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Regular article

Closed String Field Theory on a Double Layer

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Abstract. The holography principle relates the quantum gravity in the bulk, described by closed string, and the gauge theory, described by open string on the boundary with certain asymptotic conditions. Thus, it is important to understand intimate relations between open string theory and closed string theory: In the present work we propose a cubic closed string field theory, introducing a double layer to describe the closed string world-sheet as an extension of the open string world-sheet of the Witten's cubic open string. We mapped the closed string world-sheet onto the complex plane, of which the lower half plane is completely covered by the extended part of the string world-sheet. Using the Green's function on the complex plane, evaluated the Polyakov string path integral, from which we extracted the Neumann functions and the vertex operators.

Keywords: String field theory; Vertex operators; Neumann functions

1 Introduction

Since the seminal paper of Witten [1, 2], the covariant string field theory has become one of the main pillars in string theory study [3, 4, 5, 6, 7, 8, 9, 10, 11]. It is the Becchi-Rouet-Stora-Tyutin (BRST) symmetry [12, 13, 14, 15, 16, 17, 18] that enabled him to construct the relativistic gauge covariant interacting string field theory. However, since the cubic string field theory is defined on a two dimensional surface with conic singularity, it is difficult to evaluate scattering amplitudes of order higher than three. This difficulty may be dealt with as studies have shown that the Witten's cubic string field theory is continuously deformable [19, 20, 21, 22, 23] to a covariant cubic string field theory in the proper-time gauge [24] where the world-sheet is planar.

The world-sheet of Witten's cubic open string field theory, which is a conical surface, may be mapped on a disk by a well-defined conformal transformation: The spatial coordinate on the two dimensional world-sheet is restricted to $[0, \pi]$ and the world trajectories of open string end points form a unit circle. By extending the range of the spatial coordinates η_r, $r = 1, 2, 3$ to $[-\pi, \pi]$, then the image of the string stretches out the unit disk and forms a closed line, if a periodic condition is imposed; describing a closed string. A double layer may be introduced for the local patch, so that the extended parts may be put on the second layer. When the unit disk is further mapped to a complex z-plane, it becomes clearer. The unit disk can be mapped onto the upper half complex z-plane by a simple conformal transformation. Applying the same conformal transformation, the extended part of the string world-sheet is now mapped onto the lower half complex z-plane so that the entire complex z-plane is covered by the world-sheet of three interacting closed strings.

2 Closed String Theory on a Double Layer

The Witten's cubic open string field theory is described by a BRST invariant action, which is given as

$$S_{\rm open} = \int {\rm tr}\left(\Psi * Q\Psi + \frac{2g}{3}\Psi * \Psi * \Psi\right), \tag{1}$$

where the star product between the string field operators is defined as

$$\begin{aligned}(\Psi_1 * \Psi_2)[X(\sigma)] &= \int \prod_{\frac{\pi}{2}\le\sigma\le\pi} DX^{(1)}(\sigma) \prod_{0\le\sigma\le\frac{\pi}{2}} DX^{(2)}(\sigma) \\ &\prod_{\frac{\pi}{2}\le\sigma\le\pi} \delta\left[X^{(1)}(\sigma) - X^{(2)}(\pi-\sigma)\right] \Psi_1[X^{(1)}(\sigma)]\Psi_2[X^{(2)}(\sigma)]. \end{aligned}\tag{2}$$

The star product is associative and the string field action is invariant under the BRST gauge transformation

$$\delta\Psi = Q * \epsilon + \Psi * \epsilon - \epsilon * \Psi. \tag{3}$$

In terms of the normal modes, the open string coordinates X^μ, may be expanded as

$$X^\mu(\sigma) = x^\mu + 2\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}x^\mu_n \cos(n\sigma), \quad 0 \le \sigma \le \pi, \quad \mu = 0, 1, \dots, d, \tag{4}$$

and the string field Ψ may carry the group indices

$$\Psi[X] = \frac{1}{\sqrt{2}}\Psi^0[X] + \Psi^a[X]T^a, \quad a = 1, \dots, N^2 - 1, \tag{5}$$

where Ψ^0 is the $U(1)$ component and Ψ^a, $a = 1, \dots, N^2 - 1$ are the $SU(N)$ components. If we introduced three local coordinate patches, which describe free propagation of three open strings, we might have depict the string world-sheet of three-open-string interaction as by Fig. 1.

The cubic string vertex operator, which is the Fock space representation of three open string interactions, is obtained by mapping the string world-sheet onto the upper half of complex plane, where the Green's function is well-known. The mapping, called the

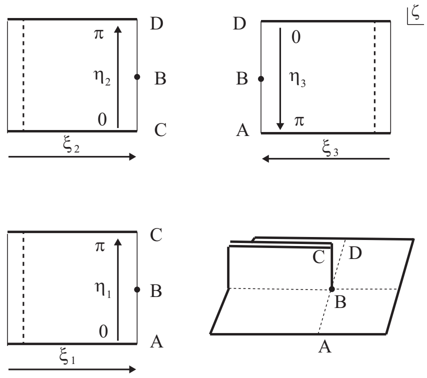

Figure 1: Local coordinate patches on the world-sheet of three-string scattering.

Schwarz-Christoffel (SC) transformation, from the world-sheet onto the upper half plane is constructed in two steps: First, we map the world-sheet onto a unit disk by a conformal transformation:

$$\begin{aligned} \omega_1 &= e^{\frac{2\pi i}{3}}\left(\frac{1+ie^{\zeta_1}}{1-ie^{\zeta_1}}\right)^{\frac{2}{3}}, \\ \omega_2 &= \left(\frac{1+ie^{\zeta_2}}{1-ie^{\zeta_2}}\right)^{\frac{2}{3}}, \\ \omega_3 &= e^{-\frac{2\pi i}{3}}\left(\frac{1+ie^{\zeta_3}}{1-ie^{\zeta_3}}\right)^{\frac{2}{3}}, \end{aligned} \tag{6}$$

where the local coordinates on the three patches are given as $\zeta_r = \xi_r + i\eta_r$, $r = 1, 2, 3$. Fig. 1 depicts three-open-string world-sheet mapped on a unit disk (ω-plane). The interaction point B, where all the three open strings meet, is mapped to the origin of the disk and the external strings are located at $e^{\frac{2\pi i}{3}}$, 1, $e^{-\frac{2\pi i}{3}}$ respectively. Then, each local coordinate patch on the unit disk is mapped onto the upper half plane by the following conformal transformation:

$$z = -i\,\frac{\omega_r - 1}{\omega_r + 1}, \quad \frac{\pi}{3} \le \arg\,\omega_r \le \frac{2\pi}{3}, \quad r = 1,\ 2,\ 3. \tag{7}$$

The three-open-string world-sheet mapped on the z-plane is described by Fig. 1. The external strings are mapped to three points on the real line

$$Z_1 = \sqrt{3}, \quad Z_2 = 0, \quad Z_3 = -\sqrt{3}. \tag{8}$$

Having mapped the world-sheet of three strings onto the upper half plane, we can adopt the well-known Green's functions on the upper half plane,

$$G_N(z, z') = \ln|z - z'| + \ln|z - z'^*|, \quad \text{for Neumann boundary condition.} \tag{9}$$

The main objective of the present work is to extend this cubic BRST invariant action of open string to that of the closed string, and to construct the three-closed-string vertex operator: The closed string coordinates X are decomposed into left-movers and right-movers

$$X(\tau, \sigma) = X_L(\tau + \sigma) + X_R(\tau - \sigma), \quad -\pi \le \sigma \le \pi \tag{10}$$

whose normal mode expansions are given as

$$X_L(\tau,\sigma) = x_L + \sqrt{\frac{\alpha'}{2}}\, p_L(\tau+\sigma) + i\sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \frac{1}{n}\alpha_n e^{-in(\tau+\sigma)}, \quad (11a)$$

$$X_R(\tau,\sigma) = x_R + \sqrt{\frac{\alpha'}{2}}\, p_R(\tau-\sigma) + i\sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \frac{1}{n}\tilde{\alpha}_n e^{-in(\tau-\sigma)}, \quad (11b)$$

where $x = x_L + x_R$.

Fig. 2 depicts world-sheet of cubic open string interaction where the ranges of η_r, $r = 1, 2, 3$ are limited to $[0, \pi]$. To describe a closed string, we may extend their ranges to $[-\pi, \pi]$,

$$|\omega_r| = \left| \frac{1 + e^{2\xi_r} - 2e^{\xi_r} \sin\eta_r}{1 + e^{2\xi_r} + 2e^{\xi_r} \sin\eta_r} \right|^{\frac{1}{3}}, \quad r = 1, 2, 3. \quad (12)$$

By extending the ranges, the images of the extended parts of string, (for $-\pi \leq \eta_r \leq 0$) stretch out of the unit disk and make closed curves (See Fig 3). Further, if we map the extended parts of the world-sheet onto the complex plane, they precisely fill the lower half complex plane. Therefore, it may be possible to describe the cubic closed string interaction by simply extending the ranges of η_r with a periodic boundary condition. We may imagine that the string world-sheet has a double layer and the extended parts are on the second layer as depicted in Fig. 4. We explored this possibility to study the cubic closed string throughout this work.

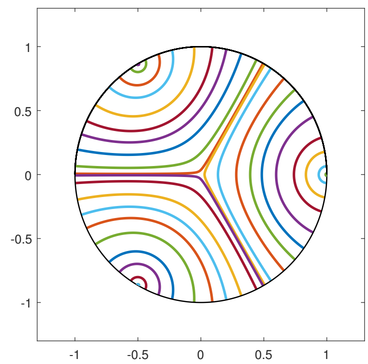

Figure 2: Interaction of the three open strings on ω-complex plane.

3 Construction of Cubic Closed String Vertex

The overlapping condition for three-closed-string interaction is expressed as

$$X^{(r)}(\sigma) = X^{(r+1)}(\pi - \sigma), \quad \text{for } \frac{\pi}{2} \leq \sigma \leq \pi, \text{ and } -\pi \leq \sigma \leq -\frac{\pi}{2}, \quad r = 1, 2, 3. \quad (13)$$

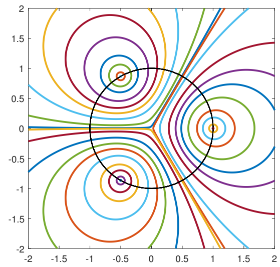

Figure 3: World-sheet of interacting three closed strings on ω-complex plane.

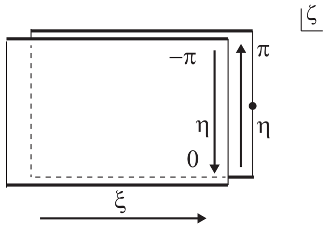

Figure 4: String world-sheet with a Double Layer

where we identify $X^{(r+3)} = X^{(r)}$. In accordance with it, the star product between two string field operators may be defined as

$$\begin{aligned}(\Psi_1 * \Psi_2)\,[X(\sigma)]_{\text{closed}} &= \int \prod_{-\pi\le\sigma\le-\frac{\pi}{2}} \prod_{\frac{\pi}{2}\le\sigma\le\pi} DX^{(1)}(\sigma) \prod_{0\le\sigma\le\frac{\pi}{2}} \prod_{-\frac{\pi}{2}\le\sigma\le 0} DX^{(2)}(\sigma) \\ &\quad \prod_{-\pi\le\sigma\le-\frac{\pi}{2}} \prod_{\frac{\pi}{2}\le\sigma\le\pi} \delta\left[X^{(1)}(\sigma) - X^{(2)}(\pi-\sigma)\right] \\ &\quad \Psi_1[X^{(1)}(\sigma)]\Psi_2[X^{(2)}(\sigma)]. \end{aligned} \tag{14}$$

It would be instructive to compare it with the previous star product of open string defined by Witten,

$$\begin{aligned}(\Psi_1 * \Psi_2)\,[X(\sigma)]_{\text{open}} &= \int \prod_{\frac{\pi}{2}\le\sigma\le\pi} DX^{(1)}(\sigma) \prod_{0\le\sigma\le\frac{\pi}{2}} DX^{(2)}(\sigma) \\ &\quad \prod_{\frac{\pi}{2}\le\sigma\le\pi} \delta\left[X^{(1)}(\sigma) - X^{(2)}(\pi-\sigma)\right] \Psi_1[X^{(1)}(\sigma)]\Psi_2[X^{(2)}(\sigma)]. \end{aligned} \tag{15}$$

The star product of three closed strings was illustrated in Fig. 5. It may be instructive to compare this star product of closed string with other previously reported star product of Witten open string presented in Fig. 6. We recognize that the region inside the dotted box of Fig. 5 of closed string corresponds to the star product of the open string.

Now, being equipped with the proper star product for the closed strings, we may write the BRST gauge invariant action for closed string:

$$S_{\text{Closed}} = \int \text{tr}\left(\Psi * \left(Q + \tilde{Q}\right)\Psi + \frac{2g}{3}\Psi * \Psi * \Psi\right), \tag{16}$$

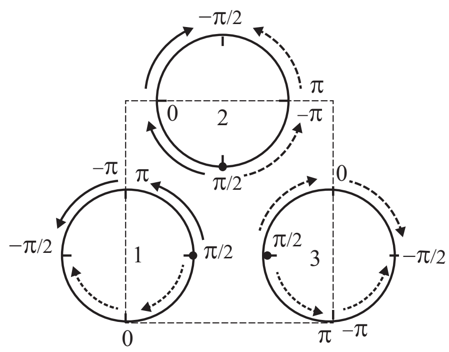

Figure 5: Star product of three closed strings.

which is invariant under the (extended) BRST gauge transformation

$$\delta\Psi = Q * \epsilon + \Psi * \epsilon - \epsilon * \Psi + \tilde{Q} * \tilde{\epsilon} + \Psi * \tilde{\epsilon} - \tilde{\epsilon} * \Psi. \tag{17}$$

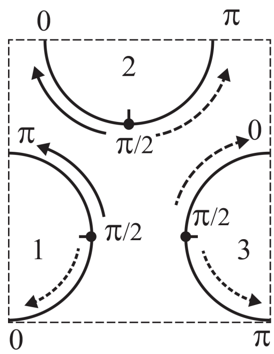

Figure 6: Star product of three open strings.

Mapping from the world-sheet coordinates, $\zeta_r = \xi_r + i\eta_r$, $r = 1, 2, 3$ to the disk is given as follows:

$$\omega_1 = e^{\frac{2\pi i}{3}} \left(\frac{1 + ie^{\zeta_1}}{1 - ie^{\zeta_1}}\right)^{\frac{2}{3}}, \tag{18a}$$

$$\omega_2 = \left(\frac{1 + ie^{\zeta_2}}{1 - ie^{\zeta_2}}\right)^{\frac{2}{3}}, \tag{18b}$$

$$\omega_3 = e^{-\frac{2\pi i}{3}} \left(\frac{1 + ie^{\zeta_3}}{1 - ie^{\zeta_3}}\right)^{\frac{2}{3}}. \tag{18c}$$

A pair of two string patches meet along the line ξ_r, $r = 1, 2, 3$ and the three string patches meet at $(\xi_r, \eta_r) = (0, \frac{\pi}{2})$, $r = 1, 2, 3$. If we choose the domains of η_r, $r = 1, 2, 3$, as $[0, \pi]$, ω, $r = 1, 2, 3$ describe the world-sheet of three open strings as given in Fig. 2. External string are at the region where $\xi_r \to -\infty$: $e^{\frac{2\pi i}{3}}$, 1, $e^{-\frac{2\pi i}{3}}$ on ω-complex plane. The images of the string world-sheet remain inside of the unit disk,

$$|\omega_r| \le 1 \quad \text{for} \;\; 0 \le \eta_r \le \pi, \;\; r = 1, 2, 3. \tag{19}$$

However, if the domains of η_r, $r = 1, 2, 3$ are extended to $[-\pi, \pi]$, their images stretch out the unit disk as earlier shown in Fig. 3 :

$$|\omega_r| \ge 1 \quad \text{for} \;\; -\pi \le \eta_r \le 0, \;\; r = 1, 2, 3. \tag{20}$$

The conformal transformation we employed before to map the complex ω-plane to the complex z-plane:

$$z_r = -i\,\frac{\omega_r - 1}{\omega_r + 1}, \quad r = 1,\; 2,\; 3. \tag{21}$$

The external strings are now located on the real line,

$$Z_1 = \sqrt{3}, \quad Z_2 = 0, \quad Z_3 = -\sqrt{3}. \tag{22}$$

Each local coordinate patch is mapped onto the upper half plane for $0 \le \eta_r \le \pi$, $r = 1, 2, 3$ (open string, see Fig. 7 and onto the lower half plane for $-\pi \le \eta_r \le 0$, $r = 1, 2, 3$). Thus, string world-sheet covers the entire complex z-plane for closed string (Fig. 8)

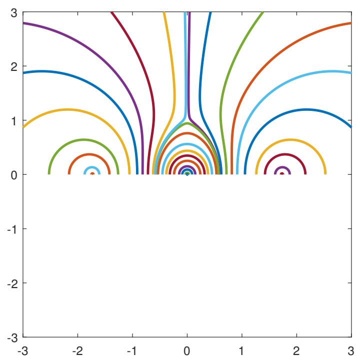

Figure 7: Cubic open string world-sheet on complex z-plane.

The relations between the local coordinates and the z-complex coordinates may be manifested through expansions of $e^{-\zeta_r}$, $r = 1, 2, 3$ near $z_r = Z_r$ (at the asymptotic region),

$$\begin{aligned}
e^{-\zeta_r} &= \frac{a_r}{(z_r - Z_r)} + \sum_{n=0} c^{(r)}_n (z_r - Z_r)^n, \qquad (23)\\
a_1 &= \frac{8}{3}, \quad a_2 = \frac{2}{3}, \quad a_3 = \frac{8}{3},\\
c^{(1)}_0 &= \frac{2\sqrt{3}}{3}, \quad c^{(1)}_1 = -\frac{5}{72}, \quad c^{(1)}_2 = \frac{5\sqrt{3}}{288}\\
c^{(2)}_0 &= 0, \quad c^{(2)}_1 = -\frac{5}{18}, \quad c^{(2)}_2 = 0,\\
c^{(3)}_0 &= -\frac{2\sqrt{3}}{3}, \quad c^{(3)}_1 = -\frac{5}{72}, \quad c^{(3)}_2 = -\frac{5\sqrt{3}}{288}.
\end{aligned}$$

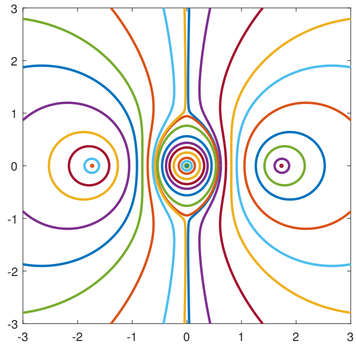

Figure 8: Cubic closed string world-sheet on complex z-plane.

These explicit expressions of expansions are useful when we calculating the Neumann functions.

4 Conclusions and Discussions

The construction of BRST invariant covariant closed string field theory has been one of the most outstanding problems in string theory. In this study, we directly extended the Witten's cubic open string field theory, and introduced double layers to open string world-sheets. The string world trajectory of open string can be mapped onto a unit disk on the complex ω-plane: The end points of the open strings form the boundary of the unit disk. The result showed that if we extend the range of spatial coordinate from $[0, \pi]$ for open string to $[-\pi, \pi]$ with periodic boundary condition for closed string, the world trajectory of string makes a closed curve in the complex ω-plane. Therefore, it is possible to describe the cubic closed string field theory, which is BRST invariant, extending the Witten's open string field theory. When we mapped the string world-sheet further on the complex z-plane, we confirmed that the extended string theory correctly describes closed string: The complex z-plane where the upper half only is covered by the open string is now fully covered and the complex z-plane is symmetric under reflection. It is not difficult to identify the overlapping condition in terms of the spatial string coordinate σ, which leads us to the extended string world-sheet of closed string. The three-closed-string vertex was constructed and found to respect the KLT structure: Their Neumann functions are completely factorized into those of corresponding open string. This is a manifestation of the KLT relation [25] of the first quantized theory at the level of the second quantized theory. If we are restricted to the spin two sector, this factorization may lead us to the recent works on the double copy theory [26, 27]. This suggests that the open-closed string field theory may be the best framework to explore the essence of the double copy theory.

Data Availability

No data are available.

Conflicts of Interest

The author declares that there is no conflict of interest.

Ethical Considerations

The author has diligently addressed ethical concerns, such as informed consent, plagiarism, data fabrication, misconduct, falsification, double publication, redundancy, and other related matters.

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