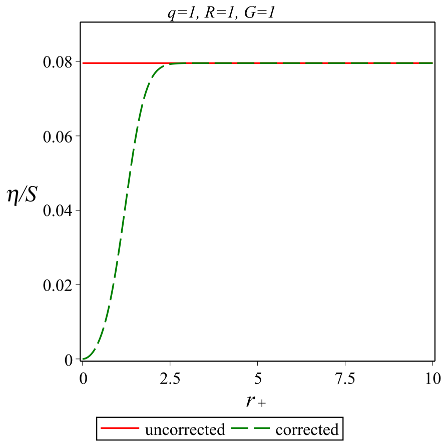

Figure 4: The first scenario. Shear viscosity to entropy ratio versus r_+ with $R = 1$, $q = 1$, and $G = 1$.

Also in Ref. [14] it has been suggested that

$$\hat{q} = \mathcal{B}S = \hat{q}_0 + \mathcal{B}e^{-\frac{\hat{q}_0}{\mathcal{B}}}, \tag{24}$$

where $\mathcal{B}$ is a constant and $\hat{q}$ denotes the jet-quenching parameter, so

$$\hat{q}_0 = \mathcal{A}S_0. \tag{25}$$

It means that non-perturbative corrections may increase the value of the jet-quenching parameter as well as the drag force. It suggests that the shear viscosity also should enhanced due to the non-perturbative corrections, hence we can write,

$$\frac{\eta}{S} = \frac{\eta_0 + \bar{\eta}}{S_0 + e^{-S_0}}, \tag{26}$$

where $\bar{\eta}$ is a positive contribution coming from the quantum corrections. Comparing with equations (22) and (24) suggests that

$$\eta = \eta_0 + \alpha e^{-\frac{\eta_0}{\alpha}}, \tag{27}$$

where α is a positive constant. In Fig. 5 we draw the behavior of the shear viscosity to entropy ratio (27). In the case of $\alpha = 0$ the result of the previous subsection is reproduced and the lower bound holds. Also the cases of $\alpha \geq \frac{1}{4\pi}$ yields to satisfying the universality. However, the values of α lower than $\frac{1}{4\pi}$ violate the universal lower bound at the small radii (see green dash dotted line of Fig. 5. A special case of this kind will be considered in the next subsection.

4.3 Lower bound

Finally, we assume that the universal lower bound is satisfied:

$$\frac{\eta}{S} = \frac{1}{4\pi}. \tag{28}$$

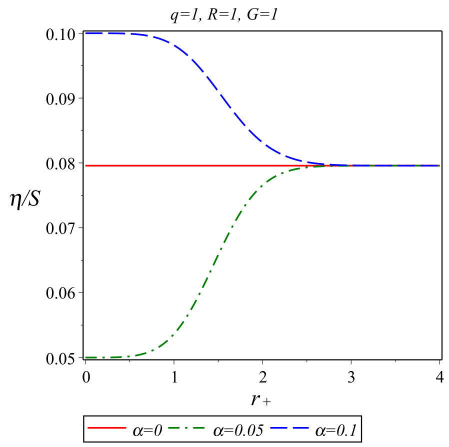

Figure 5: The second scenario. Shear viscosity to entropy ratio versus r_+ with $R = 1$, $q = 1$, and $G = 1$.

It means that in the equation (27) we should choose $\alpha = \frac{1}{4\pi}$ which yields,

$$\bar{\eta} = \frac{1}{4\pi} e^{-S_0}. \tag{29}$$

Therefore, the shear viscosity, as well as entropy, drag force and jet-quenching parameter get corrected by an exponential term due to the non-perturbative corrections.

5 Conclusion

In this paper, we considered non-perturbative quantum corrections on the shear viscosity to entropy ratio in an STU black hole background. All possibilities are studied in this paper to extract the shear viscosity of a quark-gluon plasma which is a holographic dual of an STU black hole background. We found that the universal lower bound of the shear viscosity to entropy ratio may be violated due to the non-perturbative corrections.

Data Availability

The manuscript has no associated data or the data will not be deposited.

Conflicts of Interest

The author declares that there is no conflict of interest.

Ethical Considerations

The author has diligently addressed ethical concerns, such as informed consent, plagiarism, data fabrication, misconduct, falsification, double publication, redundancy, submission, and other related matters.

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