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Regular article

Radiation from Hayward Black Hole via Tunneling Process in Einstein-Gauss-Bonnet Gravity

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Abstract. One of the most promising theories for modified gravity is the Einstein-Gauss-Bonnet (EGB) gravity. In the framework of EGB gravity, we intend to compute the Hawking radiation of a 5-dimensional Hayward black hole with a regular center and with both inner (Cauchy) and outer (event) horizons. On the basis of particles in a dynamical geometry, we provide a brief derivation of Hawking radiation as a tunneling process. The Boltzmann factor of emission at the Hawking temperature is related to the imaginary part of the action for the classically prohibited process.

Keywords: Hawking Radiation; Hayward Black Holes; Einstein-Gauss-Bonnet Gravity.

1 Introduction

As far as we know, black holes are some singular solutions of General Relativity [1]. In fact, the existence of singularity results in signaling the breaking down of general relativity and one of the possible solutions could be an incorporation of quantum theory. This is actually the right way to finally overcome the issue of singularities. However, one step to treat a class of black hole solutions could be considering regular black holes. Bardeen presented the first regular black hole model [2], according to which there are horizons but there is no singularity. Hayward [3] proposed Bardeen-like, regular spacetimes that describe the formation of a black hole from an initial vacuum region which has a finite density and pressure, vanishing rapidly at large distances and behaving as a cosmological constant at small distances. It is a basic exact model of general relativity coupled to electrodynamics, therefore Hayward black hole has been applied recently in different studies, such as Quasinormal modes of the black holes by Lin et al. in Ref. [4], the geodesic equation of a particle by Chiba and Kimura [5], wormholes from the regular black hole [6,7] with their stability [8], black hole thermodynamics [9] and related properties [10,11], and strong deflection lensing [12]. The rotating regular Hayward's metric has been reviewed as a particle accelerator in Refs. [13,14].

In the last years, many works have been done on higher dimensions gravity to recognize the low-energy limit of string theory. The EGB gravity proposed by Lanczos [15], is a very substantial higher dimensional generalization of Einstein's gravity, and after that rediscovered by David Lovelock [16]. The examination of EGB theory could provide a vast setup to understand conceptual issues related to gravity. This theory is of the order of the field equations in the EGB theory which is no higher than two and is free of ghost. There have been a lot of attempts to obtain the black hole solution, but the first exact black hole solution in the EGB gravity was obtained by Boulware and Deser [17,18]. Since then, various authors have discussed several exact black hole solutions with their thermodynamical properties [19,20]. Different black hole solutions with matter source generalizing the BoulwareDesser solution have also been found [21,22].

A crucial question in this framework is: what will be the effect of the EGB modification on the regular black holes and their properties? To answer this question, firstly a regular solution for the EGB theory is needed. In Ref. [23] the authors obtained a 5D spherically symmetric and static Hayward-like black hole solution of the EGB gravity. It turned out that the metric purposed there is an exact black hole model of EGB having minimal coupling with nonlinear electrodynamics, thereby it is a generalization of the BoulwareDesser solution.

In this paper, we are going to study the tunneling of massless particles from the event horizon of a 5D EGB-Hayward black hole. We investigate the correlation between the emission modes and the temperature of the horizon. A lot of works has been done in literature for driving Hawking temperature in quantum tunneling formalism, see for instance Refs. [24-29] and references therein. We follow the standard procedure to fill this small gap (Hawking radiation of 5D EGB-Hayward black hole via tunneling mechanism) in the literature.

This paper is organized as follows: In section 2, we have a short review of the structure of the 5D EGB-Hayward black holes with a regular center and two horizons. In section 3, we explain the tunneling process and illustrate the temperature of the 5D EGB-Hayward black hole. Section 4 contains a summary and conclusion.

2 5D exact Hayward-like black holes in EGB gravity

General Relativity with minimal coupling with nonlinear electrodynamics leads to exact spherically symmetric regular black holes [30-35]. The two most famous exact black hole solutions are Bardeen [2] and Hayward [3] regular black holes. Here, we are interested in the Hayward-like black hole solution with a regular center in the Einstein-GaussBonnet gravity in 5D spacetime. we use the metric ansatz as [36]:

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_3^2, \tag{1}$$

where $d\Omega_3^2 = d\theta^2 + \sin^2\theta(d\phi^2 + \sin^2\theta d\psi^2)$ is the metric in the 3D hypersurface with volume V_3 and $f(r)$ is the metric function:

$$f(r) = 1 + \frac{r^2}{4\alpha}\left(1 \pm \sqrt{1 + \frac{8\alpha m}{r^4 + e^4}}\right), \tag{2}$$

where, m is a constant of integration having the relation with the Arnowitt-Deser-Misner (ADM) mass M of the black hole with the relation of

$$M = \frac{3V_3}{K_5} m\,, \tag{3}$$

where V_3 is the volume of a 3-dimensional unit sphere and the metric has a coordinate singularity at the horizon. Note that e is a constant encoding the charge of the black hole and K_5 is the 5-dimensional gravitational coupling. It is found that, when $m \neq 0 \neq \alpha$, the curvature invariants are well behaved everywhere including at $r = 0$. Thus, the 5D Hayward like black holes have no intrinsic singularity; they are regular indeed. It turns out that $g^{rr} = f(r_H) = 0$ is the only coordinate singularity that implies the presence of horizon(s). After some calculations, the location of the horizon is given by the following relation

$$r_+^2 = \frac{1}{3}\left[m - 2\alpha - \frac{2^{\frac{2}{3}}(3e^4 - (m-2\alpha)^2) + \beta^2}{2^{\frac{1}{3}}\beta}\right] \tag{4}$$

with

$$\begin{aligned} \beta &= 2(m-2\alpha)^2 + 12m\alpha^2 - 18(m+4\alpha)e^4 \\ &+ \sqrt{(2(m-2\alpha)^2 + 12m\alpha^2 - 18(m+4\alpha)e^4)^2 + 4((3e^4 - (m-2\alpha))^3} \,. \end{aligned} \tag{5}$$

We can keep the value of mass m and coupling constant α to be fixed, after that, there exists a critical value of charge (e_E), in such a way that the Cauchy (r_-) and the event (r_+) horizons coincide, i.e., $r_- = r_+$ corresponding to the extremal 5D EGB-Hayward black hole with degenerate horizon radius ($r_E = r_+$) (see Ref. [37,7]). In this way, when $e < e_E$, black hole with Cauchy and event horizons exist and if the value of charge $e > e_E$, there exists only a regular spacetime but not a black hole.

3 Hawking Radiation of the 5D EGB-Hayward Black holes with Tunneling Process

Energy conservation and dynamical geometry are two main concerns in the Parikh and Wilczek method [38,39], see also [40]. The particle tunnels semiclassically from the horizon in this configuration, where particle and antiparticle are generated with zero total energy on one side of the horizon. Through a barrier which the particle's own energy has built, this tunneling takes place. As a result, the black hole's radius and mass are reduced by an amount equal to the particle's energy. We must first build the nonsingular line element on the horizon before we can compute particle tunneling. We use the Painlev coordinate transformation to solve this problem, defining t_p as by $t_p = t - f(r)$. This will result in a new nonsingular coordinate where the metric (1) takes the following form

$$ds^2 = -f(r)dt^2 + 2\sqrt{1 - f(r)}drdt + dr^2 + r^2 d\Omega_3^2 \,. \tag{6}$$

We now intend to demonstrate how the tunneling method is utilized to calculate the black hole's temperature. A particle moves from r_{in} as an initial state to a final state in r_{out}. Tunneling calculations based on the following action, for this particle's imaginary part of the action, is as follows,

$$\operatorname{Im} S = \operatorname{Im} \int_{r_{in}}^{r_{out}} p_r dr = \operatorname{Im} \int_{r_{in}}^{r_{out}} \int_0^{p_r} dp_r' dr \tag{7}$$

where $r_{in} = r + H - \epsilon$ and $r_{out} = r + H + \epsilon$, $\tilde{\omega}$ is the particles' energy which is known as a self-interaction. Putting Hamilton equation, $dp_r = \frac{dH}{\dot{r}}$, into the Eq. (7), we find

$$I = \int_{r_{in}}^{r_{out}} \int_{M}^{M-\omega} \frac{dr}{\dot{r}} dH = \int_{0}^{\omega} \int_{r_{in}}^{r_{out}} \frac{dr}{\dot{r}} (-d\tilde{\omega}) \,. \tag{8}$$

For the massless particles' tunneling, we can use Eq. (6) to derive the light-like geodesics that correspond to the transformed metric and are obtained by

$$\dot{r} = \pm 1 - \sqrt{1 - f(r)} \,, \tag{9}$$

where + and − signs indicate the outgoing and ingoing geodesics respectively. We could obtain the imaginary part of the action by taking into account the trajectories that are departing the region (outgoing). Substituting equation (9) into equation (8) we find

$$\operatorname{Im} S = -\operatorname{Im} \int_{0}^{\omega} \int_{r_{in}}^{r_{out}} \frac{dr d\tilde{\omega}}{1 - \sqrt{1 - f(r)}} \,. \tag{10}$$

Since tunneling is a near horizon process, particles formed close to the inside of the black hole horizon are placed at $r_{in} = r_+$, and they subsequently travel through the black hole horizon to reach the outside, which is located at

$$r_{out} = \frac{1}{3} \left[(m - \omega) - 2\alpha - \frac{2^{\frac{2}{3}} (3e^4 - ((m-\omega) - 2\alpha)^2) + \beta^2}{2^{\frac{1}{3}} \beta} \right]^{\frac{1}{2}} ,$$

where denotes the energy shell of the particle ω.

To calculate the imaginary part of the action, we replace Eq. (2) in Eq. (10) to find

$$\operatorname{Im} S = -\operatorname{Im} \int_{0}^{\omega} \int_{r_{in}}^{r_{out}} \frac{dr d\tilde{\omega}}{1 - \sqrt{\frac{r^2}{4\alpha} (1 \pm \sqrt{1 + \frac{8\alpha m}{r^4 + e^4}})}} \,. \tag{11}$$

Since there are two poles in this integral, we must expand the denominator in terms of r_{out} in order to determine the poles

$$\dot{r} = 1 - \sqrt{1 - [f(r_{out}) + f'(r_{out})(r - r_{out}) - \dots} \,, \tag{12}$$

where the derivative with respect to r is marked by a prime. We first calculate the first integral by using the residue calculus and expand the result to the second order of ω. Then the second integral provides us with the imaginary part of the action in terms of the mass of the black hole and the energy of the particles. Composing all the obtained results by considering how the Boltzmann factor, the imaginary part of the action, and the emission rate are related, we find finally,

$$\Gamma \simeq e^{-2 \operatorname{Im} S} = e^{-\beta\omega}. \tag{13}$$

The temperature of the black hole may indeed be easily determined since the Hawking temperature is the inverse of the Boltzmann factor, $T = \frac{1}{\beta}$. The result would be a very lengthy relation in terms of the mass of the black hole and the coupling constant, which we avoid to present it here to economize. Instead, we try to study the results numerically. A Plot of the temperature of the black hole horizon for the 5D exact Hayward-like black holes in EGB gravity as a function of black hole mass is shown in figure 1. In drawing this plot we have set $\alpha = 0.1, 0.2, 0.4, 0.6$. When the black hole mass reaches a particular mass, then the temperature increases as the black hole's mass decreases, reaching a maximum temperature.

The temperature then decreases when the mass is reduced.
On the contrary, increasing the coupling constant causes the black hole horizon's maximum temperature to rise. We plotted also the standard Hawking temperature for comparison. It is obvious that the standard Hawking temperature grows to infinity and diverges in the last stage of the evaporation, but for this 5D EGB-Hayward black hole, the coupling constant causes the temperature to become zero in the final stage of the evaporation. This is a novel observation in the 5D EGB-Hayward black hole thermodynamics.

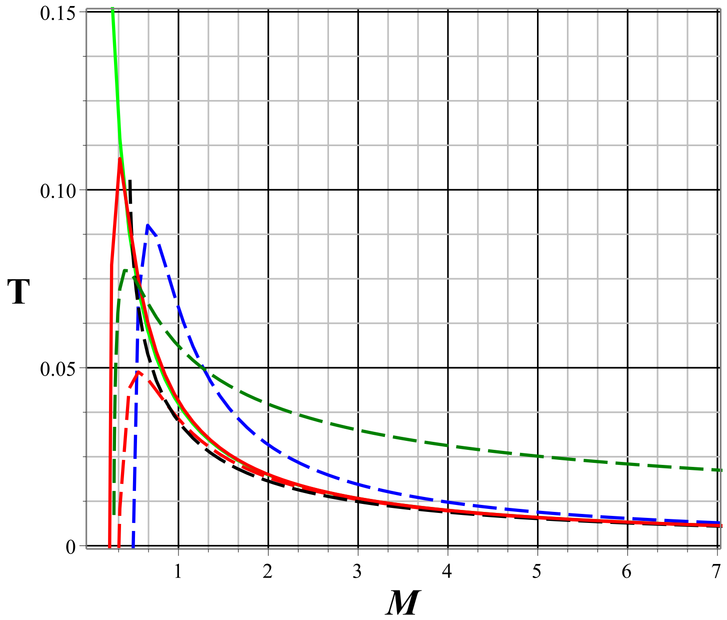

Figure 1: Temperature of the 5D exact Hayward-like black holes in EGB gravity versus the Mass. The figure is plotted with $\alpha = 0.1, 0.2, 0.4, 0.6$ from top to bottom. The green curve is for the standard Hawking temperature for a Schwarzschild black hole which as usual tends to infinity at the final stage of the evaporation process. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article [https://jhap.du.ac.ir/].)

4 Conclusions

In this work, we have focused on the issue of the Hawking radiation and temperature of a 5D exact Hayward-like black hole in Einstein-Gauss-Bonnet gravity. We have calculated the tunneling rate of the massless particles from the event horizon of 5D EGB-Hayward black hole and then the temperature of this horizon. The final equation for the temperature is a so lengthy relation in terms of the black hole mass and the Gauss-Bonnet coupling that has not been presented here, but its numerical behavior is plotted in Fig. 1 for some values of the Gauss-Bonnet coupling. We have shown that reduction the mass of the black hole through evaporation results in the increment of the temperature to a maximum value, when the black hole mass reaches a special mass. Then, by reducing the mass, the temperature reduces too. On the other hand, as a result of reducing the coupling constant, the maximum temperature of the black hole horizon increases. We concluded that in the final stage of the evaporation, while the standard Hawking temperature diverges and goes to infinity, for this 5D EGB-Hayward black hole, the presence of the Gauss-Bonnet coupling leads to have a zero temperature at the final stage of evaporation. That is, there is a remnant with a non-vanishing mass with zero temperature. This is not surprising since the Gauss-Bonnet effect is essentially a quantum effect and as has been shown, for instance in Refs. [39-43], in the presence of quantum gravitational effects this is indeed a consequence of the existence of a minimal measurable length of the order of the Planck length in the fabric of spacetime. As is known, these remnants may be a candidate for dark matter.

Data Availability

The manuscript has no associated data or the data will not be deposited.

Conflicts of Interest

Authors declare that there is no conflict of interest.

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