Journal of Holography Applications in Physics
Volume 2, Issue 2, Spring 2022, 39-46
© Available online at http://jhap.du.ac.ir
DOI: 10.22128/jhap.2022.519.1020

Journal of Holography Applications in Physics

Regular article

Holographic pole – skipping of flavor branes

Mahdi Atashi[1] . **Kazem Bitaghsir Fadafan**[1]
[1] Faculty of Physics, Shahrood University of Technology, P.O. Box 3619995161, Shahrood, Iran.
Email: m.atashi@shahroodut.ac.ir
Email: bitaghsir@shahroodut.ac.ir

Received: March 09, 2022; Revised: April 18, 2022; Accepted: April 29, 2022

Abstract. In this paper, we investigate special points in which poles of the Green's function of the D3/D7 branes are skipping. The system that we consider is $SU(N_c)\,\mathcal{N}=4$ SYM theory coupled to N_f flavors $\mathcal{N}=2$ hypermultplets. We fluctuate the scalar field and find that there is no such special values of frequency and momentum to skip the poles. This behavior is different than scalar field in the AdS black brane background.

Keywords: Pole – Skipping; D3/D7 Model; Holography; Flavor Branes.

1 Introduction

One of the important applications of holography is studying Green's functions [1]. It provides a useful tool to study the hydrodynamics regime of the boundary field theory. Recently, it was shown that in the complex plane of frequency ω and wave number k, there are special points where Green's functions are not unique [2,3]. This phenomenon is called pole skipping, and it seems that it is related to the chaotic behavior of the strongly coupled system [4]. Using out of time ordered correlation functions (OTOCs), one can probe the time evolution of operators in quantum many-body system [5]. The chaotic behavior of the system is given by Lyapunov exponent $\lambda_L = i\,2\pi T$ and butterfly velocity v_B. At special values in the complex plane (ω, k) one finds that both of $\omega_*$and $k_*$are purely imaginary as

$$\omega_* = i\,\lambda_L, \qquad k_* = i\lambda_L/v_B\ . \tag{1}$$

Such relation is quite important and motivates a connection with quantum many-body system [6,7,8]. At these points, retarded Green's functions $G_T^R(\omega,k)=\frac{b(\omega,k)}{a(\omega,k)}$ are not uniquely defined because

$$\lim_{(\omega,k)\to(\omega_*,k_*)}\big(a(\omega,k), b(\omega,k)\big) = 0. \tag{2}$$

So far, pole skipping has been studied in field theories dual to black holes. Here we extend the study to the case of adding flavors to the system. Then we couple $\mathcal{N}=4$ SYM theory to a number of flavors N_f in the quenching limit where N_f is less than N_c. The gravity setup is given by D3/D7. Recently this gravitational construction has been applied in condensed matter by studying properties of Weyl semimetals [8]. Then that is a good motivation to check if the field theory dual to D3/D7 branes leads to pole skipping. We start by reviewing scalar perturbations and showing how one finds ω_* and k_* from bulk equations in the next section.

2 Finding pole-skipping points for scalar Green's function

The scalar Green's function is obtained by solving Klein Gordon equation in the black hole space-time background. In Eddington-Finklestein coordinate, it is given by

$$ds^2 = -r^2 f(r) dt_*^2 + 2dt_* dr + r^2 d\vec{x}^2, \tag{3}$$

where r coordinate is the bulk direction and $f(r) = 1 - r_H^4/r^4$. r_H is the horizon location. The boundary is located at $r \to \infty$. One finds the scalar field EOM

$$\partial_\mu\left(\sqrt{-g}\partial^\mu \phi\right) - \sqrt{-g}\, m^2 \phi = 0, \tag{4}$$

here g is determinant of the background metric in Eq. (3), and from the equation of motion, the scalar field ϕ behaves at the boundary as

$$\phi \to a\, r^{\Delta-4} + b\, r^{-\Delta} \quad . \tag{5}$$

So that a is source and b is the condensation from AdS/CFT dictionary. The dimension of the scalar field is Δ, so

$$\Delta(\Delta - 4) = m^2 \quad . \tag{6}$$

Consider the near horizon expansion of the scalar bulk field

$$\phi(r) = \sum_{i=0} a_i\, (r - r_H)^i \tag{7}$$

Here a_i's are scalar field series expansion at each order. Plugging (7) in the equation of motion and considering order by order equation, one gets at each order the following relations

$$\mathcal{O}[(r-r_H)^0]: \quad A_{00}(\omega, k^2) a_0 + A_{01}(\omega) a_1 = 0 \tag{8.a}$$

$$\mathcal{O}[(r-r_H)^1]: \quad A_{10}(\omega, k^2) a_0 + A_{11}(\omega, k^2) a_1 + A_{12}(\omega) a_2 = 0 \tag{8.b}$$

$$\mathcal{O}[(r-r_H)^n]: \quad A_{n0}(\omega, k^2) a_0 + A_{n1}(\omega, k^2) a_1 + \cdots + A_{nn+1}(\omega) a_{n+1} = 0 \tag{8.c}$$

The first pole skipping point is found by setting (8.a) to be zero

$$A_{01}(\omega) = 0 \Longrightarrow \omega = \omega_{*1} = -i\ 2\pi T \tag{9}$$

$$A_{00}(\omega_*, k^2) = 0 \Longrightarrow k = k_{*1} \tag{10}$$

So, the (ω_{*1}, k_{*1}) is the first pole – skipping point. Similarly, the n-th pole – skipping points is $\omega_{*n} \equiv -i\ 2\pi Tn$, and the k_{*n} comes from $\det(M_n) = 0$, where

$$M_n \equiv \begin{pmatrix} A_{00} & A_{01} & 0 & 0 & \dots \\ A_{10} & A_{11} & A_{12} & 0 & \dots \\ \dots & & & & \\ A_{n0} & A_{n1} & A_{n2} & \dots & A_{nn+1} \end{pmatrix} \tag{11}$$

Several extensions have been considered, like adding higher derivative corrections [9,10], anisotropic background [11]. Constraints on the quasi-normal modes have been studied in [12].

3 D3/D7 holographic model

In this section, we introduce the holographic setup that we are interested in this study [8]. In table 1, the embedding of the D7 brane in the background of the D3 brane has been explained. There are N_c D3 branes placed along with the coordinates $(x_0, \dots, x_3)$, which describe $SU(N_c)$ $\mathcal{N} = 4$ SYM theory. The N_f D7 branes are placed along $(x_0, \dots, x_7)$ giving rise to the fundamental representation of $SU(N_c)$ as $\mathcal{N} = 2$ hypermultplets.

	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
$D3$	×	×	×	×						
$D7$	×	×	×	×	×	×	×	×		

Table 1. Embedding of D7 branes in the backgrounds of D3 branes.

By separating D7 branes from D3 branes in the transverse plane (x_8, x_9) with a distance R, one gives the mass to open strings stretched between D7 and D3 branes. The mass m is proportional to R, and its phase is the angle separation in the transverse plane. Fig. 1 shows the coordinates of D7 brane embedding. Notice that the 89 plane is transverse to the D7 brane. The scalar that we want to study is R(r). A study of quasi-normal modes in D3/D7 set up has been done in [13,14].

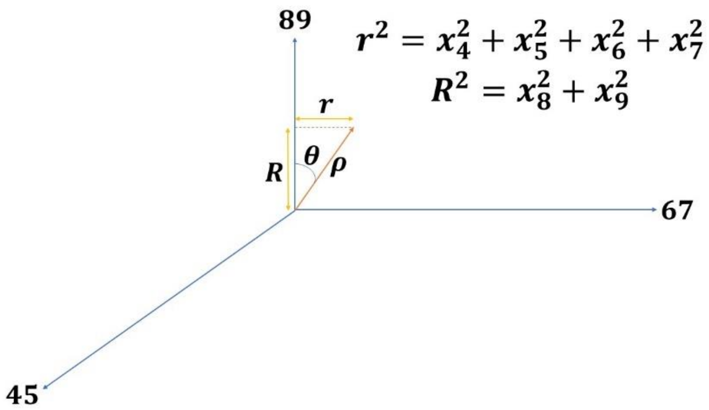

Figure 1. Coordinates of the D7 brane

The background space-time is

$$ds^2 = \frac{\rho^2}{L^2}\left(-\frac{g(\rho)}{h(\rho)}dt^2 + h(\rho)d\vec{x}^2\right) + \frac{L^2}{\rho^2}\left(dr^2 + r^2 ds_{S^3}^2 + dR^2 + R^2 d\phi^2\right), \qquad (12)$$

where $\rho^2 = r^2 + R(r)^2$, $g(\rho) = 1 - \rho_H^4/\rho^4$ and $h(\rho) = 1 + \rho_H^4/\rho^4$. Temperature is

$$T = \frac{\frac{\sqrt{2}}{\pi}\rho_H}{L^2}.$$

We do not turn on any gauge field on the D7 brane, then, the action includes only following DBI part,

$$S_{D7} = -N_f T_{D7}\int d^8\xi\sqrt{-\det(P[G])} \qquad (13)$$

Clearly, this action differs from Klein Gordon action in Eq. (4) for the scalar field. Then that is interesting to explore the physics and investigate the pole skipping for DBI action.

4 Pole – skipping in D3/D7 model

Using the symmetries of the background, we separate the scalar field as $R \equiv R(t,r,x) = e^{-i(\omega t+kx)}\mathcal{R}(r)$. The equation of motion is extracted by Eular-Lagrange as

$$0 = \partial_{R(t,r,x)}\mathcal{L} - \partial_t\partial_{\partial_t R(t,r,x)}\mathcal{L} - \partial_r\partial_{\partial_r R(t,r,x)}\mathcal{L} - \partial_x\partial_{\partial_x R(t,r,x)}\mathcal{L} \quad . \qquad (14)$$

Equation of motion is a lengthy equation, and we avoid expressing all terms here. After expanding the field around the horizon r_H as $\mathcal{R}(r) = \sum_{i=0} a_i \, (r - r_H)^i$, one can obtain the series expansion terms.
We have checked massive flavors but the situation is the same as the massless one. Then we concentrate first to explain the massless case which needs this condition $r_H = \rho_H \, \sin(\frac{\pi}{2})$, see Fig. 2. Here the expansion starts from $\mathcal{O}[(r - r_H)^2]$ as:

$$\mathcal{O}[(r - r_H)^2]: \quad A_{20}(\omega^2)a_0 = 0 \,, \tag{15.a}$$

$$\mathcal{O}[(r - r_H)^3]: \quad A_{30}(\omega^2)a_0 + (4\pi^2T^2 + \omega^2)a_1 = 0 \,, \tag{15.b}$$

$$\mathcal{O}[(r - r_H)^4]: \quad A_{40}(\omega^2, k^2)a_0 + A_{41}(\omega^2)a_1 + (16\pi^2T^2 + \omega^2)a_2 = 0 \,. \tag{15.c}$$

where coefficients are given by

$$A_{20} = \omega^2$$

$$A_{30} = \pi^2T^2 - \omega^2;$$

$$A_{40} = -16k^2 + 184\pi^2T^2 + 517\,\omega^2;$$

$$A_{41} = 29\pi^2T^2 + 8\omega^2;$$

This is not like Eq. (8) where coefficients depend linearly on ω and k. There is no first pole skipping at the first order of expansion in Eq. (15), because k is free and setting ω to zero is enough to kill the first order. But solving at the next order gives a pure imaginary value for ω as $\omega_{*3} = -2i\,\pi T$ while still k is a free parameter. From eq.(15c), one obtains pure imaginary points

$$\omega_{*4} = -4i\,\pi T; \quad k_{*4} = -i\sqrt{\frac{1057}{2}}\,\pi T.$$

But that is not clear for us if it leads to a pole skipping in theory. Actually, it needs careful analysis that we leave it for further study.

For massive case, the coordinate relation is $r_H = \rho_H \, \sin(\alpha); \alpha \neq \pi/2$ and the series expansion starts from zeroth-order

$$\mathcal{O}[(r - r_H)^0]: \quad A_{00}(\omega^2, k^2)a_0 + A_{01}a_1 + A_{02}a_2 = 0 \tag{16.a}$$

$$\mathcal{O}[(r - r_H)^1]: \quad A_{10}(\omega^2, k^2)a_0 + A_{11}(\omega^2, k^2)a_1 + A_{12}a_2 + A_{13}a_3 = 0 \tag{16.b}$$

We list the coefficients as

$$A_{00}(\omega^2, k^2) = 8(-9k^2 + 96\,\pi^2T^2 + 25\omega^2)$$
$$A_{01} = -249\,(\pi T)^3$$
$$A_{02} = 45\,(\pi T)^4$$
$$A_{10}(\omega^2, k^2) = -16(171k^2 - 864\,\pi^2T^2 - 2075\,\omega^2)$$

$A_{11}(\omega^2, k^2) = -24\,\pi T\,(45k^2 + 108\,\pi^2 T^2 - 125\omega^2)$
$A_{12} = -4680\,(\pi T)^4$
$A_{22} = 2025\,(\pi T)^5$

As it is clear from the above expressions, some coefficients do not depend on ω and k. From $A_{00}(\omega^2, k^2) = 0$ we found the following condition

$$k_* = \pm\frac{1}{3}\sqrt{96\pi^2 T^2 + 25\omega^2}, \tag{17}$$

while there is not any condition on ω. Also, A_{01}, A_{02} don't vanish anywhere. Similarly, from $A_{10}(\omega^2, k^2) = 0$ and $A_{11}(\omega^2, k^2) = 0$, we found that

$$\omega_* = (\pm 2.8038\, i\,)\, T,\ \ k_* = (\pm 6.7471\, i)\, T\ . \tag{18}$$

But the A_{12} and A_{22} don't vanish anywhere.

5 Results and Discussion

We found that studying pole-skipping of scalar fields in D3/D7 is not like [2]. As it was reviewed in the second section, near horizon analysis leads to pole skipping points. But the near horizon series coefficients for massless and massive scalars in D3/D7 are completely different, at least there is no first pole skipping in Eqs. (15) and (16). Probably they are mentioning non-hydrodynamic modes in the dual theory, however, it needs more investigation. We are working on this question to better understand the nature of the dual field theory.

6 Conclusions

That would be interesting to investigate possible relation between quantum many-body chaotic behavior dual to D3/D7 branes. One should calculate the Lyapunov exponent and butterfly velocity in this background. Here we have only studied scalar perturbation transverse to the D7 brane, but that would be interesting to see if pseudo scalar shows the same behavior. This field plays an important role in studying Weyl semimetals that have been done in [8], currently, this study is under investigation. Considering gauge and tensor perturbations are also important. One may conclude that hydrodynamic that comes from flavors in D3/D7 is not the same as the $SU(N_c)$ gauge theory governing the Klein Gordon equation.

Acknowledgement

Mahdi Atashi was supported by a grant from Basic Sciences Research Fund (No. BSRF-phys-399-09)

References

[1] D. T. Son and A. O. Starinets, "Minkowski space correlators in AdS / CFT correspondence: Recipe and applications", JHEP **09**, 042 (2002) [arXiv:hep-th/0205051 [hep-th]].

[2] M. Blake, R. A. Davison and D. Vegh, "Horizon constraints on holographic Green’s functions", JHEP **01**, 077 (2020) [arXiv:1904.12883 [hep-th]].

[3] M. Natsuume and T. Okamura, "Nonuniqueness of Green’s functions at special points", JHEP **12**, 139 (2019) [arXiv:1905.12015 [hep-th]].

[4] M. Natsuume and T. Okamura, "Holographic chaos, pole-skipping, and regularity", PTEP **2020**, 013B07 (2020) [arXiv:1905.12014 [hep-th]].

[5] S. Grozdanov, K. Schalm and V. Scopelliti, "Black hole scrambling from hydrodynamics", Phys. Rev. Lett. **120**, 231601 (2018) [arXiv:1710.00921 [hep-th]].

[6] H. Lee, and H. Liu, "A quantum hydrodynamical description for scrambling and many-body chaos Mike Blake", JHEP **10**, 127 (2018) arXiv: 1801.00010

[7] M. Blake, R. A. Davison, S. Grozdanov and H. Liu, "Many-body chaos and energy dynamics in holography", JHEP **10**, 035 (2018) [arXiv:1809.01169 [hep-th]].

[8] K. Bitaghsir Fadafan, A. O'Bannon, R. Rodgers and M. Russell, "A Weyl semimetal from AdS/CFT with flavour", JHEP **04**, 162 (2021) [arXiv: 2013.11434]

[9] S. Grozdanov, "On the connection between hydrodynamics and quantum chaos in holographic theories with stringy corrections", JHEP **01**, 048 (2019) [arXiv:1811.09641 [hep-th]].

[10] X. Wu, "Higher curvature corrections to pole-skipping", JHEP **12**, 140 (2019) [arXiv:1909.10223 [hep-th]].

[11] K. Sil, "Pole skipping and chaos in anisotropic plasma: a holographic study", JHEP **03**, 232 (2021) [arXiv:2012.07710 [hep-th]].

[12] N. Abbasi and M. Kaminski, "Constraints on quasinormal modes and bounds for critical points from pole-skipping", JHEP **03**, 265 (2021) [arXiv:2012.15820 [hep-th]].

[13] C. Hoyos-Badajoz, K. Landsteiner and S. Montero, "Holographic meson melting", JHEP **04**, 031 (2007) [arXiv: hep-th/0612169]

[14] M. Kaminski, K. Landsteiner, F. Pena-Benitez, J. Erdmenger, C. Greubel and P. Kerner, "Quasinormal modes of massive charged flavor branes", JHEP **03**, 117 (2010) [arXiv:0911.3544 [hep-th]]