Journal of Holography Applications in Physics
Volume 3, Issue 1, Winter 2023, 57-70
©Available online at http://jhap.du.ac.ir
DOI:10.22128/jhap.2023.651.1038
Online ISSN: 2783-3518

Journal of Holography Applications in Physics

Regular article

Bouncing universe for deformed non-minimally coupled inflation model

Sudhaker Upadhyay
Department of Physics, K.L.S. College, Magadh University, Nawada-805110, Bihar, India.
Email: sudhakerupadhyay@gmail.com

Received: February 02, 2023; Revised: March 01, 2023; Accepted: March 10, 2023.

Abstract. In this paper, we consider a non-minimally coupled gravity model to study the bouncing universe. The holographic principle has various effects on the bouncing universe. We choose some suitable new variables and achieve the new Hamiltonian and Lagrangian which have harmonic oscillator forms. The corresponding Lagrangian is deformed by non-commutative geometry. In order to have a solution for the bouncing universe we specify the potential in the equation state. In that case, we draw the equation of state in terms of time and show that the equation of state crosses -1. Such bouncing behavior leads us to apply some conditions on θ and β from non-commutative geometry. Here, also we can check the stability of the system due to the deformation of the non-minimally coupled to the gravity model. In order to examine the stability of the system we obtain the variation of pressure with respect to density energy. Also, we draw the variation of pressure with respect to energy density and show the condition of stability.

Keywords: Non-Commutative Geometry; Non-Minimally Coupled Gravity; Bouncing Universe; Dark Energy.

1 Introduction

The moving from the accelerated collapse era to the expanding era without a singularity lead us to have a bouncing universe. The bounce realization due to the holographic principle in the early universe studied recently [1]. The holographic dark energy plays a crucial role in describing the late-time universe, and found in agreement with observations [2, 3, 4, 5, 6]. In that case, many cosmological solutions for the bouncing universe tested in the last decades [7]. The subject of bouncing was investigated in several models as brane-world scenarios of cosmology. Here, if the bulk space is taken to be a charged AdS black hole, then we have the bouncing universe [8]. In this case, brane makes a contracting to expanding phase transition. As we know from the singularity theorem in four dimensions, such a bounce cannot occur because of a certain energy condition. Hence, in order to produce the bouncing universe, it is necessary that the bulk geometry must contribute a negative energy density to the effective stress-energy tensor of the brane configuration [9]. As we know, several authors give further developments for these bouncing brane-world scenarios [10, 11, 12, 13, 14, 15, 16]. Now, it is believed that the universe is in an accelerated expansion phase. For this cosmic acceleration, we have seen some observations from type Ia supernovae [17, 18, 19] associated with Large Scale Structure (LSS) [20, 21] and Cosmic Microwave Background (CMB) anisotropy [22, 23]. On the other hand, the dark energy with negative pressure lead us to this cosmic acceleration. In order to have such phenomena, the theories trying to modify Einstein equations. On the other hand, in modern cosmology, the dark energy can be explained by the equation of state parameter as $\omega = \frac{p}{\rho}$, where p and ρ are pressure and energy density, respectively. In that case, for the accelerated expansion the equation of state dark energy must be satisfied by $\omega < -\frac{1}{3}$.

As we know, there are two puzzles from cosmological constant as dark energy. One of them is that, the cosmological constant is about 120 orders of magnitude smaller than its natural expectation. The other is the cosmological coincidence problem. In order to solve these problems, various models of dark energy have been introduced such as quintessence [24, 25, 26] or k-essence [27], scaler-field dark energy model including tachyon [28, 29], ghost condensate [30, 31] and quintom [32, 33, 34, 35, 36, 37, 38, 39]. Also, there is a unified dark energy-dark matter model based on the Chaplygin gas equation of state and its extensions [40, 41, 42, 43, 44, 45, 46, 47, 48, 49]. We note here the analysis of the dark energy properties from recent observation mildly favors the model with ω crossing -1 in the near past. In that case there are other proposals to describe the accelerating expansion of the universe including interacting dark energy models [50, 51], brane-world models [52, 53], and holographic dark energy models [54, 55, 56, 57]. The difficulty of realizing ω crossing over -1 in quintessence and phantom-like models for the first time has been shown by Refs. [33, 36, 37]. Because when the equation of state ω approaches -1, the dark energy perturbation would be divergent [58, 59, 60, 61]. The most important point here is that the quintom scenario of dark energy very well understands the nature of dark energy with ω across -1 [62, 63].

All the above physical information gives us the motivation to work non-minimally coupled gravity with deformed phase space. As we know, such non-minimal coupling is very successful [64, 65]. A simple example of such a model is the Higgs boson which acts as the inflation field in the form of $\xi\varphi^2 R$. In that case, φ and R are Higgs field and Ricci scaler respectively. Here, we are going to study the dark energy and bouncing universe ω with the usual metric formalism in a non-minimally coupling model. We deformed the non-minimally coupling to the gravity model. Then, we compare the results of the deformed and non-deformed corresponding models to each other. Here, we note that the bouncing universe and dark energy in non-minimally coupled gravity with deformed phase space lead us to have some new results. Here, another deformed approach will be Finsler geometry. In the future one

can realize the relation between the above deformed method, non-commutative geometry, and Finsler geometry.
So, in section 2 we introduce the non-minimal coupling to the gravity model. Also, we take Friedmann-Roberson-Walker (FRW) background and write the corresponding Lagrangian. In section 3, first of all we assume $\xi = \frac{-1}{6}$, $\kappa = 0$ and write the corresponding Lagrangian. In order to deform such a model we chose some changes in variables. Also, again we obtain the Lagrangian and Hamiltonian of the non-minimal model by new variables. The corresponding Hamiltonian will be a form of the harmonic oscillator, and it is useful to investigate gravitational theories. On the other hand, we review some non-commutative (NC) geometries [66]. We transform the classical phase space variables of x_i, y_i, P_{x_i} and P_{y_i}. Also, we achieve some deformed algebra for the new variables which is satisfied by anti-commutative relations. Here, the NC geometry helps us to write the Hamiltonian and Lagrangian for the non-minimal coupling to gravity model with the new variable. In section 4, we have some cosmology solutions for the deformed non-minimally model and investigate the dark energy and bouncing universe. The equation of state plays important role in the bouncing universe. Here also, we have some figures for the equation of state cross -1. In the last section, we have some results and suggestions for the deformed model in the bouncing universe.

2 Lagrangian of non-minimal coupling to gravity

Now, we consider the general model of inflation with non-minimal coupling which has the following action,

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}(M^2 + \xi\varphi^2)g^{\mu\nu}R_{\mu\nu} - \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V(\varphi)\right], \tag{1}$$

where M, $R_{\mu\nu}$, ξ, and φ are mass scales, Ricci tensor, dimensionless coupling constant, and the inflation field respectively. If we back to the Einstein frame, we need the following conformal transformation,

$$g_{\mu\nu} \to \Omega(\varphi)^{-1}g_{\mu\nu}, \tag{2}$$

where

$$\Omega(\varphi) = \frac{M^2 + \xi\varphi^2}{M_p^2}. \tag{3}$$

We do not need such transformation because we take a general form of action (1), and employ the FRW metric background with $\kappa = 0$. Here, we try first to study the bouncing universe and dark energy to the usual non-minimal coupling model (1). In order to have some cosmology solutions, one can write the corresponding Lagrangian with the following equation,

$$\mathcal{L} = 3M^2(\kappa a - a\dot{a}^2) + 3\xi(-a\dot{a}^2\varphi^2 - 2a^2\dot{a}\varphi\dot{\varphi} + \kappa a\varphi^2) + \frac{1}{2}a^3\dot{\varphi}^2 - V(\varphi)a^3, \tag{4}$$

where we used,

$$\begin{aligned}\sqrt{-g} &= a^3,\\ R &= 6(\frac{\ddot{a}}{a} + \frac{\kappa + \dot{a}^2}{a^2}).\end{aligned} \tag{5}$$

The equation (4) is the Lagrangian of non-minimal coupling to gravity. In the next section, we are going to take the above Lagrangian and apply some transformations to the phase space variables.

3 Deformation of non-minimal coupling to gravity model

Now, we take the equation (4) and assume $\xi = \frac{-1}{6}$ and $\kappa = 0$, to rewrite the Lagrangian as following,

$$\mathcal{L} = 3M^2(-a\dot{a}^2) + \frac{1}{2}(a\dot{a}^2\varphi^2 + 2a^2\dot{a}\varphi\dot{\varphi} + a^3\dot{\varphi}^2) - V(\varphi)a^3. \quad (6)$$

Here, we choose the following change of variables,

$$\begin{array}{lll} x_1 = a\sinh(\frac{\varphi a}{\sqrt{2}}), & x_2 = a\cosh\sqrt{3a}M, & x_3 = \sinh(\sqrt{V(\varphi)}a^{\frac{3}{2}}), \\ y_1 = a\cosh(\frac{\varphi a}{\sqrt{2}}), & y_2 = a\sinh\sqrt{3a}M, & y_3 = \cosh(\sqrt{V(\varphi)}a^{\frac{3}{2}}), \end{array} \quad (7)$$

and

$$\begin{array}{l} x_4 + y_4 = 1 + \sqrt{V(\varphi)}a^{\frac{3}{2}}, \\ x_4 - y_4 = 1 - \sqrt{V(\varphi)}a^{\frac{3}{2}}. \end{array} \quad (8)$$

We put the equations (7) and (8) into the equation (6) and obtain the following Lagrangian,

$$\mathcal{L} = \sum_{i=1}^{4}((\dot{x}_i^2 - \dot{y}_i^2) + (x_i^2 - y_i^2)). \quad (9)$$

By using the equation $\mathcal{H} = P_{x_i}\dot{x}_i + P_{y_i}\dot{y}_i - \mathcal{L}$ one can write the corresponding Hamiltonian as,

$$\mathcal{H} = \frac{1}{4}\sum_{i=1}^{4}((P_{x_i}^2 - P_{y_i}^2) + \omega_i^2(x_i^2 - y_i^2)), \quad (10)$$

where

$$\begin{array}{l} P_{x_i} = \frac{\partial\mathcal{L}}{\partial\dot{x}_i} = 2\dot{x}_i, \\ P_{y_i} = \frac{\partial\mathcal{L}}{\partial\dot{y}_i} = -2\dot{y}_i. \end{array} \quad (11)$$

It is obvious that the Hamiltonian (10) has a harmonic oscillator form with the following frequency,

$$\omega_i^2 = -4. \quad (12)$$

Here, we note that the Hamiltonian (10) is very useful to obtain the solution of any cosmological system. Now, we are going to deform the non-minimal coupling to gravity system. In order to do such a process, we need to give some transformations to the classical phase space. So, at the first, we review such transformation as an NC geometry. Therefore, before applying NC geometry to the new Hamiltonian (10), we give some explanation of such geometry. As we know, in commutative space we have the following Poisson brackets,

$$\{x_i, x_j\} = 0, \quad \{P_{x_i}, P_{x_j}\} = 0, \quad \{x_i, P_{x_j}\} = \delta_{ij}, \quad (13)$$

where $i = 1, 2$. But in NC geometry the Moyal product plays an important role in the deformation phase space variables. Also, the quantum effects can be dissolved by the following Moyal brackets,

$$\{f, g\}_\alpha = f \star_\alpha g - g \star_\alpha f. \quad (14)$$

The brackets (14) are based on the Moyal product which is given by,

$$(f \star_\alpha g)(x) = exp[\frac{1}{2}\alpha^{ab}\partial_a^{(1)}\partial_b^{(2)}]f(x_1)g(x_2)|_{x_1=x_2=x}. \quad (15)$$

By using the equations (14) and (15), one can find the following relation,

$$\{x_i, x_j\}_\alpha = \theta_{ij}, \quad \{x_i, P_j\}_\alpha = \delta_{ij} + \sigma_{ij}, \quad \{P_i, P_j\} = \beta_{ij}. \tag{16}$$

So, in that case, the transformations of the classical phase space variables are given by,

$$\hat{x}_i = x_i + \frac{\theta}{2}P_{y_i}, \quad \hat{y}_i = y_i - \frac{\theta}{2}P_{x_i}, \quad \hat{P}_{x_i} = P_{x_i} - \frac{\beta}{2}y_i, \quad \hat{P}_{y_i} = P_{y_i} + \frac{\beta}{2}x_i, \tag{17}$$

and

$$\{\hat{y}, \hat{x}\} = \theta, \quad \{\hat{x}, \hat{P}_x\} = \{\hat{y}, \hat{P}_y\} = 1 + \sigma, \quad \{\hat{P}_y, \hat{P}_x\} = \beta, \tag{18}$$

where $\sigma = \frac{\beta\theta}{2}$. In order to construct the deformed Hamiltonian for the non-minimal coupling to gravity model, we apply the new variables (17) on the Hamiltonian (10). Hence, the Hamiltonian (10) in deformed form will be as,

$$\hat{\mathcal{H}} = \frac{1}{4}\sum_{i=1}^{4}((P_{x_i}^2 - P_{y_i}^2) - \gamma_i^2(y_i P_{x_i} + x_i P_{y_i}) + \tilde{\omega}_i^2(x_i^2 - y_i^2)), \tag{19}$$

where ω_i^2 can be changed by the following equation,

$$\tilde{\omega}_i^2 = \frac{{\omega_i}^2 - \frac{\beta^2}{4}}{1 - {\omega_i}^2\frac{\theta^2}{4}}, \quad {\gamma_i}^2 = \frac{\beta - {\omega_i}^2\theta}{1 - {\omega_i}^2\frac{\theta^2}{4}}. \tag{20}$$

Here, two parameters of NC geometry θ and β play an important role in the deformation of theory. When we investigate the bouncing universe, we will explain the role of two parameters θ and β with some figures. The above deformed Hamiltonian lead us to have a new Lagrangian which is a deformed form of the original Lagrangian of non-minimal coupling to the gravity model.
Now, for the corresponding model, we arrange the deformed Lagrangian as,

$$\hat{\mathcal{L}} = \frac{1}{4}\sum_{i=1}^{4}((P_{x_i}^2 - P_{y_i}^2) - \hat{\gamma}_i^2(y_i P_{x_i} + x_i P_{y_i}) - \hat{\omega}_i^2(x_i^2 - y_i^2)), \tag{21}$$

where

$$\hat{\omega}_i^2 = \frac{{\omega_i}^2 + \frac{\beta^2}{4}}{1 + {\omega_i}^2\frac{\theta^2}{4}}, \quad \hat{\gamma}_i^2 = \frac{\beta + {\omega_i}^2\theta}{1 + {\omega_i}^2\frac{\theta^2}{4}}. \tag{22}$$

We use the equations (11) and (21) to obtain the deformed Lagrangian with respect to a, $\dot{a}$, φ, $\dot{\varphi}$ and $V(\varphi)$ which is given by,

$$\hat{\mathcal{L}} = -3M^2 a\dot{a}^2 + \frac{1}{2}(a\dot{a}^2\varphi^2 + 2a^2\dot{a}\varphi\dot{\varphi} + a^3\dot{\varphi}^2) - \lambda_1^2 V(\varphi)a^3 + \lambda_2^2(\frac{\dot{\varphi}a^3}{\sqrt{2}} + \frac{a^2\dot{a}\varphi}{\sqrt{2}} - \sqrt{3}M\dot{a}a^{\frac{3}{2}}), \tag{23}$$

where

$$\lambda_1^2 = \frac{1 - \frac{\beta^2}{16}}{1 + \theta^2}, \quad \lambda_2^2 = \frac{2\theta - \frac{1}{2}\beta}{1 - \theta^2}. \tag{24}$$

The above Lagrangian is the deformed shape of the original Lagrangian given by the equation (6). So the above deformed Lagrangian shows us how the NC parameters affect the corresponding theory. We note here, for the non-minimal model there were several papers investigating the bouncing universe, and they have shown that the equation of state crosses -1. But, we use the deformed non-minimal coupling to gravity model and obtain the equation of state. The corresponding equation of state for the deformed case leads us to investigate the bouncing universe.

4 Bouncing universe of deformed non-minimal coupling to gravity model

According to the conservation of energy,

$$E_{\hat{\mathcal{L}}} = P_a \dot{a} + P_\varphi \dot{\varphi} - \hat{\mathcal{L}} \tag{25}$$

where

$$P_a = \frac{\partial \hat{\mathcal{L}}}{\partial \dot{a}}, \quad P_\varphi = \frac{\partial \hat{\mathcal{L}}}{\partial \dot{\varphi}} \tag{26}$$

Here, by using definition of Hubble parameter $H = \frac{\dot{a}}{a}$, and assumption $E_{\hat{\mathcal{L}}} \equiv 0$, we will arrive at,

$$H^2 = \frac{1}{6M^2}(H\varphi + \dot{\varphi})^2 + \frac{\lambda_1^2}{3M^2} V(\varphi). \tag{27}$$

The Euler-Lagrangian equation helps us to obtain the following equations,

$$\ddot{\varphi} + 3H\dot{\varphi} + (2H^2 + \dot{H})\varphi + \sqrt{2}\lambda_2^2 H + \lambda_1^2 V^{'}(\varphi) = 0, \tag{28}$$

and

$$\dot{H} = -\frac{2(H\varphi + \dot{\varphi})^2 + \sqrt{2}\lambda_2^2(H\varphi + \dot{\varphi}) + \lambda_1^2 V^{'}(\varphi)\varphi}{6M^2}. \tag{29}$$

And then, by using $\dot{H} = -\frac{1}{2}(\rho + p)$, one can rewrite the equation (29) as,

$$p + \rho = \frac{\frac{2}{3}(H\varphi + \dot{\varphi})^2 + \frac{\sqrt{2}}{3}\lambda_2^2(H\varphi + \dot{\varphi}) + \frac{\lambda_1^2}{3} V^{'}(\varphi)\varphi}{M^2}. \tag{30}$$

Then, by using $p + \rho = (1 + \omega)\rho$ and assuming $\dot{\varphi} = 0$ and $\omega = -1$, from the equation (24), and $\theta^2 = \beta^2 = 0$, one can obtain the following equation,

$$\frac{2}{3}(H\varphi)^2 + \frac{\sqrt{2}}{3}(2\theta - \frac{1}{2}\beta)(H\varphi) + \frac{1}{3}V^{'}(\varphi)\varphi = 0. \tag{31}$$

The bouncing behavior of deformed non-minimally coupled to gravity leads us to arrange the suitable potential of the system. For this reason, we need to check the necessary condition for the bouncing universe, such conditions help us to obtain the corresponding potential. As we know, during the contracting phase, the scale factor $a(t)$ is decreasing, which means $\dot{a} < 0$. For the expanding phase, we have $\dot{a} > 0$. Also, we note here for a period of time at the bouncing point and around this point, we have $\dot{a} = 0$ and $\ddot{a} > 0$ respectively. At the bouncing point, we have a transition from $H < 0$ to $H \geq 0$ [49].

In Fig. 1, we show the numerical solution of the equation (29) where $H < 0$ to $H > 0$ transition illustrated. We can see the modified parameters are necessary to have a bouncing universe. Green dotted line of Fig. 1 shows that at a special time, the Hubble expansion parameter changes the sign from plus to minus. In absence of modification, the Hubble expansion parameter is a constant.

We can also find analytical expression, in such case, the suitable bouncing universe can be obtained by,

$$H = \frac{-\frac{\sqrt{2}}{2}\lambda_2^2 \pm \sqrt{\frac{1}{2}\lambda_2^4 - 2\lambda_1^2 V'(\varphi)\varphi}}{2\varphi}. \tag{32}$$

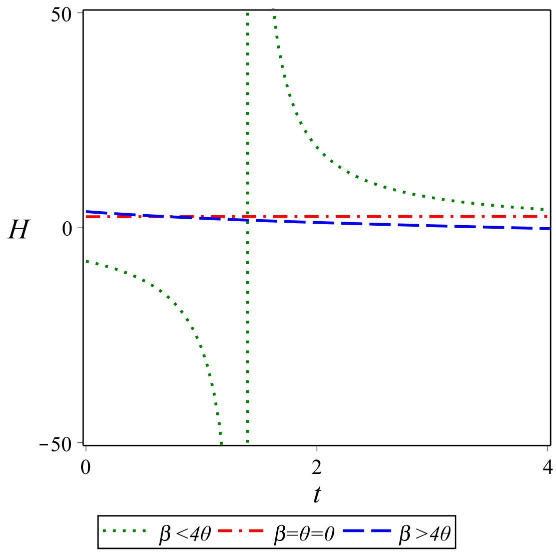

Figure 1: H with respect to t, where $M = 1$, $\varphi = 0.5$, $\theta = 2$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

In order to have a bouncing universe we have to consider three cases, which help us to obtain the exact potential for the non-minimally coupled to gravity model.
The first case: if we assume $H = 0$, one can obtain $V^{'}(\varphi) = 0$, therefor $V(\varphi) = constant$.
The second case: if the Hubble parameter H is positive, then $V^{'}(\varphi) < 0$ as a result $V(\varphi) \propto \varphi^{-n}$.
The third case: if H is negative then there is no condition for potential, therefore we suppose $V(\varphi) \propto \varphi^{-n}$. The pointed potential gives us the opportunity to study the cosmology evolution of the equation of state for the proposed model.
In order to explore the possibility of ω across -1, we have to check out $\frac{d}{dt}(p + \rho) \neq 0$ when $\omega \neq -1$,

$$\frac{d}{dt}(p+\rho) = \frac{4}{3}(H\varphi+\dot{\varphi})(\dot{H}\varphi+H\dot{\varphi}+\ddot{\varphi})+\frac{\sqrt{2}}{3}\lambda_2^2(\dot{H}\varphi+H\dot{\varphi}+\ddot{\varphi})+\frac{\lambda_1^2}{3}(V^{''}(\varphi)\dot{\varphi}\varphi+V^{'}(\varphi)\dot{\varphi}), \quad (33)$$

by assuming $\dot{\varphi} = \ddot{\varphi} = 0$, one can rewrite following equation,

$$\frac{d}{dt}(p + \rho) = \frac{4}{3}(H\varphi)(\dot{H}\varphi) + \frac{\sqrt{2}}{3}\lambda_2^2(\dot{H}\varphi), \quad (34)$$

therefore $\frac{d}{dt}(p + \rho) \neq 0$, otherwise $H\varphi = -\frac{\sqrt{2}}{4}\lambda_2^2$ (it means that $H\varphi$ is constant). This is not true because $H\varphi$ is a function of time. So, the above result proves the equation of state crosses -1, therefore we have a bouncing universe. The equation of state given by,

$$\omega = \frac{\frac{(H\varphi)^2}{6} + \frac{\sqrt{2}}{3}\lambda_2^2 H\varphi - \frac{\lambda_1^2}{\varphi^2} - \lambda_1^2 V(\varphi)}{\frac{(H\varphi)^2}{6} + \lambda_1^2 V(\varphi)}. \quad (35)$$

In the plots of Fig. 1, we draw the equation of state parameter in terms of time to see the effect of deformation parameters separately. In the left plot, we can see the effect of β while in the right plot, we can see the effect of β.

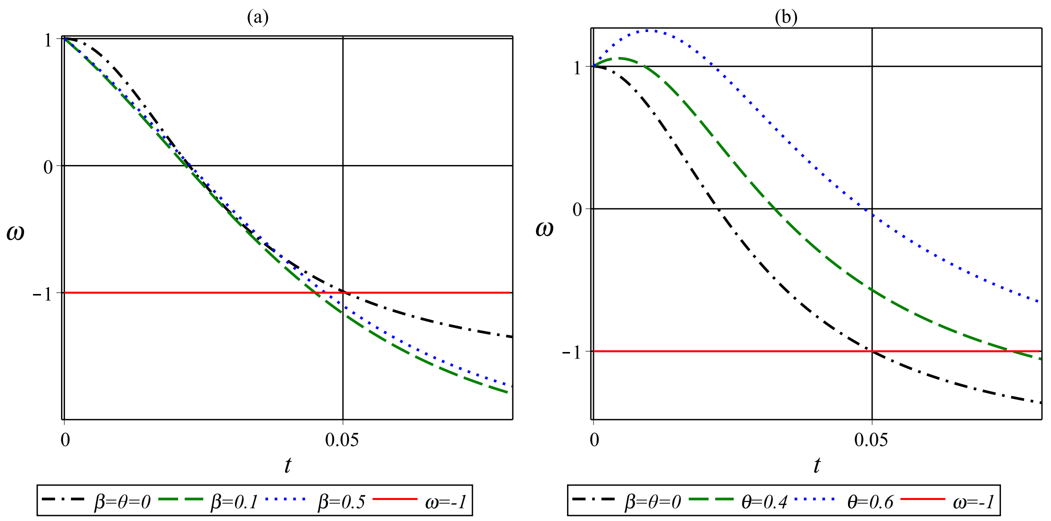

Figure 2: The variation ω with respect to t, where $V(\varphi) = \frac{V_0}{2\varphi^2}$, $\varphi = 0.5$, $V_0 = 0.5$. (a) $\theta = 2$; (b) $\beta = 0.1$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

It is illustrated that the variation with β is infinitesimal while increasing θ increases the value of the ω (see Fig. 2 (b)). We can see from Fig. 2 that the equation of state parameter yields a constant at the late time. It is shown that in presence of the deformation parameters, the equation of state parameter across -1 at an earlier time. The values of parameters are fixed according to the stability condition which is discussed below.
Here, also we can check the stability of the system due to the deformation of the non-minimally coupled to the gravity model. In order to examine the stability of the system we need to obtain the variation of pressure with respect to the density energy of the system. Hence, we investigate the stability of non-minimal coupling to gravity with deformation phase space variables. In that case, we want to consider the stability by useful functions as $C_s^2 = \frac{dp}{d\rho}$. In order to have stability with a non-minimal deformation model such function must become more than zero. So, if we want to investigate the stability of the system, the above mentioned function can be expressed by the sound speed in perfect liquid [67]. By using the equations (27), (30) and $H^2 = \frac{\rho}{3}$, one can obtain the sound speed,

$$\begin{aligned} \frac{dp}{d\rho} &= \frac{\frac{1}{3}(H\varphi + \dot{\varphi})(\dot{H}\varphi + H\dot{\varphi} + \ddot{\varphi}) + \frac{\sqrt{2}}{3}\lambda_2^2(\dot{H}\varphi + H\dot{\varphi} + \ddot{\varphi})}{(H\varphi + \dot{\varphi})(\dot{H}\varphi + H\dot{\varphi} + \ddot{\varphi}) + \lambda_1^2 V'(\varphi)\dot{\varphi}} \\ &+ \frac{\frac{\lambda_1^2}{3}(V''(\varphi)\dot{\varphi}\varphi + V'(\varphi)\dot{\varphi}) - \lambda_1^2 V'(\varphi)\dot{\varphi}}{(H\varphi + \dot{\varphi})(\dot{H}\varphi + H\dot{\varphi} + \ddot{\varphi}) + \lambda_1^2 V'(\varphi)\dot{\varphi}}. \end{aligned} \tag{36}$$

If we assume $\dot{\varphi} = \ddot{\varphi} = 0$, then $c_s^2 \equiv C_s$ lead us to have following condition,

$$\frac{1}{3} + \frac{\sqrt{2}}{3}\frac{\lambda_2^2}{H\varphi} > 0. \quad (37)$$

So, the above condition for the stability of non-minimally coupled to gravity in case of deformed phase space leads us to have $4\theta > \beta$. Here, we draw the variation of pressure with respect to the variation of energy density in terms of time ($C_s = \frac{dp}{d\rho}$) and show that how the stability corresponds to two parameters θ and β from NC geometry. It is illustrated by Fig. 3 that the non-deformed case is completely stable as well as the deformed case with condition $\beta = 4\theta$ (see the solid red line of Fig. 3).

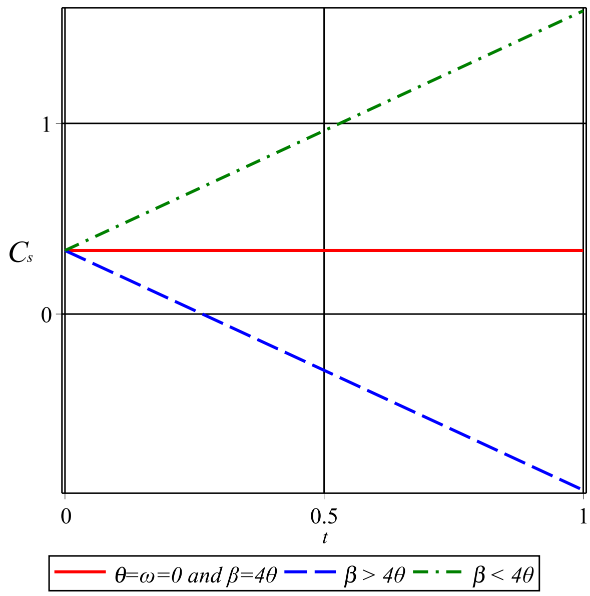

Figure 3: The variation $\frac{dp}{d\rho}$ with respect to t, where $V(\varphi) = \frac{V_0}{2\varphi^2}$, $\varphi = 0.5$, $V_0 = 0.5$, $\theta = 0.5$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

5 Conclusion

In this paper, we introduced non-minimally coupled gravity and wrote Lagrangian. Also, by using some suitable variables, we obtained new Hamiltonian and Lagrangian. In order to deform the above mentioned system we arranged the harmonic oscillator form of Hamiltonian. We used the NC geometry and deformed the new Hamiltonian and Lagrangian. Now, we take advantage of the dark energy solution and investigated the bouncing universe in a deformed non-minimal coupling to the gravity model. The condition of the bouncing universe in deformed model leads us to arrange the potential in the equation of state. In order to have a bouncing solution we drew the equation of state in terms of time. It shows that the equation of state crossing -1. Also, we obtained some suitable relation between the θ and β parameters of NC geometry to satisfy by bouncing universe. We found a special condition ($\beta = 4\theta$) where the stability of the deformed model behaves like a non-deformed case. In the case of $\beta \leq 4\theta$ the deformed model is completely stable. Otherwise, the model is initially stable which yields to unstable phase at a late time. Hence it is possible to have a stable/unstable phase transition. In order to have a solution of dark energy, we arranged the potential in the equation of state of deformed non-minimally coupled to gravity. In that case, we draw the equation of state in terms of time and shown that the equation of state crosses -1. Finally, we employed the pressure and energy density and investigated the stability of deformed non-minimally coupled to gravity. Here, we obtained the relation between θ and β in NC for approving the stability of the system. Also here, we had some figures for describing stability with some special values of parameters of NC geometry. The relation between the above deformation of the corresponding model and Finsler geometry will be important in future for the describing some parameters in physics. Also, such a relation gives us more degree of freedom to compare the obtained results of theory with data. As we know the results from Finsler geometry have a relation with non-commutative geometry. So, we hope to continue this research for the different cosmological models in the future. For instance, recently bouncing universe was considered in the contexts of generalized cosmic Chaplygin gas and variable modified Chaplygin gas, now it is interesting to consider the bouncing universe in the context of extended Chaplygin gas [68, 69, 70, 71, 72] and use the method of this paper. It is also possible to use the same method of this paper to study the holographic Barrow dark energy model [73, 74].

Data Availability

The manuscript has no associated data or the data will not be deposited.

Conflicts of Interest

The author declares that there is no conflict of interest.

Ethical Considerations

The author has diligently addressed ethical concerns, such as informed consent, plagiarism, data fabrication, misconduct, falsification, double publication, redundancy, submission, and other related matters.

Funding

This research did not receive any grant from funding agencies in the public, commercial, or non-profit sectors.

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