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Regular article

Implications of quantum corrections on the thermodynamics of charged AdS black hole

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Abstract. This paper investigates the quantum corrections to the thermodynamic properties of charged AdS black holes. The corrections that we examine arise because of quantum fluctuations in space-time geometry, which corresponds to thermal fluctuations in thermodynamics. First, we compute the leading order corrections to entropy, and later we plot the corrected entropy as a function of event horizon radius for various values of correction parameter α to explore the effect of quantum corrections on the entropy of black holes analytically.

Keywords: Hawking Entropy; Free Energy; Stability.

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1 Introduction

Black holes are laboratories to test the connection between thermodynamics, quantum mechanics, and the classical theory of general relativity. It unifies the three pillars of physics i.e. thermodynamics, quantum mechanics, and classical theory of general relativity (GR) in a whimsical manner. Black hole thermodynamics came into existence with the seminal work of Hawking and Beckenstien [1, 2, 3, 4, 5]. Later on, Bardeen formulated four laws of black hole thermodynamics [2], which gave black hole thermodynamics a significant place in black hole mechanics. The quantitative relation for the entropy of black holes was first derived by Jacob Bekenstein [6]. It is now a well-established fact that the entropy of any black hole equals one-fourth of the Area of the event horizon in Plank units [7, 8, 9, 10]. It has been found that on reducing the size of the black hole, quantum mechanics starts disturbing the system to modify the thermodynamics of the black hole via quantum fluctuations. Quantum fluctuations in debts the variation in space-time topology and result in the change in thermodynamics of the equilibrium state of a black hole. Several attempts have been made to discuss these leading order corrections, for example, the first-order corrections to Schwarzschild black hole were studied in [11], and it was found that entropy gets corrected by the logarithmic function of a product of entropy and square of Hawking temperature. Again in [12], the effect of thermal fluctuations on BTZ, string theoretic, and many others whose microscopic degrees of freedom are described by underlying CFT have been examined. The effect of thermal fluctuations on the thermodynamics of BTZ black hole was studied in [13, 14, 15, 16, 17]. Moreover, the role of thermal fluctuations in the thermodynamics of the BTZ black hole was also studied using the partition function approach [18]. In [19, 20], the studies were made to examine the effect of small statistical thermal fluctuations on the entropy of black hole by taking into account the background matter field. Furthermore, the logarithmic corrections to the dilatonic black hole were computed in [21]. Meanwhile, in [22], the Rademancher expansion of partition function was used to study the leading order corrections to black hole thermodynamics. Also, the logarithmic corrections were studied extensively for string black hole correspondence [23, 24, 25, 26]. From all these papers, we can see that entropy gets corrected by logarithmic terms, in fact, in [27] Das et al. proved that entropy in any thermodynamic system gets corrected by logarithmic functions only, irrespective of the nature of the system.

The detailed analysis of the thermodynamics of black holes suggested that the quantum gravity approach to the thermodynamics of black holes is inevitable. This resulted in the corrections to various thermodynamic quantities by quantum effects. One such approach which is worth mentioning is GUP-corrected thermodynamics for all black objects and the existence of remnants by Mir Faizal et al. [28]. The correction (of the form $\alpha \ln A$) to the Gödel black hole has been discussed in [29]. These corrections of the type of $\alpha \ln A$ which are logarithmic in nature, were also discussed by Kaul and Majmuzdar in [30]. The effect of quantum gravity corrections on the thermodynamics of black holes using Cardy formalism has been studied in [18]. These corrections ought to be logarithmic and could be easily followed from [12]. One can also find the detailed effect of Quantum corrections on the thermodynamics of black holes from K. Nozari et al. [31]. Sudhaker in [32] carried quantum corrections to the thermodynamics of quasi-topological black holes. One excellent text regarding quantum corrections to black holes is by Khireddine Nouicer [33]. Further, there is ample literature available on logarithmic corrections to black hole thermodynamics as it has been the trending research area in the recent past, [32, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46]. We will carry out the same procedure to study the effect of quantum corrections on the thermodynamics of charged AdS black hole.

In the present paper, we investigate the effects of quantum corrections on the thermodynamics of charged AdS black holes. We first compute the leading order corrections to the entropy of the black hole, which originally equals one-fourth of area of the black hole horizon. We, later on, do the comparative analysis between uncorrected and corrected entropy density via a plot between entropy density and event horizon radius. It is found that large-sized black holes , entropy is an increasing function of the event horizon radius. At a small event horizon radius, the entropy of charged AdS black hole increases (decreases) asymptotically corresponding to a negative (positive) value of the correction parameter. The entropy, once modified, hints that the entire thermodynamics is likely to undergo deformation accordingly. We shall therefore calculate more corrected thermodynamic quantities. After entropy, we study the influence of quantum corrections on internal energy. We analyze that at a small event horizon radius i.e., for small-sized black holes internal energy increases (decreases) asymptotically corresponding to positive (negative) values of the correction parameter. Following the same trend, we examine the effects of quantum corrections on the free energy, pressure enthalpy, Gibbs free energy and specific heat. From the derivation of all these thermodynamic equations of states, it is found that quantum corrections are significant only at a small event horizon radius, while the thermodynamics of large-sized black holes remain unaltered.
The organization of the paper is that in the next section 2, we Compute the corrected entropy of charged AdS black hole subjected to quantum corrections. Then in section 3, we derive corrected equations of states and do a comparative analysis of these equations of states (uncorrected and corrected). The stability of the black hole is investigated in section 4 Finally in section 5, we summarize our results under the heading Conclusion.

2 First order corrected entropy of charged AdS Black Hole

Let's have a brief review of charged AdS black hole. First of all write down the action for a four-dimensional asymptotically AdS space-time coupled to Maxwell's equations as follows [47].

$$I=\int d^4x\sqrt{-g}\Big(R+\frac{6}{l^2}-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}\Big) \tag{1}$$

where l represents the AdS radius, and $F_{\mu\nu}$ is electromagnetic field strength and is defined as $F_{\mu\nu}=\nabla_\nu A_\mu-\nabla_\mu A_\nu$. The corresponding field equations for this system can be written as $G_{\mu\nu}=\frac{3g_{\mu\nu}}{l}+F^{\mu\tau}F^{\tau}_{\nu}-\frac{g_{\mu\nu}F^{\tau\rho}F_{\tau\rho}}{8}$ and $\sqrt{-g}\nabla_\nu F^{\mu\nu}=0$. The metric for our system, i.e. charged AdS black hole, is given by

$$ds^2=-f(r)dt^2+\frac{dr^2}{f(r)}+r^2d\Omega_k^2, \tag{2}$$

while

$$d\Omega_k^2=d\theta^2+\frac{1}{k}\sin^2(\sqrt{k}\theta)d\phi^2, \tag{3}$$

and metric function $f(r)$ is

$$f=k-\frac{2M}{r}+\frac{Q^2}{r^2}+\frac{r^2}{l^2}. \tag{4}$$

We can find the value of the event horizon radius by setting $f(r) = 0$ and then taking the positive root of the following equation

$$r_+^4 + l^2 r_+^2 - 2Ml^2 r_+ + l^2 Q^2 = 0. \tag{5}$$

From the given metric we can define Hawking's temperature by using the formula

$$T_H = \frac{f'(r)}{4\pi} \tag{6}$$

which yields the following relation for the Hawking temperature of charged AdS black hole.

$$T_H = \frac{l^2(r_+^2 - Q^2) + 3r_+^4}{4\pi l^2 r_+^3}. \tag{7}$$

Now, we will move on to understand the effects of quantum fluctuations on entropy. Entropy, a dynamical thermodynamic quantity, plays a central role in the evolution of thermodynamic systems. Boltzmann derived the expression for entropy from the statistical point of view and found that entropy is a logarithmic function of degrees of freedom, which usually depends on the volume of the system. But Bekenstien proposed the idea that for black holes, degrees of freedom are proportional to the area of the black hole, i.e. bulk degrees of freedom are projected on the boundary surface enclosing that volume. This constitutes the principle of holographic duality. The holographic principle in turn allows quantum corrections to occur at very small distances. The corrections we specifically study here are basically the extension of work carried out by Behnam Pourhassan and Mir Faizal [48]. The corrections are of the form

$$S = S_0 + \alpha \ln A, \tag{8}$$

where S_0 denotes the exact entropy of black hole , A stands for the area of event horizon radius and α quantifies the amount of influence that quantum fluctuations induce in the entropy of the black hole. Now charged AdS black hole has spherical symmetry with the area equal to the area of a sphere of radius r. Therefore the corrected entropy for charged AdS black hole is,

$$S = \pi r_+^2 + \alpha \ln 4\pi r_+^2. \tag{9}$$

To analyze the effect of perturbations due to quantum fluctuations on the entropy of charged AdS black hole, the above-derived expression is plotted against the event horizon radius for different values of correction parameter α. It is evident from the plot that by removing corrections, i.e., in the limit, $\alpha = 0$, an unperturbed entropy density is reproduced. This assures us that we are on the right track. Moreover, it may be emphasized that quantum corrections are significant only at a small event horizon radius. Far from the singularity, quantum corrections fail to affect the entropy of charged AdS black holes. At small values of event horizon radius, entropy increases infinitely for a negative value of the correction parameter, while the positive value of the correction parameter leads to a negative asymptotic value of entropy that is physically not allowed.

3 Perturbed equations of states

Now we have found that quantum fluctuations modify the charged AdS black hole entropy. As such entire thermodynamics is likely to undergo modification. Let's derive corrected

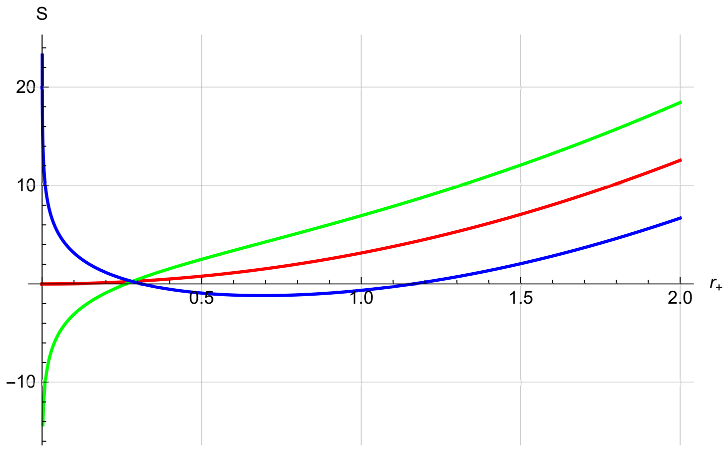

Figure 1: Entropy vs. the black hole horizon radius. Here, $\alpha = 0$ denoted by the red line, $\alpha = 0.5$ denoted by green line, and $\alpha = -0.5$ denoted by blue line.

expressions for various thermodynamic parameters. One of the most important thermodynamical parameters is internal energy (U) which is, in fact, a state function and is defined in thermodynamics as follows

$$U = \int T_H dS. \tag{10}$$

Plugging the corresponding values of corrected entropy and Hawking temperature, the expression for corrected internal energy is obtained as,

$$U = \frac{l^2[\alpha\left(Q^2 - 3r_+^2\right) + 3\pi r_+^2\left(Q^2 + r_+^2\right)] + 3\pi r_+^6 + 9\alpha r_+^4}{6\pi l^2 r_+^3}. \tag{11}$$

To get a clear idea of the effects of quantum corrections at a very small scale, we plot the obtained expression against the event horizon radius for different correction parameter values. When the corrections are switched off, i.e. α tends to zero, we get an uncorrected expression back exactly matching the undeformed curve for internal energy as predicted by black hole thermodynamics (without taking into account the effect of quantum corrections). Plugging in the negative value of the correction parameter produces negative asymptotic behavior in our system at a very small event horizon radius, thereby decreasing the mass or, in other words increasing the temperature. A positive value of the deformation parameter α results in a positive asymptotic value of internal energy and hence increases the mass of the black hole infinitely. Consequently, the temperature of the black hole decreases to its lowest possible limit.

Furthermore, we discuss the effect of quantum corrections on the system's Free energy. It is also a state function and represents one of the thermodynamic potentials, used to study the stability of the system. To attain stability, the system must minimize its free energy. Here we will first compute the leading order corrections to the free energy and later plot the corrected free energy against the event horizon radius. We will see what values of the correction parameter minimize the free energy. The corrected version of free energy for charged AdS black hole is given by

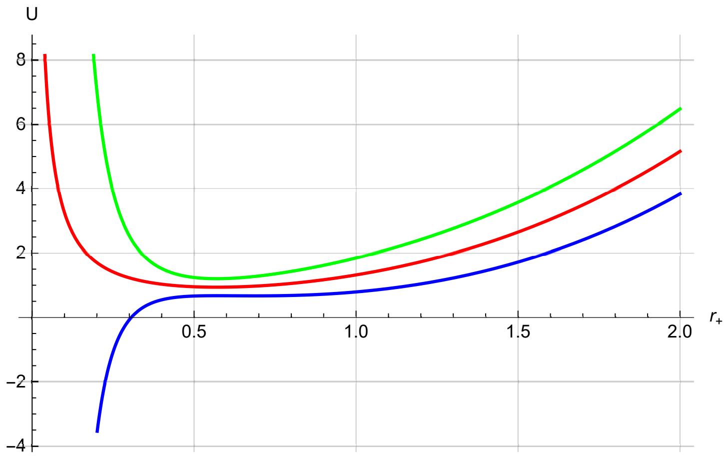

Figure 2: Internal energy vs. the black hole horizon radius for $l = 1, Q = .8$. Here, $\alpha = 0$ denoted by the red line, $\alpha = 1.5$ denoted by green line, and $\alpha = -1.5$ denoted by blue line.

$$\begin{aligned} F &= \frac{1}{12\pi l^2 r_+^3}[l^2(2\alpha Q^2 + 9\pi Q^2 r_+^2 + 3\pi r_+^4 - 6\alpha r_+^2) \\ &+ 3\alpha \ln 4\pi r_+^2 [l^2\left(Q^2 - r_+^2\right) - 3r_+^4] - 3\pi r_+^6 + 18\alpha r_+^4]. \end{aligned} \tag{13}$$

Here it may be emphasized that leading order corrections in the above expression represent the measure of the effect of quantum corrections on the equilibrium value of free energy. Fig. 3 represents the plot for corrected Free energy. From Fig. 3, we observe two critical points, both in the first quadrant. Between the critical points, the free energy curve shows a dip corresponding to negative values of the correction parameter, implying enhanced stability in this region and thereby paving the way for the extraction of energy for full work via a black hole heat engine. Before the first critical point, the positive values (negative values) of the correction parameter decrease (increase) the free energy asymptotically at small event horizon distances. For large-sized black holes, free energy is a decreasing function of the event horizon radius, which is evident from the plot after a second critical point.

We shall now discuss how the equation of state for our system (i.e., Pressure) gets affected by quantum fluctuations. The corrected expression for Pressure (P) is given by.

$$P = \frac{[l^2\left(3Q^2 - r_+^2\right) + 3r_+^4][\alpha \ln 4\pi r_+^2 + \pi r_+^2]}{16\pi^2 l^2 r_+^6}. \tag{14}$$

The higher-order corrections represent the effect of quantum corrections on the Pressure about the equilibrium. The behaviour of the uncorrected pressure curve resembles the Van der Wall fluid. However, quantum fluctuations try to induce the idea of gas behaviour in the AdS black hole, as evident from the figure 4. The negative values of the perturbation parameter enhance the deviation of Van der Waals fluid from the ideal gas behaviour. In

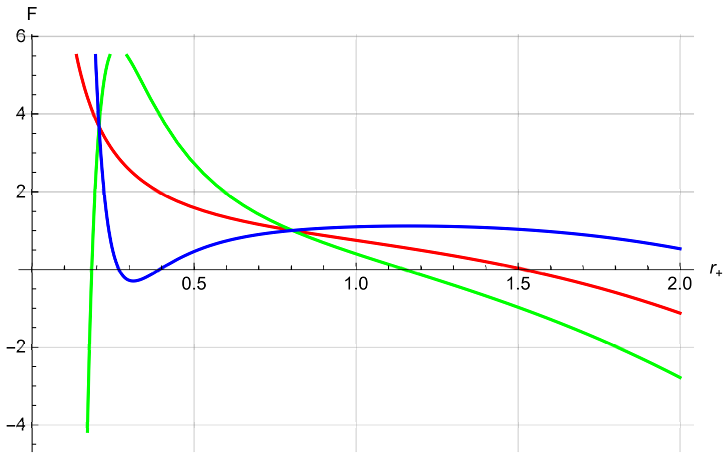

Figure 3: Free energy vs. the black hole horizon radius for $l = 1, Q = 0.8$. Here, $\alpha = 0$ is denoted by the red line, $\alpha = 1.5$ is denoted by the green line, and $\alpha = -1.5$ is denoted by the blue line.

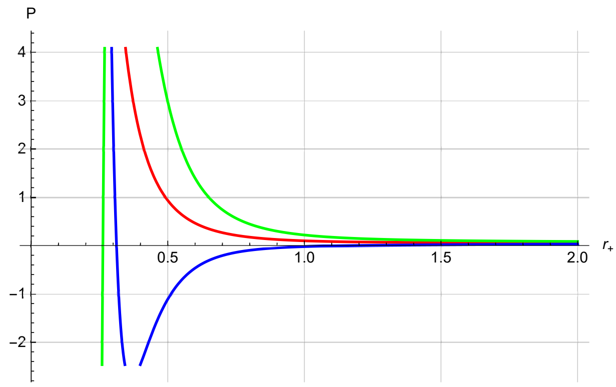

Figure 4: Pressure vs. the black hole horizon radius for $l = 1, Q = 0.8$. Here, $\alpha = 0$ is denoted by the red line, $\alpha = 1.5$ is denoted by the green line, and $\alpha = -1.5$ is denoted by the blue line.

contrast, the positive values decrease the deviation between the Van der Waals fluid and the ideal gas and compel the AdS black hole to behave like the ideal gas.

we see that at a large event horizon radius, the pressure remains constant irrespective of the nature of the correction parameter. However, at a small event horizon radius, quantum fluctuations play a dominant role. Due to the negative value of the correction parameter, there is a dip in pressure at a small event horizon. Before this dip, pressure increases and tends to a positive asymptotic value and hence creating high pressure for a small value of event horizon radius. This implies that if any object falls into a black hole, the tidal forces will be so strong that the object will get high spaghettification. Furthermore, for the higher values of the negative correction parameter, we see the depth of dip increase and there occurs an asymptotic divergence. Moreover, in the limit, α tends to zero, we retrieve the uncorrected pressure. However, it may be noted that at a particular point, a divergence occurs in the pressure. We will make use of the above-corrected quantities to derive further thermodynamic quantities. One such quantity is enthalpy which is a state function. The corrected enthalpy (H) for a charged AdS black hole is given by.

$$H = \frac{l^2[2\alpha Q^2 + 9\pi Q^2 r_+^2 + 5\pi r_+^4 - 6\alpha r^2]}{12\pi l^2 r_+^3} + \frac{\alpha \ln 4\pi r_+^2 [l^2 \left(3Q^2 - r_+^2\right) + 3r_+^4] + 9r^4[2\alpha + \pi r_+^2]}{12\pi l^2 r_+^3} \tag{15}$$

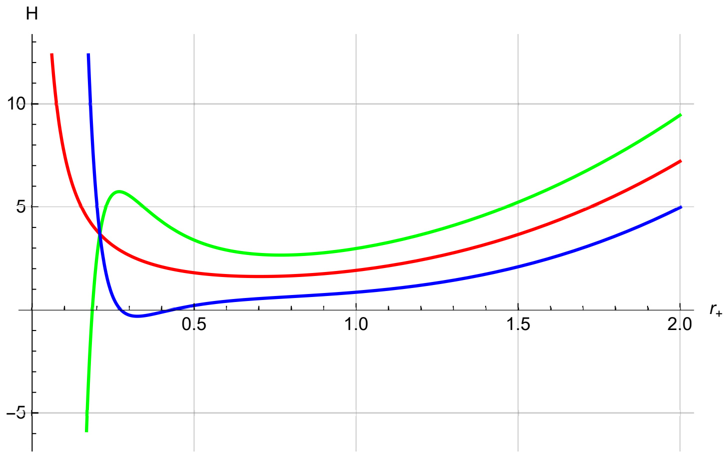

Figure 5: Enthalpy vs. the black hole horizon radius for $l = 1, Q = .8$. Here, $\alpha = 0$ is denoted by the red line, $\alpha = 1.5$ is denoted by the green line, and $\alpha = -1.5$ is denoted by the blue line.

We examine the effects of quantum corrections on the enthalpy by doing the qualitative analysis via plot for corrected enthalpy versus event horizon radius. One may notice from the plot 5 that the nature of the correction parameter is immaterial at a large event horizon radius. However, the behaviour of enthalpy is highly sensitive to the nature of the correction parameter at a minimal event horizon radius. Corresponding to the correction parameter's positive value, there is a very decrease in enthalpy asymptotically. With negative correction values [parameter, we observe that enthalpy increases asymptotically.
Let us now discuss the higher-order corrections on the Gibbs free energy, which gives us an idea of the maximum work we can draw from the system. A decrease in Gibbs's free energy in thermodynamics equals the practical work one can draw from the system. The corrected version for expression of the Gibbs free energy reads as follows.

$$G = \frac{l^2[\alpha Q^2 + 6\pi Q^2 r_+^2 + \pi r_+^4 - 3\alpha r_+^2]}{6\pi l^2 r_+^3} + \frac{\alpha \ln 4\pi r_+^2 [l^2 \left(3Q^2 - 2r_+^2\right) - 3r_+^4] + 9\alpha r_+^4}{6\pi l^2 r_+^3} \tag{16}$$

We plot the obtained expression as a function of the event horizon radius. From the plot 6, we observe that Gibbs free energy shows asymptotic behaviour as r_+ tends to zero. We found two critical points at a small event horizon radius. Between these two critical points, quantum fluctuations compel Gibbs free energy to undergo a drastic change. In this region, a negative value of the correction parameter leads to a dip in Gibbs free energy. Furthermore, one can also observe that the more the negative value of the correction parameter, the more the depth of dip, i.e., Gibbs free energy decreases more and more. This implies that we can

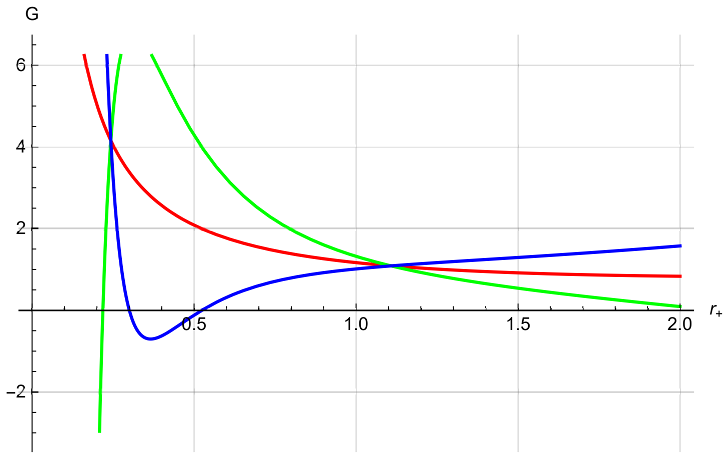

Figure 6: Gibbs free energy vs. the black hole horizon radius for $l = 1, Q = 0.8$. Here, $\alpha = 0$ is denoted by the red line, $\alpha = 1.5$ is denoted by the green line, and $\alpha = -1.5$ is denoted by the blue line.

draw the maximum work from here, which shows the sign of stability in this region. On the other hand, the positive value of the correction parameter produces quite the opposite result to that of the negative correction parameter. Before the first critical point, one can examine that the negative (positive) value of the correction parameter produces a positive (negative) asymptotic value of Gibbs free energy.

4 Phase transition

We investigate the stability of black holes whimsically by computing the specific heat. The positivity of specific heat is directly related to the stability of black hole. The expression for specific heat at constant volume and specific heat at constant pressure are reckoned as follows,

$$C_v = \frac{2\left(\alpha + \pi r_+^2\right)\left(l^2\left(r_+^2 - Q^2\right) + 3r_+^4\right)}{l^2\left(3Q^2 - r_+^2\right) + 3r_+^4}. \tag{17}$$

$$C_p = \frac{r_+^2(l^2\left(4\alpha - 9\pi Q^2 + 5\pi r_+^2\right) + 3r_+^2(8\alpha + 9\pi r_+^2)) + \alpha \ln 4\pi r_+^2(l^2\left(r_+^2 - 9Q^2\right) + 3r_+^4)}{l^2(9Q^2 - 3r_+^2) + 9r_+^4} \tag{18}$$

The expressions obtained are plotted against the event horizon radius in Fig. 7 and fig.8. Further, we also find the ratio of two specific heats as follows

$$\gamma = \frac{\alpha \ln 4\pi r_+^2\left(l^2\left(9Q^2 - r_+^2\right) - 3r_+^4\right) - r_+^2\left(l^2\left(4\alpha - 9\pi Q^2 + 5\pi r_+^2\right) + 3r_+^2\left(8\alpha + 9\pi r_+^2\right)\right)}{6\left(\alpha + \pi r_+^2\right)\left(l^2\left(Q^2 - r_+^2\right) - 3r_+^4\right)}. \tag{19}$$

From the ratio, one can notice that quantum corrections are trying to change the behaviour to behave like an ideal gas. To check the occurrence of phase transition, the quantity Q is

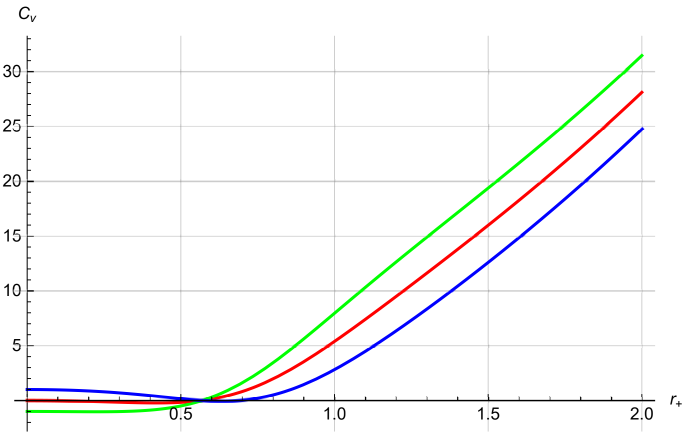

Figure 7: Specific heat vs. the black hole horizon radius for $l = 1, Q = 0.8$. Here, $\alpha = 0$ is denoted by the red line, $\alpha = 1.5$ is denoted by the green line, and $\alpha = -1.5$ is denoted by the blue line.

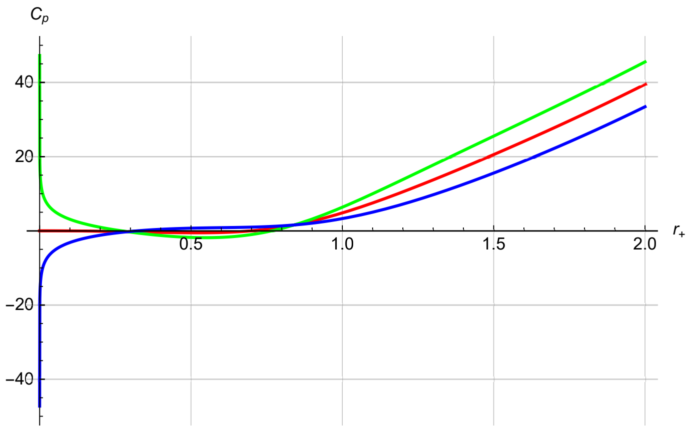

Figure 8: Specific heat vs. the black hole horizon radius for $l = 1, Q = 0.8$. Here, $\alpha = 0$ is denoted by the red line, $\alpha = 1.5$ is denoted by the green line, and $\alpha = -1.5$ is denoted by the blue line.

eliminated in equation 17 by exploiting eqns.5 and 15. It turns out that an unperturbed black hole is unstable, and quantum fluctuation induces some stable regions for the value of $\alpha \geq 0.5$. The phase transition is seen to occur at specific values of horizon radius.

5 Conclusion

In conclusion, this study delved into the intricacies of the thermodynamic behaviour of a charged AdS black hole under the influence of quantum fluctuations. The primary focus was on calculating the leading-order quantum corrections to the entropy of the charged AdS black hole. We plotted the corrected entropy against the event horizon radius for varying correction parameter α values by comparing corrected and uncorrected entropy densities.

Notably, in the limit where α approaches zero, the uncorrected entropy curve was faithfully replicated, affirming the accuracy of our approach. Furthermore, it became evident that quantum corrections wield substantial influence only when the event horizon radius is small. Far removed from the singularity, these corrections exhibited negligible impact on the entropy of the charged AdS black hole. Notably, for lower event horizon radius values, the entropy assumed positive values, corresponding inversely to the negative values of the correction parameter. Conversely, a positive correction parameter yielded a forbidden negative asymptotic value. Beyond the study of entropy, our analysis extended to corrections in internal energy. Like the entropy analysis, corrected internal energy was graphed against the event horizon for different correction parameter values. Strikingly, as the corrections were attenuated (α approaching zero), the uncorrected internal energy expression was accurately restored, aligning with the predictions of black hole thermodynamics. Subsequently, exploring the effects of the correction parameter on internal energy, we observed distinct behaviours based on the correction parameter's sign. Negative (positive) values led to negative (positive) asymptotic trends, resulting in reduced mass or increased temperature and an infinitely increasing black hole mass with plummeting temperature. The scrutiny then extended to free energy, which exhibited two critical points in the first quadrant. The inter-critical region showcased a dip indicating improved stability, suggesting potential for energy extraction through a black hole heat engine. Quantum corrections influenced the equilibrium pressure, driving AdS black holes toward ideal gas behaviour. Enthalpy, too, was scrutinized, revealing high sensitivity to the correction parameter's nature at minuscule event horizon radii. The enthalpy's asymptotic trends mirrored the correction parameter's increasing or decreasing behaviour. The analysis then encompassed the corrected Gibbs free energy, unveiling intriguing critical points where a negative correction parameter induced a dip, implying maximal work extraction potential. Lastly, the study explored the effects of quantum corrections on specific heat, unraveling stability regions and hinting at the possibility of phase transitions induced by quantum fluctuations. In essence, this research demonstrates the profound implications of quantum corrections on the thermodynamic characteristics of charged AdS black holes, unraveling a complex interplay between quantum fluctuations and thermodynamic properties.

Authors' contributions

All authors have the same contribution.

Data Availability

No data are available.

Conflicts of Interest

The authors declare that there is no conflict of interest.

Ethical Considerations

The authors have diligently addressed ethical concerns, such as informed consent, plagiarism, data fabrication, misconduct, falsification, double publication, redundancy, submission, and other related matters.

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References

[1] J.D. Bekenstein, "Black holes and the second law", Lett. Nuovo Cimento **4**, 737 (1972). DOI: 10.1007/BF02757029

[2] J.M. Bardeen, B. Carter, S.W. Hawking, "The four laws of black hole mechanics", Commun. Math. Phys. **31**, 161 (1973). DOI: 10.1007/BF01645742

[3] J.D. Bekenstein, "Generalized second law of thermodynamics in black-hole physics", Phys. Rev. D **9**, 3292 (1974). DOI: 10.1103/PhysRevD.9.3292

[4] S.W. Hawking, "Black hole explosions?", Nature **248**, 30 (1974). DOI: 10.1038/248030a0

[5] S.W. Hawking, "Particle creation by black holes", Commun. Math. Phys. **43**, 199 (1975). DOI: 10.1007/BF02345020

[6] J.D. Bekenstein, "Black Holes and Entropy", Phys. Rev. D **7**, 2333 (1973). DOI: 10.1103/PhysRevD.7.2333

[7] A. Strominger, C. Vafa, "Microscopic Origin of the Bekenstein-Hawking Entropy", Phys. Lett. B **379**, 99 (1996). DOI: 10.1016/0370-2693(96)00345-0

[8] A. Ashtekar, J. Baez, A. Corichi, K. Krasnov, "Quantum Geometry and Black Hole Entropy", Phys. Rev. Lett. **80**, 905 (1998). DOI: 10.1103/PhysRevLett.80.904

[9] S. Carlip, "Black Hole Entropy from Conformal Field Theory in Any Dimension", Phys. Rev. Lett. **82**, 2828 (1999). DOI: 10.1103/PhysRevLett.82.2828

[10] S. N. Solodukhin, "Conformal description of horizon's states", Phys. Lett. B **454**, 213 (1999). DOI: 10.1016/S0370-2693(99)00398-6

[11] D. Fursaev, "Temperature and Entropy of a Quantum Black Hole and Conformal Anomaly", Phys. Rev. D **51**, 5352 (1995). DOI: 10.1103/PhysRevD.51.R5352

[12] S. Carlip, "Logarithmic Corrections to Black Hole Entropy from the Cardy Formula", Class. Quant. Grav. **17**, 4175 (2000). DOI: 10.1088/0264-9381/17/20/302

[13] Nadeem ul islam, Prince A. Ganai and S. Upadhyay, "Thermal fluctuations to the thermodynamics of a non-rotating BTZ black hole", Prog. Theor. Exp. Phys. **2019**, 103B06 (2019). DOI: 10.1093/ptep/ptz113

[14] Nadeem ul islam, and Prince A. Ganai, "Quantum corrections to thermodynamics of BTZ black hole", International Journal of Modern Physics A, **34**, 1950063 (2019). DOI: 10.1142/S0217751X19500635

[15] S. Upadhyay, Nadeem ul islam and Prince A. Ganai, "A modified thermodynamics of rotating and charged BTZ black hole", Journal of Holography Applications in Physics **2** (1), 25-48 (2022). DOI: 10.22128/jhap.2021.454.1004

[16] Nadeem ul Islam and Prince A. Ganai, "Quantum corrections to AdS black hole in massive gravity", International Journal of Modern Physics A **34**, 1950225 (2019). DOI: 10.1142/S0217751X19502257

[17] Nadeem ul Islam and Prince A. Ganai, "First-order corrected thermodynamic potentials characterizing BTZ black hole in massive gravity", International Journal of Modern Physics A **35**, 2050080 (2020). DOI: 10.1142/S0217751X20500803

[18] T. R. Govindarajan, R. K. Kaul, V. Suneeta, "Logarithmic correction to the Bekenstein-Hawking entropy of the BTZ black hole", Class. Quant. Grav. **18**, 2877 (2001). DOI: 10.1088/0264-9381/18/15/303

[19] R. B. Mann and S. N. Solodukhin, "Universality of Quantum Entropy for Extreme Black Holes", Nucl. Phys. B **523**, 293 (1998). DOI: 10.1016/S0550-3213(98)00094-7

[20] A. J. M. Medved and G. Kunstatter, "Quantum corrections to the thermodynamics of charged 2D black holes", Phys. Rev. D **60**, 104029 (1999). DOI: 10.1103/PhysRevD.60.104029

[21] J. Jing, M-L Yan, "Statistical Entropy of a Stationary Dilaton Black Hole from Cardy Formula", Phys. Rev. D **63**, 24003 (2001). DOI: 10.1103/PhysRevD.63.024003

[22] D. Birmingham, S. Sen, "An Exact Black Hole Entropy Bound", Phys. Rev. D **63**, 47501 (2001). DOI: 10.1103/PhysRevD.63.047501

[23] S. N. Solodukhin, "Entropy of Schwarzschild black hole and string-black hole correspondence", Phys. Rev. D **57**, 2410 (1998). DOI: 10.1103/PhysRevD.57.2410

[24] A. Sen, "Logarithmic corrections to Schwarzschild and other non-extremal black hole entropy in different dimensions", JHEP **04**, 156 (2013). DOI: 10.1007/JHEP04(2013)156

[25] A. Sen, "State Operator Correspondence and Entanglement in AdS_2/CFT_1", Entropy **13**, 1305 (2011). DOI: 10.3390/e13071305

[26] D. A. Lowe and S. Roy, "Punctuated eternal inflation via AdS/CFT duality", Phys. Rev. D **82**, 063508 (2010). DOI: 10.1103/PhysRevD.82.063508

[27] S. Das, P. Majumdar and R. K. Bhaduri, "General Logarithmic Corrections to Black Hole Entropy", Class. Quant. Grav. **19**, 2355 (2002). DOI: 10.1088/0264-9381/19/9/302

[28] M. Faizal and M. M. Khalil, "GUP-corrected thermodynamics for all black objects and the existence of remnants", International Journal of Modern Physics A **30**, 1550144 (2015). DOI: 10.1142/S0217751X15501444

[29] A. Pourdarvish, J. Sadeghi, H. Farahani, and B. Pourhassan, "Thermodynamics and Statistics of Godel Black Hole with Logarithmic Correction", Int. J. Theor. Phys. **52**, 3560 (2013). DOI: 10.1007/s10773-013-1658-4

[30] R. K. Kaul and P. Majumdar, "Logarithmic Correction to the Bekenstein-Hawking Entropy", Phys. Rev. Lett. **84**, 5255 (2000). DOI: 10.1103/PhysRevLett.84.5255

[31] K. Nozari and S. H. Mehdipour, "Black Holes Remnants in Extra Dimensions and Dark Matter", International Journal of Modern Physics A **21**, 4979 (2006). DOI: 10.1142/S0217751X06031570

[32] S. Upadhyay, "Quantum corrections to thermodynamics of quasitopological black holes", Physics Letters B **775**, 130 (2017). DOI: 10.1016/j.physletb.2017.10.059

[33] K. Nouicer, "Quantum-corrected black hole thermodynamics to all orders in the Planck length", Phys. Lett. B **646**, 63 (2007). DOI: 10.1016/j.physletb.2006.12.072

[34] B. Pourhassan, M. Faizal, S. Upadhyay, L. Al Asfar, "Thermal Fluctuations in a Hyperscaling Violation Background", Eur. Phys. J. C **77**, 555 (2017). DOI: 10.1016/j.physletb.2006.12.072

[35] S. H. Hendi, S. Panahiyan, S. Upadhyay, and B. Eslam Panah, "Charged BTZ black holes in the context of massive gravity's rainbow", Phys. Rev. D **95**, 084036 (2017). DOI: 10.1103/PhysRevD.95.084036

[36] S. Upadhyay, "Leading-order corrections to charged rotating AdS black holes thermodynamics", Gen. Rel. Grav. **50**, 128 (2018). DOI: 10.1007/s10714-018-2459-0

[37] S. Upadhyay, S. H. Hendi, S. Panahiyan, B. Eslam Panah, "Thermal fluctuations of charged black holes in gravity's rainbow", Prog. Theor. Exp. Phys. **2018**, 09E01 (2018). DOI: 10.1093/ptep/pty093

[38] B. Pourhassan, S. Upadhyay, H. Saadat, H. Farahani, "Quantum gravity effects on Horava-Lifshitz black hole", Nuclear Physics B **928**, 415 (2018). DOI: 10.1016/j.nuclphysb.2018.01.018

[39] S. Soroushfar, R. Saffari, and S. Upadhyay, "Thermodynamic geometry of a black hole surrounded by perfect fluid in Rastall theory", Gen. Rel. Grav. **51**, 130 (2019). DOI: 10.1007/s10714-019-2614-2

[40] B. Pourhassan, H. Farahani, S. Upadhyay, "Thermodynamics of Higher Order Entropy Corrected Schwarzschild-Beltrami-de Sitter Black Hole", International Journal of Modern Physics A **34**, 1950158 (2019). DOI: 10.1142/S0217751X19501586

[41] S. Upadhyay, B. Pourhassan, "Logarithmic corrected Van der Waals black holes in higher dimensional AdS space", Prog. Theor. Exp. Phys. **2019**, 013B03 (2019). DOI: 10.1093/ptep/pty145

[42] S. Soroushfar, and S. Upadhyay, "Accretion disks around a static black hole in $f(R)$ gravity", Eur,Phys. J. Plus, 135, 388 (2020). DOI: 10.1140/epjp/s13360-020-00329-4

[43] Y. H Khan,, Prince A. Ganai and S. Upadhyay, "Quantum corrected thermodynamics and P-V criticality of self-gravitating Skyrmion black holes", Prog. Theor. Exp. Phys. **2020**, 103B06 (2020). DOI: 10.1093/ptep/ptaa135

[44] S. Soroushfar, and S. Upadhyay, "Phase transition of a charged AdS black hole with a global monopole through geometrical thermodynamics", Phys. Lett. B **804**, 135360 (2020). DOI: 10.1016/j.physletb.2020.135360

[45] B. Pourhassan, and S. Upadhyay, "Perturbed thermodynamics of charged black hole solution in Rastall theory", Eur. Phys. J. Plus **136**, 311 (2021). DOI: 10.1140/epjp/s13360-021-01271-9

[46] S. Upadhyay, S. Soroushfar and R. Saffari, "Perturbed thermodynamics and thermodynamic geometry of a static black hole in $f(R)$ gravity", Modern Physics Letters A **36**, 2150212 (2021). DOI: 10.1142/S0217732321502126

[47] M. Banados, C. Teitelboim and J. Zanelli, "The Black Hole in Three Dimensional Space Time", Phys. Rev. Lett. **69**, 1849-1851 (1992). DOI: 10.1103/PhysRevLett.69.1849

[48] B. Pourhassan, M. Faizal, "Thermal Fluctuations in a Charged AdS Black Hole", EPL 111, 40006 (2015). DOI: 10.1209/0295-5075/111/40006