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Regular article

Magnetic charge effects on thermodynamic phase transition of modified anti de Sitter Ayón-Beato-García black holes with five parameters

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Abstract. In this paper, we choose a generalized Ayón-Beato-García (ABG) magnetic charged black hole with five parameters to investigate the possibility of thermodynamic phase transition and coexistence of different gas/liquid/solid phases of this black hole. In fact, this work is an extension of our recent work where ABG black hole with three parameters were used to seek the phase transition. In this work, we obtain other physical values on the parameters with respect to our previous work, where the phase transition happens together with the coexistence point of different phases in the phase space.

Keywords: Black Holes; Thermodynamics; Phase Transition; Magnetic Charge; Non Singularity.

1 Introduction

After that Ayón, Beato and García (ABG) [1] considered a nonlinear electromagnetic field to produce a nonsingular magnetic black hole which applies to modeling central black holes of galaxies. Other authors try to extend his model with more parameters [2–4]. For instance, Cai and Miao [5] take on a generalized ABG black hole solution which is dependent on five parameters named as the mass, the magnetic charge, and three dimensionless parameters, which are related to nonlinear electrodynamic fields. This kind of black hole returns to a regular black hole under special conditions. In the introduction of our previous work [6], we described applications of this type of the black hole more, and we studied its thermodynamics phase transitions but with some symmetries on parameters of this black hole. In this work, we like to be free of the restrictions on the parameters of the generalized ABG black hole, and we investigate its thermodynamic phase transition. In the black hole thermodynamics, to produce that equation of state in the usual way, we need a suitable pressure that is affect on the black hole by its environment. This is done by applying a negative cosmological parameter originating from anti de Sitter vacuum space. Fortunately, magnetic charge of this kind of black hole can produce a variable cosmological parameter itself, and there is no need to use an unknown cosmological constant similar to the well-known Schwarzschild de Sitter one. In this way, Hawking and Page discovered a first-order phase transition for black holes in anti de Sitter space-time [7]. Other types of phase transitions have been followed

in other works [8–13]. Since the cosmological constant has been suggested as thermodynamic pressure [14–17], the attention have been attracted to black hole thermodynamics in extended phase space [18–24]. The layout of this work is as follows.
In section 2, we define the metric of generalized ABG magnetic black holes with five parameters briefly. In section 3, we investigate the thermodynamics perspective of the model. In section 4, we study the possibility of the black hole phase transition and the coexistence of different phases. We dedicate the last section to the summary and conclusion.

2 Generalized ABG magnetic black hole

Consider the following nonlinear Einstein-Maxwell action functional [1]:

$$S = \int d^4x\sqrt{-g}\Big[\frac{R}{16\pi} - \frac{L(P)}{4\pi}\Big] \tag{1}$$

in which, $R = g_{\mu\nu}R^{\mu\nu}$ is Ricci scalar and $g = |detg_{\mu\nu}|$ is absolute value of determinant of metric tensor field. Nonlinear electromagnetic field lagrangian density $L(P) = 2PH_P - H(P)$ is minimally coupled to the gravity where $P \equiv \frac{1}{4}P_{\mu\nu}P^{\mu\nu}$ is a gauge invariant scalar. $P_{\mu\nu} \equiv \frac{F_{\mu\nu}}{H_P}$ is nonlinear antisymmetric tensor and $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ is electromagnetic tensor field, where A_μ is electromagnetic potential. $H(P)$ is a structure function of nonlinear electrodynamic field and $H_p = \frac{dH(P)}{dP}$ [1]. According to [5], one can infer that the above model has a spherically symmetric static black hole metric field as,

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2(d\theta^2 + \sin\theta^2 d\varphi^2) \tag{2}$$

in which,

$$f(r) = 1 - \frac{2mr^{\frac{\alpha\gamma}{2}-1}}{(q^\gamma + r^\gamma)^{\alpha/2}} + \frac{q^2r^{\frac{\beta\gamma}{2}-2}}{(q^\gamma + r^\gamma)^{\beta/2}} \tag{3}$$

is metric potential with m and q are the mass and the magnetic charge parameters respectively. Other three dimensionless parameters α, β and γ are associated to nonlinear electrodynamic source fields $F_{\mu\nu}$. By assuming $\alpha\beta \geqslant 6$, $\beta\gamma \geqslant 8$, and $\gamma > 0$ the solution (3) reduces to regular black hole solutions [25]. For particular choices of $\alpha = 3$, $\beta = 4$, and $\gamma = 2$ the generalized ABG metric field (3) returns to original ABG black hole solution [1] and it goes to other generalized ABG black hole solutions by setting $\gamma = 2$ [6]. In this work, we consider the metric form (3) to study thermodynamic phase transition of the black hole under consideration.

3 Thermodynamic perspective

In order to participate a negative cosmological parameter due to the AdS/CFT correspondence in our study, the metric potential (3) can be rewritten similar to Schwarzschild-AdS form apparently such that

$$f(r) = 1 - \frac{2M(r)}{r} - \frac{1}{3}\Lambda(r)r^2 \tag{4}$$

where $M(r)$ is the mass function and $\Lambda(r)$ is the variable cosmological parameter:

$$M(r) = \frac{mr^{\frac{\alpha\gamma}{2}}}{(r^\gamma + q^\gamma)^{\alpha/2}}, \quad \Lambda(r) = -\frac{3q^2r^{\frac{\beta\gamma}{2}-4}}{(r^\gamma + q^\gamma)^{\beta/2}}. \tag{5}$$

Since the cosmological parameter plays the role of the pressure of the AdS space, which influences the black hole, $P(r)$ is defined as the variable pressure such that

$$P(r) = \frac{-\Lambda(r)}{8\pi} = \frac{3q^2 r^{\frac{\beta\gamma}{2}-4}}{8\pi(r^\gamma + q^\gamma)^{\beta/2}} \tag{6}$$

By rewriting the equation (4) in terms of the mass and pressure parameters and by solving the event horizon equation as $f(r_+) = 0$, we can obtain the enthalpy equation of the black hole as

$$M = H = U + PV \tag{7}$$

in which the enthalpy H is equal to the black hole mass M, so the internal energy U and the thermodynamic volume V are defined respectively by

$$U = \frac{r_+}{2}, \quad V = \frac{4\pi}{3} r_+^3. \tag{8}$$

As we know, the thermodynamic volume is a conjugate quantity of the thermodynamic pressure, and it is different from the geometric volume of the black hole, but in this case the thermodynamic volume takes the same form as the geometric volume. By regarding the first law of thermodynamic as $dU = TdS - PdV$ and thermodynamic parameters obtained earlier, we are able to define the Bekenstein entropy of the black hole under consideration as follows.

$$S(r_+) = \int \left[1 + 8\pi r_+^2 P(r_+)\right] \frac{dr_+}{2T(r_+)} \tag{9}$$

where

$$T_H(r_+) = \frac{f'(r_+)}{4\pi} = \frac{1}{4\pi r_+} - \frac{q^2 r_+^{\frac{\beta\gamma}{2}-2}\left[2(q^\gamma + r_+^\gamma) + (\alpha - \beta)\gamma q^\gamma\right](q^\gamma + r_+^\gamma)^{-\beta/2} + \alpha\gamma q^\gamma}{8\pi r_+ (q^\gamma + r_+^\gamma)} \tag{10}$$

is the Hawking temperature of the AdS ABG black hole. It is defined by surface gravity on the black hole horizon r_+. In the latter equation we substituted $m(r_+)$ obtained from the horizon equation $f(r_+) = 0$. We now investigate the possibility of the thermodynamic phase transition of the AdS ABG black hole.

4 Equation of state and phase transitions

By assuming the relation of $x = \frac{r_+}{q}$, the dimensionless form of mass function and cosmological parameter (5) are obtained as follows,

$$M(x) = \frac{m x^{\frac{\alpha\gamma}{2}}}{(1 + x^\gamma)^{\alpha/2}}, \quad \Lambda(x) = -\frac{3x^{\frac{\beta\gamma}{2}-4}}{q^2(1 + x^\gamma)^{\beta/2}}, \tag{11}$$

and the thermodynamic pressure takes the following form:

$$p = q^2 P(x) = \frac{3x^{\frac{\beta\gamma}{2}-4}}{8\pi(1 + x^\gamma)^{\beta/2}}. \tag{12}$$

By substituting the relation of $x = r_+/q$ and (12) into (10), the dimensionless equation of state is obtained in terms of the specific volume v such that

$$t = qT(x) = pv + \frac{2(1 + x^\gamma) + \alpha\gamma}{8\pi x(1 + x^\gamma)}, \tag{13}$$

where

$$v = \frac{[(\beta-\alpha)\gamma - 2]\,x - 2x^{\gamma+1}}{3(1+x^{\gamma})}. \tag{14}$$

As the equation of state reveals the thermodynamic behavior of an ordinary thermodynamic system, and this is valid for black holes also. In this regard, the equation of state plays an important role in studying the thermodynamic behavior of a black hole. For this purpose, (13) is employed to calculate the critical points within solving the equations of $\frac{\partial t}{\partial v}\Big|_p = 0$ and $\frac{\partial^2 t}{\partial v^2}\Big|_p = 0$, which takes the following form due to the usage of the chain rules to solve the critical equations:

$$\frac{\partial t}{\partial x}\Big|_p = 0, \quad \frac{\partial^2 t}{\partial x^2}\Big|_p = 0. \tag{15}$$

By substituting (13) and (14) into (15), one can obtain parametric forms for the critical points which for simplicity we substitute ansatz

$$x_c = 1 \tag{16}$$

into them as critical radius of black hole such that

$$p_c = \frac{3}{16\pi}\left(\frac{8}{\gamma} - 6 + 2\alpha - \alpha\gamma^2\right) \tag{17}$$

with

$$\beta = \frac{\alpha^2(\gamma^4 - 2\gamma^2) - 16\alpha\gamma - 32}{\gamma[\alpha(\gamma^3 - 2\gamma) + 6\gamma - 8]}. \tag{18}$$

By using these conditions in the temperature (13) and the volume (14), we obtain their critical values, respectively as follows:

$$t_c = \frac{\alpha\gamma^2 + 2\alpha\gamma + 8}{8\pi} \tag{19}$$

and

$$v_c = -\frac{\gamma}{3}\left(\frac{2\alpha\gamma^2 + 3\alpha\gamma + 12}{\alpha\gamma^3 - 2\alpha\gamma + 6\gamma - 8}\right). \tag{20}$$

By substituting (16) into the the horizon equation $f(x) = 0$, we obtain

$$w = \frac{2m}{q} = 2^{\frac{\alpha-\beta}{2}}(1 + 2^{\frac{\beta}{2}}). \tag{21}$$

To obtain numeric values of the metric parameters of (α, β, γ), we keep the positivity condition on the critical volume v_c and $\gamma > 0$ with ansatz

$$\alpha = 0 \tag{22}$$

for which we obtain

$$p_c = \frac{3}{4\pi v_c}, \quad t_c = \frac{1}{\pi}, \quad \gamma = \frac{4v_c}{2+3v_c}, \quad \beta = \frac{(2+3v_c)^2}{2v_c}, \quad w = 1 + 2^{-\frac{(2+3v_c)^2}{4v_c}}. \tag{23}$$

By using the above numerical values into the equation of state (13) and (14) reads

$$v = \frac{2x}{3}\left(\frac{1 + 3v_c - x^{\frac{4v_c}{2+3v_c}}}{1 + x^{\frac{4v_c}{2+3v_c}}}\right), \quad t = pv + \frac{1}{4\pi x}. \tag{24}$$

By substituting $v_c = 1$ into the above equation of state, we plot p-v and t-v diagrams which are shown in figure 1-a and 1-b. For further investigation on the phase transition of this ABG magnetic black hole under consideration, the heat capacity and the Gibbs free energy are examined. To do so we substitute $r_+ = qx$ into the equation (9) to obtain a dimensionless form for the entropy such that

$$ds = \frac{dS(x)}{q^2} = (1 + 8\pi p x^2)\frac{dx}{2t} \tag{25}$$

which by applying (24) the equation (25) reads to the following series form.

$$ds = 2\pi x - \frac{16\pi^2 p x^3}{3} + \frac{80\pi^2 p x^{19/5}}{3} - \frac{80\pi^2 p x^{23/5}}{3} + \frac{512\pi^3 p^2 x^5}{9} + \frac{80\pi^2 p x^{27/5}}{3} - \frac{3200\pi^3 p^2 x^{29/5}}{9} + O(x^6). \tag{26}$$

By integrating (26) we obtain

$$s = \pi x^2 - \frac{4\pi^2 p x^4}{3} + \frac{50\pi^2 p x^{24/5}}{9} - \frac{100\pi^2 p x^{28/5}}{21} + \frac{25\pi^2 p x^{32/5}}{6} + \frac{256\pi^3 p^2 x^6}{27} - \frac{8000\pi^3 p^2 x^{34/5}}{153} + O(x^7). \tag{27}$$

Gibbs free energy as another thermodynamic variable is suitable in the study of the phase transition for this modified ABG black hole, and it is obtained from the relation of $G = M - TS$, in which M refers to the black hole enthalpy or its ADM mass. By substituting the above equation of state and the entropy equation we obtain a dimensionless form for the Gibbs free energy such that

$$\begin{aligned} g = \frac{G}{q} &= \frac{1 + 2^{-25/4}}{2(1 + x^{4/5})} - ts \\ &= (-86016000\pi^3 p^3 x^{43/5} + 359661568\pi^3 p^3 x^{39/5} + 6854400\pi^2 p^2 x^{41/5} - 35251200\pi^2 p^2 x^{37/5} \\ &\quad + 72729600\pi^2 p^2 x^{33/5} - 12343296\pi^2 p^2 x^{29/5} - 2570400\pi p x^{31/5} + 367200\pi p x^{27/5} \\ &\quad - 489600\pi p x^{23/5} - 959616\pi p x^{19/5} - 62390272\pi^3 p^3 x^7 + 2924544\pi^2 p^2 x^5 - 5757696\pi p x^3 \\ &\quad - 616896 x^{9/5} + 9639(2)^{3/4} - 616896x + 1233792)/(2467584x^{4/5} + 2467584). \end{aligned} \tag{28}$$

We plot a diagram of the Gibbs free energy versus the temperature at constant pressures where for pressures higher than the critical one $p > p_c$, the diagram has a cross point means coexistence of two phases. Usually, this is called as swallowtail form of the diagram. This phenomena is also appeared in the diagram of the Gibbs energy versus the pressure at constant temperatures and coexistence of two phases happens at temperatures below the critical one $t < t_c$. The Gibbs free energy diagrams are shown in the figure 2-a and 2-b respectively. We end our study about the possibility of phase transition of modified ABG magnetic black hole with five parameters by calculating the heat capacity of the black hole. This is obtained as follows.

$$c_p = t\left(\frac{\partial s}{\partial t}\right)_p = \frac{6\pi x^2(1 + 8\pi p x^2)(1 + x^{4/5})^2}{8\pi p x^2\left[4 - x^{4/5}(1 + x^{4/5})\right] - 3\left[1 + x^{4/5}(2 + x^{4/5})\right]} \tag{29}$$

where we substitute (24) and (25). Change of sign of the above heat capacity shows a phase transition for the black hole so that positive (negative) heat capacity reveals the absorber (heater) phase of the black hole. Diagram of the heat capacity at constant pressure is plotted versus the specific volume. This shows change in the values of the heat capacity from positive absorber to negative (heater) values for pressures higher than the critical one.

5 Conclusion

In this work, we used a modified nonsingular ABG black hole with five parameters to study thermodynamic phase transition by calculating the equation of state, Gibbs energy, and heat capacity. In fact, non-singularity of this black hole comes from a hypothetical magnetic monopole charge, and this black hole plays an important role in the modeling of the Galaxies. Our calculations show that for $P-V$ diagrams, the black hole has three different phases at $t < t_c$ where one phase behaves as an ideal gas with no any phase transition, but two other phases participate in the Hawking Page phase transition. This happened because of the maximum point of the diagrams where an evaporating AdS modified ABG black hole reaches to vacuum AdS space finally. Diagram of $T-V$ at constant pressures shows that the black hole has three phases for $p < p_c$ where one phase behaves as regular means that by raising the specific volume, the temperature increase. Two other phases of the black hole participate in a small to large black hole phase transition because of a minimum point of the diagram. The coexistence of these phases is studied by plotting the Gibbs free energy in which the crossing point of two branches of the diagram shows a swallowtail form. Change of sign of the heat capacity shows positions in phase space where the evaporating black hole can be a heater/absorber of the thermal energy. This work, with respect to our similar previous work has extensions on the black hole parameters, which causes to phase transition of the black hole. In our previous work, we use some restrictions on the black hole parameters (with three parameters) corresponding to model for which Cai and Miao were studied quasi normal modes [5]. As an extension of this work, we like to study the effects of these black hole parameters in the cooling-heating phase or Jule-Thomson expansion of this black hole.

References

[1] E. Ayon-Beato and A. Garcia, "Regular black hole in general relativity coupled to nonlinear electrodynamics", Phys. Rev. Lett. **80**, 5056 (1998).

[2] L. Balart and E.C. Vagenas, "Regular black holes with a nonlinear electrodynamics source", Phys. Rev. D **90**, 124045 (2014) [arXiv:1408.0306 [gr-qc]].

[3] M. S. Ma, "Magnetically charged regular black hole in a model of nonlinear electrodynamics", Ann. Phys. **362**, 529 (2015) [arXiv:1509.05580 [gr-qc]].

[4] E. L. Junior, M. E. Rodrigues, and M. J. Houndjo, "Regular black holes in f(T) Gravity through a nonlinear electrodynamics source", J. Cosmol. Astropart. Phys. **10**, 060 (2015) [arXiv:1503.07857 [gr-qc]].

[5] X. Ch. Cai and Y. G. Miao, "Quasinormal modes and shadows of a new family of Ayón-Beato and García black holes", Phys. Rev. D **103**, 124050 (2021).

[6] E. Ghasemi and H. Ghaffarnejad, "Thermodynamic Phase Transition of Generalized Ayon-Beato Garcia Black Holes with Schwarzschild anti de Sitter space time perspective", [arXiv:2201.08389].

[7] S. W. Hawking and D. N. Page, "Thermodynamics of black holes in anti-de Sitter space", Commun. ath. Phys. **87**, 577 (1983).

[8] D. Kubiznak, R. B. Mann, "P-V criticality of charged AdS black holes". J. High Energy Phys. **1207**, 033 (2012).

[9] V. P. Maslov, "Zeroth-order phase transitions". Math. Notes **76**, 697-710 (2004).

[10] S. Gunasekaran, D. Kubiznak, and R. B. Mann, "Extended phase space thermodynamics for charged and rotating black holes and Born-Infeld vacuum polarization", J. High Energy Phys. **1211**, 110 (2012).

[11] N. Altamirano, D. Kubiznak, and R. B. Mann, "Reentrant phase transitions in rotating antide Sitter black holes", Phys. Rev. D **88**, 101502 (2013).

[12] N. Altamirano, D. Kubiznak, and R. B. Mann, Z. Sherkatghanad, "Kerr-AdS analogue of tricritical point and solid/liquid/gas phase transition", Class.Quant.Grav. **31**, 042001 (2014).

[13] S. W. Wei and Y. X. Liu, "Triple points and phase diagrams in the extended phase space of charged Gauss-Bonnet black holes in AdS space", Phys. Rev. D **90**, 044057 (2014).

[14] D. Kastor, S. Ray and J. Traschen, "Enthalpy and the mechanics of AdS black holes", Class. Quantum Gravity **26**, 195011 (2009).

[15] M. Cvetic, G.W. Gibbons and D. Kubiznak, "Black hole enthalpy and an entropy inequality for the thermodynamic volume", C. N. Pope, Phys. Rev. D **84**, 024037 (2011).

[16] B. P. Dolan, "The cosmological constant and black-hole thermodynamic potentials", Class. Quant. Gravity **28**, 125020 (2011).

[17] B. P. Dolan, "Pressure and volume in the first law of black hole thermodynamics", Class. Quant. Gravity **28**, 235017 (2011).

[18] B. P. Dolan, "Where is the PdV Term in the Fist Law of Black Hole Thermodynamics? In Open Questions in Cosmology", Olmo, G.J., Ed.; InTech: Rijeka, Croatia, 2012.

[19] A. Belhaj, M. Chabab, H. El Moumni, and M. B. Sedra, "On thermodynamics of AdS black holes in arbitrary dimensions", Chin. Phys. Lett. **29**, 100401 (2012).

[20] H. Ghaffarnejad, "Classical and Quantum Reissner-Nordstrom Black Hole Thermodynamics and first order Phase Transition", Astrophys. and Space Sci. **361**, 7, 6 (2016); [physics.gen-ph/1308.1323].

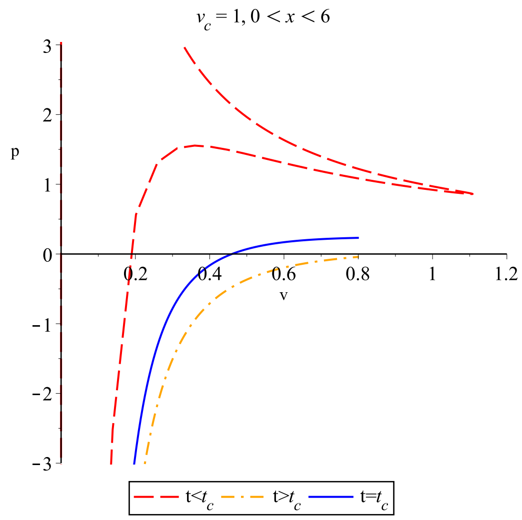
[21] H. Ghaffarnejad and M. Farsam, "Reissner-Nordstrom black holes statistical ensembles and first order thermodynamic phase transition", Advances in High Energy Physics **2019**, 2539217 (2019).

[22] H. Ghaffarnejad, E. Yaraie and M. Farsam, "Thermodynamic phase transition for Quintessence Dyonic Anti de Sitter Black Holes", Eur. Phys. J. Plus, **135**, 179 (2020).

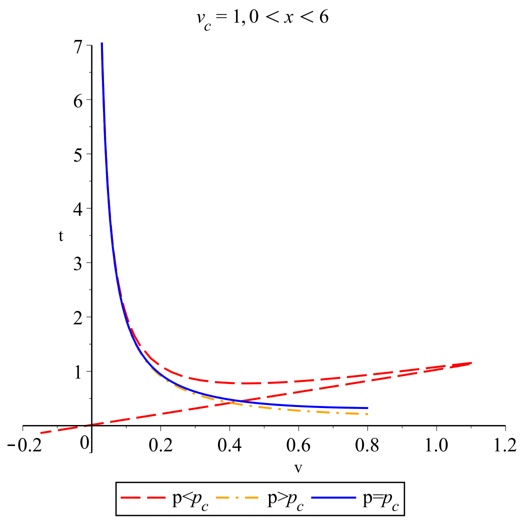
[23] H. Ghaffarnejad, "Thermodynamics of 4D dS/AdS Gauss-Bonnet black holes from consistent gravity theory in presence of cloud of strings"; [arxiv:gr-qc/2105.12729]

[24] S. W. Wei and Y.X. Liu. "Critical phenomena and thermodynamic geometry of charged Gauss-Bonnet AdS black holes", Phys. Rev. D **87**, 044014 (2013).

[25] E. Ayón-Beato and A. Garcia, "Four-parametric regular black hole solution", Gen. Relativ. Gravit. **37**, 635 (2005).

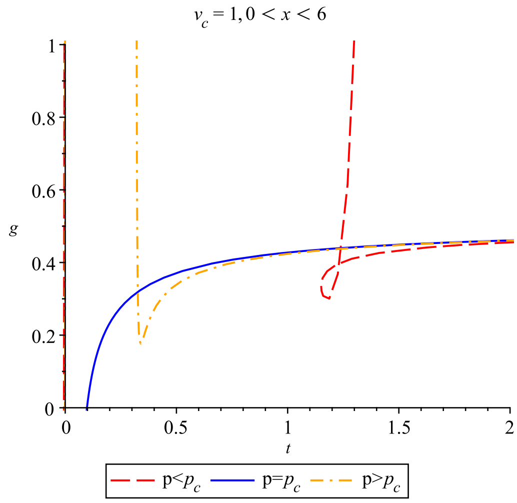


(a)



(b)

Figure 1: $P - V$ and $T - V$ curves at phase space.



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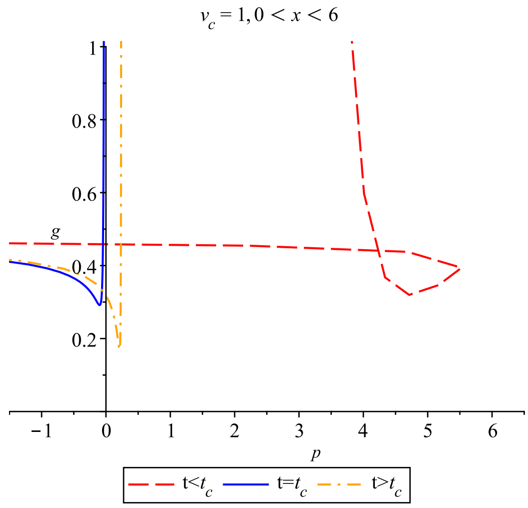
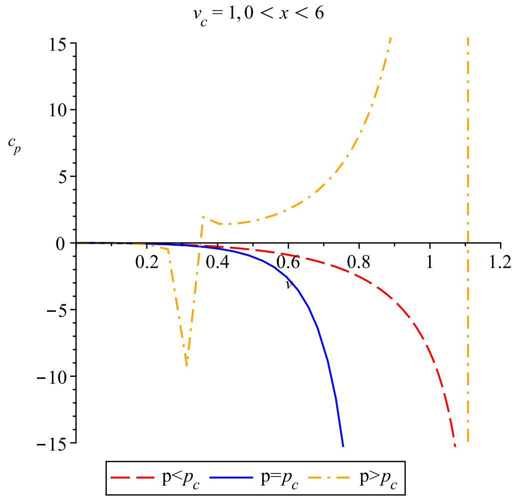


(b)

Figure 2: $G-T$ and $G-P$ curves at phase space.



(a)

Figure 3: Diagram of heat capacity at constant pressure versus the specific volume.