



Regular article

Digital Holography for Intelligent Computational Imaging: Advances in Fractional Fourier Transform-Based Reconstruction, Phase Retrieval, Quantitative Phase Imaging, and Deep Learning

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Abstract. Digital holography has evolved from a conventional wavefront-recording technique into an intelligent computational imaging paradigm capable of recovering complex optical fields and extracting quantitative information from phase-sensitive measurements. The rapid integration of artificial intelligence (AI), fractional Fourier transform (FrFT) methods, phase-retrieval algorithms, quantitative phase imaging (QPI), and digital holographic microscopy (DHM) has substantially expanded the capabilities of holographic systems in biomedical imaging, optical metrology, and label-free diagnostics. This review provides a comprehensive analysis of the recent development of AI-enhanced digital holography, with particular emphasis on the mathematical foundations of hologram formation, Fourier-domain and Fresnel reconstruction, fractional-domain processing, iterative phase retrieval, deep learning architectures, and quantitative phase analysis. Classical and emerging computational strategies are critically compared in terms of reconstruction fidelity, computational efficiency, phase accuracy, interpretability, and application-specific adaptability. A numerical digital holography framework is further considered to examine the complete processing chain from complex object generation and off-axis hologram formation to FFT-, Fresnel-, and FrFT-based reconstruction, phase retrieval, optical path difference estimation, and thickness mapping. The comparative analysis demonstrates that FFT-based reconstruction provides the strongest object-structure preservation under the investigated conditions, whereas Fresnel propagation offers greater physical relevance for depth-dependent reconstruction and numerical refocusing. The FrFT framework provides a flexible intermediate representation but remains highly dependent on fractional-order optimization. In parallel, AI-based approaches demonstrate considerable potential for automated phase

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recovery, aberration correction, image enhancement, object classification, and quantitative cellular analysis. The review identifies current limitations associated with data dependence, model generalization, phase ambiguities, computational complexity, experimental calibration, and the lack of standardized benchmarks. Finally, future research directions are outlined toward physics-informed learning, self-supervised holographic reconstruction, multimodal quantitative imaging, adaptive fractional-domain processing, and explainable AI-enabled digital holographic systems for reliable biomedical and industrial applications.

Keywords: Digital Holography; Artificial Intelligence; Fractional Fourier Transform; Quantitative Phase Imaging; Digital Holographic Microscopy; Deep Learning.

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1 Introduction

Digital holography has become one of the most influential optical imaging technologies for quantitatively capturing the amplitude and phase information of light waves without requiring mechanical scanning or physical contact with the specimen. Since the advent of digital image sensors and high-performance computational platforms, conventional holography has evolved into a versatile computational imaging technique capable of reconstructing three-dimensional structures with high spatial resolution and exceptional measurement accuracy. Compared with traditional microscopic imaging approaches, digital holography provides label-free, non-destructive, and real-time visualization of microscopic objects, making it particularly attractive for biomedical imaging, industrial inspection, materials science, microfluidics, optical metrology, and precision manufacturing.

Among the numerous holographic imaging modalities, digital holographic microscopy (DHM) has received considerable attention because of its ability to perform quantitative phase imaging (QPI). Unlike intensity-based microscopy, DHM directly measures optical path length variations induced by transparent or semi-transparent specimens, thereby enabling accurate characterization of cellular morphology, thickness, refractive index distribution, membrane fluctuations, and dynamic biological processes without fluorescent labeling. These advantages have established DHM as an indispensable tool for investigating living cells, microorganisms, blood components, tissues, and engineered materials while minimizing phototoxicity and sample preparation requirements.

Recent advances in computational optics have significantly transformed holographic reconstruction methodologies. Classical Fourier transform (FT)-based reconstruction algorithms have progressively been enhanced through fractional Fourier transform (FrFT), wavelet analysis, compressive sensing, iterative phase retrieval, and optimization-based reconstruction techniques. In particular, the FrFT has emerged as a powerful mathematical framework because of its capability to provide an additional fractional domain for signal representation, leading to improved focusing accuracy, noise suppression, image enhancement, and robust reconstruction in both inline and off-axis digital holography. These developments have substantially increased reconstruction quality while simultaneously reducing computational complexity.

The rapid development of artificial intelligence (AI), particularly machine learning (ML) and deep learning (DL), has introduced a new paradigm in computational holography. Instead of relying exclusively on analytical optical models, modern AI algorithms are capable of learning complex mappings directly from holographic data, thereby enabling automatic phase retrieval, aberration correction, hologram reconstruction, denoising, super-resolution, segmentation, feature extraction, and object classification. Convolutional neural networks (CNNs), autoencoders, generative adversarial networks (GANs), recurrent neural networks (RNNs), transformers, and physics-informed neural networks (PINNs) have demonstrated remarkable performance in overcoming many limitations associated with conventional iterative reconstruction algorithms, including long computational times, sensitivity to noise, and dependence on carefully selected optical parameters.

The convergence of AI with digital holography has accelerated the development of intelligent holographic imaging systems across numerous scientific disciplines. In biomedical research, AI-assisted DHM enables automated identification of cancer cells, malaria-infected erythrocytes, bacteria, sperm cells, microorganisms, stem cells, and circulating tumor cells using quantitative phase information. Intelligent holographic imaging has also been successfully employed for COVID-19 screening, apoptosis and necroptosis detection, tissue characterization, drug response monitoring, and label-free cellular phenotyping. Beyond

healthcare, AI-powered holographic imaging has found important applications in industrial defect inspection, tribological wear analysis, optical encryption, biometric authentication, environmental monitoring, particle tracking, crystallography, and microfluidic visualization.

Simultaneously, the emergence of computer-generated holography (CGH), optical scanning holography, terahertz holography, polarization-sensitive holography, and multiplexed holographic imaging has considerably expanded the capabilities of computational optical systems. These technologies increasingly incorporate learning-based optimization strategies to improve hologram synthesis, phase reconstruction, adaptive focusing, image quality enhancement, and three-dimensional visualization. Consequently, digital holography has evolved from a purely optical recording technique into an intelligent computational imaging framework driven by advanced mathematical modeling and data-centric learning algorithms.

Although substantial progress has been achieved, several challenges continue to hinder the widespread deployment of AI-enabled holographic systems. Large annotated datasets remain limited, reducing the generalizability of learning-based models across different imaging platforms. Deep neural networks often operate as black-box models, raising concerns regarding interpretability and clinical reliability. Furthermore, balancing reconstruction accuracy, computational efficiency, real-time processing, robustness against experimental noise, and hardware implementation remains an active research problem. Standardized evaluation protocols, publicly available benchmark datasets, and explainable AI methodologies are therefore essential to facilitate fair comparison among reconstruction algorithms and accelerate translation into practical imaging systems.

Guan *et al.* [1] proposed a hybrid clustering and fractional Fourier transform (FrFT)-based reconstruction framework for off-axis digital holography. Their method effectively suppresses reconstruction noise while improving image quality and computational robustness for holographic phase recovery. Liu *et al.* [2] introduced a learnable fractional Fourier transform integrated with deep neural networks for computer-generated holography. The proposed data-driven framework significantly enhances hologram reconstruction accuracy and demonstrates the potential of trainable optical transforms in computational imaging. Topcu and Tozburun [3] developed a hybrid FrFT-based workflow that combines filtering and image reconstruction within a single digital holography pipeline. Their unified approach reduces computational complexity while preserving high-frequency image information and reconstruction fidelity. El-Shafai *et al.* [4] presented a software-based optical scanning holography framework for secure biometric authentication and modulation classification. Their work highlights the integration of holographic imaging with intelligent security algorithms for next-generation biometric systems. Wang and Yu [5] proposed a phase retrieval algorithm that combines off-axis digital holography with the fractional Fourier transform. Their approach achieves improved phase estimation accuracy and effectively suppresses twin-image interference during holographic reconstruction. Asoudegi [6] investigated layered and orbital angular momentum multiplexed holography using deep learning techniques. The study demonstrated that artificial intelligence can efficiently optimize hologram generation, multiplexing capacity, and optical information transmission.

Huang *et al.* [7] developed a Gaussian-assisted inline holography framework for complex-valued three-dimensional atomic spectroscopy. Their methodology enables high-resolution volumetric imaging with accurate phase retrieval and enhanced spectral characterization. Pereselkov *et al.* [8] applied the fractional Fourier transform to hologram formation for moving acoustic sources. Their study extends holographic reconstruction concepts beyond optical imaging and demonstrates the applicability of FrFT in dynamic acoustic holography. Kumar *et al.* [9] proposed a lensless Fourier-transform multiplexed digital holography technique for simultaneous hologram acquisition. Their method simplifies optical configurations while improving imaging efficiency and multiplexing capability. Zhang *et al.* [10]

presented a lensless Fourier-transform terahertz digital holography system for real-time full-field phase imaging. Their work demonstrates the feasibility of high-speed, non-contact terahertz holographic imaging with quantitative phase measurement. Pandey and Hennelly [11] investigated the influence of quantization noise in lensless Fourier digital holography and proposed effective noise reduction techniques. Their research established important theoretical foundations for improving reconstruction accuracy in digital holographic imaging. Dong *et al.* [12] introduced a multi-illumination lensless Fourier-transform digital holographic interferometry method for precise surface shape measurement. Their approach significantly improves measurement accuracy and enhances the reliability of optical metrology applications. Zhang *et al.* [13] developed a reflective lensless Fourier-transform terahertz digital holography framework for full-field imaging. Their technique extends holographic imaging to reflective objects, offering improved performance for nondestructive testing and industrial inspection.

Chen *et al.* [14] employed digital holography to quantitatively analyze cancer cell morphology under high-temperature stimulation. Their study demonstrated that quantitative phase imaging enables label-free assessment of cellular morphological variations, highlighting the potential of holography in cancer diagnostics and thermal therapy monitoring. Kumar *et al.* [15] developed a quantitative polarization digital holographic microscope capable of simultaneously measuring amplitude, phase, and polarization information. Their approach improves quantitative characterization of biological specimens and optical materials with enhanced imaging accuracy. Moon and Javidi [16] presented an AI-driven digital holographic microscopy framework for label-free quantitative cellular analysis. By integrating deep learning with digital holography, their method provides accurate cell identification while supporting low-cost and field-dedeployable biomedical diagnostic platforms. Huang *et al.* [17] explored the integration of digital holography with crystallography for mineral crystallization analysis. Their work demonstrated that holographic imaging offers valuable quantitative information for studying crystal growth dynamics and microstructural evolution. Guo *et al.* [18] proposed an off-axis digital holographic microscope with an extended field of view. Their optical configuration significantly enlarges the imaging area while maintaining high spatial resolution, making it suitable for large-scale biological specimen analysis. Ge *et al.* [19] introduced a compressive dark-field digital holography framework for spatio-temporal microfluidic reconstruction. Their methodology enables accurate visualization of dynamic microfluidic processes while reducing data acquisition requirements. Farhat *et al.* [20] investigated the morphological and optical characterization of NRAS-mutant melanoma cells using quantitative phase imaging and digital holographic microscopy. Their results demonstrate the effectiveness of holographic microscopy for label-free characterization of cancer cells with high morphological sensitivity. Li *et al.* [21] proposed a recurrent iterative phase-shifting strategy for single-exposure incoherent self-interference digital holography. The developed algorithm enhances phase reconstruction accuracy while reducing computational artifacts associated with single-shot holographic imaging.

Zhang *et al.* [22] presented a fractional Fourier transform-based off-axis digital holographic imaging technique. Their reconstruction framework improves image quality and phase recovery by optimizing the fractional transform domain for holographic processing. Li *et al.* [23] developed a robust off-axis digital holographic reconstruction algorithm based on Kronecker interpolation and self-optimized fractional Fourier transform. Their approach effectively suppresses reconstruction errors and significantly enhances holographic image quality under challenging imaging conditions. Lam *et al.* [24] utilized telecentric digital holographic microscopy and machine learning to quantitatively evaluate cancer cell morphology and motility. Their study demonstrated that combining quantitative phase imaging with intelligent classification provides accurate label-free discrimination of cancer cell phe-

notypes. Nguyen *et al.* [25] proposed a deep learning-based background detection method for automatic phase aberration compensation in digital holographic microscopy. Their algorithm substantially improves reconstruction quality by eliminating phase distortions without manual intervention. Zeng *et al.* [26] presented a comprehensive review of deep learning applications in digital holography. Their work summarized recent advances in hologram reconstruction, phase retrieval, image enhancement, and quantitative phase imaging while highlighting emerging research challenges and future opportunities.

Wenbo *et al.* [27] reviewed the application of deep learning techniques in the phase processing of digital holographic microscopy. Their study demonstrated that convolutional neural networks and other learning-based approaches substantially improve phase reconstruction accuracy while reducing computational complexity. Verduijn *et al.* [28] developed a deep learning framework to discriminate apoptosis and necroptosis using digital holographic microscopy. Their label-free methodology achieved reliable cell death classification, demonstrating the potential of artificial intelligence in biomedical diagnostics. Dubey *et al.* [29] combined partially spatially coherent digital holographic microscopy with machine learning for quantitative analysis of human spermatozoa under oxidative stress. Their approach provides an efficient, non-invasive tool for reproductive health assessment and cellular morphology analysis. Kim *et al.* [30] proposed a deep learning-based image processing framework to improve three-dimensional profile quality in digital holographic microscopy. Their algorithm effectively enhances image resolution while suppressing reconstruction artifacts, leading to more accurate quantitative measurements. Zhang *et al.* [31] introduced a fast phase retrieval technique for off-axis digital holographic microscopy using deep neural networks. Their model significantly accelerates holographic reconstruction while maintaining high reconstruction accuracy, enabling real-time holographic imaging. O'Connor *et al.* [32] developed a deep learning-based framework for cell identification and disease diagnosis using spatio-temporal cellular dynamics acquired through compact digital holographic microscopy. Their methodology demonstrates the effectiveness of artificial intelligence for automated biomedical screening and diagnostic decision support. O'Connor *et al.* [33] proposed a portable digital holographic deep learning system for rapid COVID-19 screening using red blood cell morphology. Their study highlights the capability of combining holographic microscopy with artificial intelligence for rapid and field-deployable infectious disease detection. Kim *et al.* [34] presented a smartphone-based digital holographic microscopy platform integrated with deep learning for volumetric monitoring of airborne particulate matter. Their low-cost imaging system enables real-time environmental monitoring with high measurement accuracy. Lee *et al.* [35] developed a deep learning algorithm for accurate three-dimensional tracking of microparticles using digital holographic microscopy. Their method provides rapid and robust estimation of particle trajectories, benefiting microfluidics and particle dynamics research. Go *et al.* [36] proposed a machine learning-assisted digital in-line holographic microscopy system for automatic identification of erythrocytes. Their label-free framework achieves accurate classification of red blood cells and demonstrates the applicability of artificial intelligence in hematological analysis. Di *et al.* [37] introduced a deep learning-based quantitative phase imaging method using holographic microscopy. Their approach substantially improves phase reconstruction accuracy while reducing imaging artifacts, facilitating high-precision cellular imaging. Byeon *et al.* [38] developed a deep learning-enhanced digital in-line holographic microscopy technique that simultaneously achieves high spatial resolution and an extended field of view. Their method significantly improves holographic image reconstruction for large-scale biological observations. Valentino *et al.* [39] integrated polarization-sensitive digital holographic microscopy with machine learning for automated textile fiber discrimination. Their framework demonstrates that intelligent holographic imaging can accurately classify complex materials, extending holography's applications beyond biomedical

imaging.

Belashov *et al.* [40] investigated photoinduced necrosis in HeLa cells using digital holographic microscopy combined with machine learning. Their label-free framework enabled quantitative monitoring of cellular responses, demonstrating the effectiveness of intelligent holographic imaging in biomedical research. Bedrossian *et al.* [41] developed a machine learning algorithm for identifying and tracking bacteria in three dimensions using digital holographic microscopy. Their approach achieved accurate bacterial localization and tracking, highlighting the potential of holography for rapid microbiological analysis. Cheremkhin *et al.* [42] reviewed recent machine learning techniques applied to digital holography and diffractive optics. Their study emphasized the growing importance of artificial intelligence in hologram generation, image reconstruction, optical design, and computational imaging. Dar *et al.* [43] presented a comprehensive overview of artificial intelligence and digital holography in biomedical microscopy. Their review demonstrated that integrating deep learning with digital holographic imaging significantly enhances disease diagnosis, cellular characterization, and quantitative biomedical analysis. Sun *et al.* [44] proposed a machine learning framework for multi-class cell line classification using digital holographic microscopy. Their methodology accurately discriminates multiple cell types from quantitative phase images, illustrating the effectiveness of intelligent holographic imaging for automated biomedical diagnostics. Deshpande *et al.* [45] integrated acoustic emission analysis, digital holographic microscopy, and machine learning for wear classification in tribological systems. Their hybrid methodology enables accurate monitoring of wear mechanisms, extending holographic imaging applications to industrial condition monitoring. Trujillo and Garcia-Sucerquia [46] developed a deep learning-based framework for automatic detection and counting of phase objects in raw holograms acquired through digital holographic microscopy. Their approach eliminates manual segmentation and significantly improves object recognition accuracy in complex holographic datasets. Matthews *et al.* [47] proposed a real-time three-dimensional tracking system for swimming microorganisms by integrating digital holographic microscopy with deep learning. Their framework provides accurate motion analysis and demonstrates the capability of AI-assisted holography for dynamic biological imaging. Go *et al.* [48] developed a machine learning-assisted in-line digital holographic sensing system for identifying unstained malaria-infected red blood cells. Their label-free methodology achieved reliable malaria detection, highlighting the potential of holographic microscopy for rapid point-of-care diagnostics. Liu *et al.* [49] introduced a deep learning-based holographic polarization microscopy framework for quantitative imaging of polarization information. Their technique improves reconstruction quality while enabling simultaneous characterization of amplitude, phase, and polarization features. Memmolo *et al.* [50] discussed the synergy between holographic imaging and machine learning for computational optical imaging. Their work highlighted how artificial intelligence enhances hologram reconstruction, feature extraction, automated interpretation, and intelligent image analysis across diverse scientific applications. Lin *et al.* [51] proposed a deep learning-based framework for automatic phase distortion identification and compensation in digital holographic microscopy. Their algorithm substantially improves reconstruction accuracy and reduces computational errors, enabling robust quantitative phase imaging. Melanthota *et al.* [52] reviewed recent advances in deep learning-based image processing techniques for optical microscopy. Their survey highlighted emerging neural network architectures that improve image restoration, segmentation, super-resolution, and quantitative biomedical analysis, providing valuable insights for future AI-driven holographic imaging systems. Ramya *et al.* [53] investigated the holographic, thermodynamic, and topological properties of AdS Einstein–Power–Yang–Mills black holes using generalized entropy. The study demonstrated how generalized entropy frameworks can provide deeper insights into black-hole thermodynamics, phase structure, and topological characteristics.

Existing review articles generally concentrate on isolated aspects of digital holography, including phase retrieval algorithms, quantitative phase imaging, biomedical microscopy, or deep learning applications. However, a comprehensive review that systematically integrates recent developments in fractional Fourier transform-based reconstruction, AI-assisted holographic imaging, digital holographic microscopy, computer-generated holography, quantitative phase imaging, and emerging biomedical applications remains limited. Moreover, the rapid growth of research published during 2024–2026 necessitates an updated synthesis of contemporary methodologies, application domains, current limitations, and future research opportunities.

Motivated by these observations, this review presents a comprehensive overview of recent developments in artificial intelligence-enhanced digital holography. The review first summarizes the fundamental principles of digital holography and quantitative phase imaging before discussing modern reconstruction algorithms based on Fourier and fractional Fourier transforms. Subsequently, recent advances in machine learning, deep learning, and physics-informed computational imaging for hologram reconstruction, phase retrieval, and image enhancement are critically examined. Finally, biomedical, industrial, and security applications are analyzed, followed by a comparative discussion of existing techniques, unresolved challenges, and promising future research directions toward next-generation intelligent holographic imaging systems.

2 Objectives of the Review

The rapid evolution of digital holography, quantitative phase imaging, and artificial intelligence has created new opportunities for intelligent computational imaging. Although numerous studies have reported significant advances in holographic reconstruction, digital holographic microscopy, fractional Fourier transform (FrFT), and deep learning, a comprehensive review integrating these developments remains limited. Therefore, the primary objectives of this review are summarized as follows:

- To present the fundamental principles of digital holography, digital holographic microscopy, quantitative phase imaging, and computer-generated holography, highlighting their importance in modern computational optical imaging.
- To critically review recent holographic reconstruction techniques, including Fourier transform, fractional Fourier transform, iterative phase retrieval, optimization-based reconstruction, and compressive sensing approaches.
- To examine the role of artificial intelligence, machine learning, deep learning, convolutional neural networks, generative adversarial networks, transformers, and physics-informed neural networks in improving hologram reconstruction, phase retrieval, image enhancement, and automated feature extraction.
- To summarize recent biomedical applications of intelligent digital holography, including cancer diagnosis, quantitative phase imaging, blood cell analysis, microorganism identification, infectious disease screening, and label-free cellular imaging.
- To investigate emerging applications of digital holography in optical security, biometric authentication, microfluidics, crystallography, industrial inspection, particle tracking, and computational microscopy.

- To compare existing computational methodologies with respect to reconstruction accuracy, computational complexity, noise robustness, image quality, and real-time implementation capability.
- To identify current challenges associated with large-scale datasets, computational efficiency, hardware implementation, model interpretability, and clinical translation of AI-assisted holographic systems.
- To highlight future research directions involving explainable artificial intelligence, foundation models, edge intelligence, multimodal learning, quantum-inspired imaging, and next-generation computational holography.

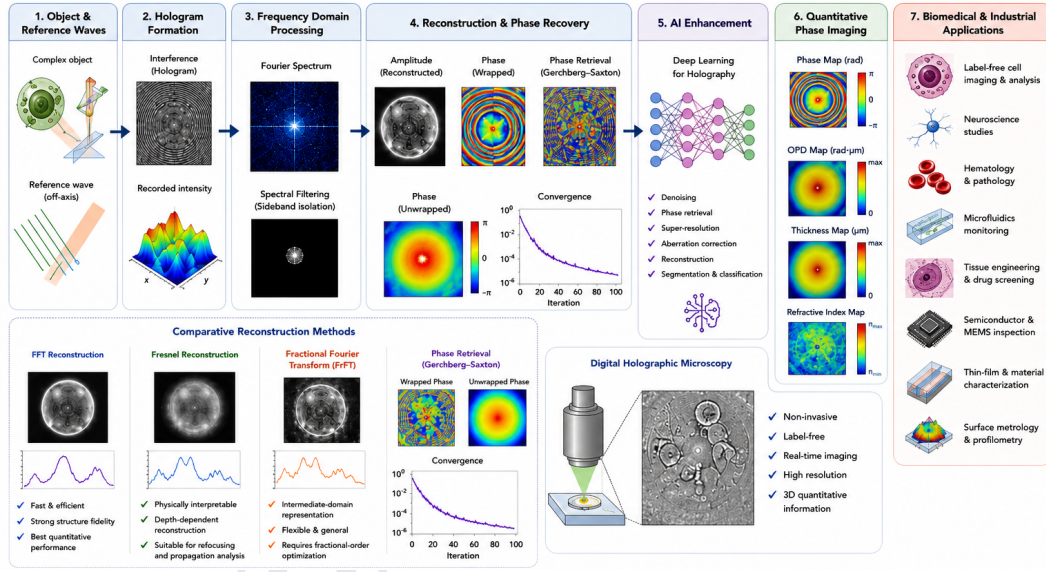


Figure 1: Graphical abstract illustrating the AI-enhanced digital holography framework, integrating complex object and reference wave generation, hologram formation, frequency-domain processing, numerical reconstruction, phase retrieval, artificial intelligence enhancement, quantitative phase imaging, and biomedical and industrial applications.

3 Novelty of the Review

Unlike previous review articles that primarily focus on either digital holography, digital holographic microscopy, quantitative phase imaging, or artificial intelligence independently, this review presents a unified perspective that integrates recent advances across these rapidly evolving research domains. The distinctive contributions of this review are summarized as follows.

- A comprehensive synthesis of recent developments (primarily 2024–2026) in digital holography, fractional Fourier transform, digital holographic microscopy, and artificial intelligence.

- A unified discussion of classical optical reconstruction methods together with modern data-driven computational approaches based on machine learning and deep learning.
- A detailed comparison of holographic reconstruction algorithms, including Fourier transform, fractional Fourier transform, phase retrieval, optimization-based methods, and learning-based reconstruction techniques.
- A systematic review of deep learning architectures, including convolutional neural networks, autoencoders, generative adversarial networks, vision transformers, and physics-informed neural networks for holographic image reconstruction and quantitative phase imaging.
- A comprehensive analysis of biomedical applications, covering cancer diagnosis, blood cell classification, cell morphology analysis, microorganism detection, infectious disease screening, and label-free quantitative imaging.
- An extensive overview of emerging applications in optical security, biometric authentication, crystallography, microfluidics, industrial metrology, and computational microscopy.
- A critical discussion of current limitations, including data scarcity, computational cost, model interpretability, robustness, generalization capability, and real-time implementation challenges.
- A future research roadmap outlining opportunities in explainable artificial intelligence, foundation models, digital twins, edge computing, multimodal holographic imaging, and quantum-enhanced computational optics.

Overall, this review bridges the gap between computational optics and artificial intelligence by providing a comprehensive roadmap for developing next-generation intelligent holographic imaging systems for biomedical, industrial, and scientific applications. As illustrated in Fig. 1, the proposed review framework integrates optical wave propagation, hologram formation, frequency-domain processing, advanced reconstruction methods, artificial intelligence, quantitative phase imaging, and biomedical and industrial applications.

Table 1 summarizes the major milestones in the evolution of digital holography, highlighting the transition from conventional Fourier-based reconstruction techniques to modern computational imaging frameworks. Early investigations primarily focused on minimizing quantization errors and improving reconstruction fidelity in lensless holography, whereas more recent studies have introduced terahertz imaging, multiplexed holography, and advanced optical recording strategies. These developments collectively demonstrate the continuous progression of digital holography toward high-resolution, real-time, and application-oriented imaging systems suitable for biomedical, industrial, and scientific investigations. Table 2 presents representative studies on hologram formation methodologies employed in recent digital holographic imaging systems. The reported approaches illustrate the evolution from classical optical wavefront recording toward computationally optimized hologram generation using fractional Fourier transform, Gaussian-assisted propagation models, orbital angular momentum multiplexing, and learning-based optical frameworks. These advancements significantly improve wavefront encoding, phase accuracy, optical efficiency, and reconstruction robustness while expanding holography's applicability to three-dimensional imaging, optical communication, and computational microscopy. Table 3 provides a comparative overview of contemporary holographic reconstruction algorithms by categorizing them according to their

Table 1: Representative studies on the historical development of digital holography.

Ref.	Author (Year)	Main Contribution	Significance
[11]	Pandey and Hennelly (2011)	Analyzed quantization noise in lensless Fourier digital holography.	Provided theoretical foundation for improving reconstruction quality.
[12]	Dong <i>et al.</i> (2017)	Developed multi-illumination lensless Fourier transform holographic interferometry.	Improved precision surface measurement.
[10]	Zhang <i>et al.</i> (2022)	Introduced real-time terahertz Fourier transform digital holography.	Enabled non-contact quantitative phase imaging.
[13]	Zhang <i>et al.</i> (2022)	Presented reflective Fourier-transform terahertz holography.	Extended holography to reflective object inspection.
[9]	Kumar <i>et al.</i> (2025)	Proposed lensless Fourier transform multiplexed holography.	Reduced optical complexity while increasing imaging efficiency.

Table 2: Recent developments in hologram formation and optical recording techniques.

Ref.	Author	Methodology	Application
[8]	Pereselkov <i>et al.</i>	Fractional Fourier transform for acoustic hologram formation.	Dynamic acoustic source imaging.
[7]	Huang <i>et al.</i>	Gaussian-assisted inline holography.	Three-dimensional atomic spectroscopy.
[5]	Wang and Yu	FrFT-assisted phase retrieval.	Off-axis digital holography.
[6]	Asoudegi	Deep learning-based layered OAM holography.	Optical communication.
[2]	Liu <i>et al.</i>	Learnable fractional Fourier transform.	Computer-generated holography.

computational principles and artificial intelligence integration. Conventional Fourier-based approaches remain computationally efficient but often encounter limitations in noise suppression and phase recovery. In contrast, fractional Fourier transform methods, iterative phase retrieval algorithms, and deep learning-based reconstruction techniques achieve superior image quality, enhanced reconstruction accuracy, and improved computational adaptability. The recent emergence of hybrid AI-FrFT frameworks further demonstrates the growing convergence between computational optics and artificial intelligence, enabling robust, automated, and high-speed holographic reconstruction for next-generation imaging applications. Overall, the studies summarized in Tables 1–3 demonstrate the rapid evolution of digital holography from conventional optical recording techniques toward intelligent computational imaging systems. The integration of advanced mathematical reconstruction algorithms with artificial intelligence has substantially improved image quality, phase retrieval accuracy, computational efficiency, and application versatility. These advances provide the foundation for the subsequent discussion on deep learning-assisted holographic microscopy, quantitative phase imaging, and emerging biomedical applications.

Table 3: Comparison of recent holographic reconstruction techniques.

Method	Representative ences	Refer- ences	AI-based	Advantages
Fourier Transform	[11], [12], [10]		No	Simple implementation and fast reconstruction.
Fractional Fourier Transform	[1], [2], [3], [22], [23]		Partially	Improved focusing accuracy and noise suppression.
Phase Retrieval	[5], [21]		No	High phase accuracy with reduced twin-image artifacts.
Deep Learning Reconstruction	[25], [31], [51]		Yes	Fast reconstruction and automatic aberration correction.
Machine Learning Reconstruction	[26], [50], [42]		Yes	Automated feature extraction and intelligent reconstruction.
Hybrid AI-FrFT	[1], [2], [3]		Yes	High reconstruction accuracy with reduced computational cost.

4 Fractional Fourier Transform in Digital Holography

The fractional Fourier transform (FrFT) has emerged as a versatile mathematical framework that extends the conventional Fourier transform by introducing an additional fractional-order domain for signal representation. Unlike classical Fourier analysis, the FrFT enables continuous transitions between the spatial and frequency domains, providing greater flexibility for wavefront manipulation, hologram reconstruction, and optical signal processing. Recent advances demonstrate that FrFT-based algorithms significantly improve focusing accuracy, phase estimation, noise suppression, and image reconstruction quality in both inline and off-axis digital holography. Furthermore, the integration of FrFT with optimization algorithms and artificial intelligence has enhanced computational efficiency and reconstruction robustness, making it a fundamental component of next-generation computational holographic imaging systems.

5 Phase Retrieval Algorithms

Accurate phase recovery remains one of the most critical challenges in digital holography because image quality strongly depends on precise reconstruction of the complex optical wavefront. Conventional phase retrieval techniques are generally based on iterative optimization, transport-of-intensity equations, or interferometric measurements; however, these approaches often suffer from convergence limitations, sensitivity to experimental noise, and high computational cost. Recent developments in learning-based phase retrieval have introduced data-driven models capable of directly estimating phase information from holograms with improved accuracy and computational efficiency. Consequently, modern phase retrieval algorithms have become essential for quantitative phase imaging, holographic microscopy, and real-time optical sensing.

6 Computer-Generated Holography

Computer-generated holography (CGH) represents a computational paradigm in which holograms are synthesized numerically rather than optically recorded. Recent developments in numerical wave propagation, optimization methods, and deep learning have substantially improved hologram synthesis by enhancing image fidelity, reducing computational complexity, and minimizing reconstruction artifacts. Modern CGH techniques support dynamic three-dimensional visualization, augmented and virtual reality displays, optical communication, beam shaping, and adaptive optical manipulation. The incorporation of learning-based hologram generation has further accelerated real-time holographic display technologies and intelligent computational imaging.

7 Deep Learning Architectures for Digital Holography

Deep learning has emerged as an important computational framework for digital holography, particularly for hologram reconstruction, phase estimation, image enhancement, denoising, segmentation, and quantitative feature extraction. Unlike conventional reconstruction techniques that explicitly rely on optical propagation models and carefully selected numerical parameters, learning-based approaches formulate holographic reconstruction as a data-driven inverse problem. A neural network is trained to establish a nonlinear mapping between the recorded holographic measurement and a desired representation of the optical field, such as the reconstructed amplitude, phase, or quantitative phase image.

Several deep learning architectures have been investigated for holographic imaging, including convolutional neural networks (CNNs), residual networks, encoder-decoder and U-Net architectures, autoencoders, generative adversarial networks (GANs), vision transformers (ViTs), and physics-informed neural networks (PINNs). CNN-based models can extract spatially localized features from holographic intensity distributions, whereas residual and encoder-decoder architectures facilitate the recovery of fine-scale structural information while maintaining efficient feature propagation. GAN-based approaches introduce an adversarial learning mechanism that can improve the perceptual quality of reconstructed images, although their performance may depend strongly on the representativeness of the training data. More recently, transformer-based architectures have provided an alternative mechanism for modelling long-range spatial dependencies, while physics-informed approaches incorporate physical constraints or governing optical relationships into the learning process.

The general learning formulation can be expressed as

$$\hat{Y} = f_{\theta}(X), \quad (7.1)$$

where X denotes the recorded holographic measurement, Y represents the corresponding target reconstruction, f_{θ} is a parameterized neural network, and $\hat{Y}$ is the predicted reconstruction. The network parameters are obtained by minimizing an appropriate objective function, “ latex

$$\theta^* = \arg \min_{\theta} \mathcal{L}(Y, f_{\theta}(X)), \quad (7.2)$$

where $\mathcal{L}$ represents the training loss. Depending on the reconstruction objective, the loss function may incorporate pixel-wise fidelity, structural similarity, perceptual consistency, or physics-based constraints. A representative composite loss may be written as

$$\mathcal{L} = \lambda_1 \mathcal{L}_{\text{MSE}} + \lambda_2 \mathcal{L}_{\text{SSIM}} + \lambda_3 \mathcal{L}_{\text{per}}, \quad (7.3)$$

where λ_1 , λ_2 , and λ_3 are non-negative weighting coefficients that determine the relative contributions of the individual loss components, while $\mathcal{L}_{\text{MSE}}$, $\mathcal{L}_{\text{SSIM}}$, and $\mathcal{L}_{\text{per}}$ denote the mean-squared-error, structural-similarity, and perceptual loss terms, respectively. For digital holography, the input-output relationship can be formulated more explicitly as

$$X = I_H(x, y), \quad Y = [A_O(x, y), \phi_O(x, y)], \quad (7.4)$$

where I_H denotes the recorded hologram, while A_O and ϕ_O represent the amplitude and phase of the desired object field. Depending on the application, the target Y may instead correspond to a reconstructed intensity image, unwrapped phase distribution, optical path difference, thickness map, or other quantitative representation.

An important advantage of learning-based reconstruction is that the computationally intensive training stage can be separated from the subsequent inference stage. Once a network has been trained, a new holographic measurement can be processed through a forward network evaluation rather than repeating the complete numerical reconstruction procedure. Nevertheless, the practical benefit depends on the network architecture, hardware, input size, training strategy, and deployment environment. In addition, the training process itself may require substantial computational resources and a sufficiently representative dataset.

The reconstruction quality of a deep learning model is strongly dependent on the correspondence between the training and application domains. Variations in illumination, wavelength, sensor characteristics, optical aberrations, speckle, noise statistics, object morphology, propagation distance, and experimental alignment can introduce domain shifts between training and test data. Consequently, high performance on a particular dataset does not necessarily imply equivalent performance for experimentally acquired holograms or previously unseen biological specimens. Dataset diversity, independent testing, and quantitative validation are therefore essential for assessing the generalization capability of a learning-based holographic reconstruction model.

In the present work, deep learning is considered as an intelligent reconstruction framework complementary to conventional Fourier-, Fresnel-, fractional-Fourier-, and phase-retrieval-based approaches. Although deep learning methods have the potential to provide rapid inference and robust reconstruction under appropriately represented imaging conditions, such advantages are task- and dataset-dependent. Therefore, claims regarding superior accuracy, noise robustness, or computational efficiency should be supported by quantitative comparisons performed under identical input data, reconstruction targets, and evaluation metrics. In the absence of a dedicated experimentally trained and independently tested deep learning model in the present study, these advantages are discussed as potential capabilities of learning-based holographic reconstruction rather than as experimentally established superiority over the conventional methods evaluated here. The advantages of deep-learning-based holographic reconstruction reported in the literature are not directly established by the numerical benchmark presented in this review. The present numerical experiment evaluates conventional FFT, Fresnel, FrFT, and phase-retrieval approaches using a controlled synthetic hologram. Therefore, statements regarding superior accuracy, noise robustness, or computational efficiency of deep-learning methods are interpreted as literature-supported capabilities rather than experimentally demonstrated advantages of the present study. A direct AI-versus-conventional comparison requires an independently trained and tested model using identical holographic inputs, reference targets, evaluation metrics, and computational conditions.

8 Machine Learning Algorithms

Machine learning algorithms provide efficient computational tools for extracting quantitative information from holographic images through automated classification, regression, clustering, and anomaly detection. Supervised and unsupervised learning techniques have been successfully applied to cell recognition, microorganism identification, particle tracking, optical metrology, and defect inspection using quantitative phase information. Compared with traditional image processing approaches, machine learning offers improved generalization capability, reduced operator dependency, and enhanced decision-making performance across diverse holographic imaging applications.

9 Quantitative Phase Imaging

Quantitative phase imaging (QPI) is one of the most significant applications of digital holography because it enables label-free measurement of optical path length variations produced by transparent specimens. Unlike conventional intensity-based microscopy, QPI simultaneously provides quantitative information regarding cell morphology, refractive index distribution, membrane dynamics, thickness, and intracellular structures. Recent computational developments integrating artificial intelligence and advanced reconstruction algorithms have substantially improved measurement precision, imaging throughput, and automated interpretation, thereby expanding QPI applications in life sciences, pathology, and biomedical engineering.

10 Digital Holographic Microscopy

Digital holographic microscopy (DHM) combines interferometric imaging with computational reconstruction to provide non-contact, label-free, three-dimensional visualization of microscopic specimens. Owing to its capability to simultaneously reconstruct amplitude and phase information, DHM enables quantitative characterization of living cells, microorganisms, biomaterials, and microfluidic systems with high temporal and spatial resolution. Recent developments in computational optics, artificial intelligence, and quantitative phase imaging have significantly improved reconstruction quality, imaging speed, and automated analysis, establishing DHM as a powerful platform for intelligent biomedical imaging.

11 Biomedical Applications

The integration of digital holography with artificial intelligence has considerably expanded the scope of biomedical imaging by enabling rapid, label-free, and quantitative analysis of biological specimens. Recent investigations demonstrate successful applications in cancer diagnosis, blood cell classification, malaria detection, COVID-19 screening, bacterial identification, sperm analysis, tissue characterization, stem cell monitoring, and microorganism tracking. By combining quantitative phase imaging with machine learning and deep learning, intelligent holographic systems achieve high diagnostic accuracy while reducing dependence on chemical staining and manual interpretation. These advances position AI-assisted digital holography as a promising technology for precision medicine, digital pathology, and next-generation clinical diagnostics.

12 Mathematical Formulation

Digital holography is based on the interference between an object wave and a coherent reference wave. The sensor records the resulting interference intensity, from which the complex optical field is subsequently recovered through numerical reconstruction. The mathematical framework considered in this study includes hologram formation, off-axis spectral filtering, Fourier-domain reconstruction, Fresnel propagation, fractional Fourier transform (FrFT) reconstruction, Gerchberg–Saxton (GS) phase retrieval, quantitative phase imaging (QPI), and learning-based reconstruction. Particular attention is given to the distinction between the recorded intensity hologram and the complex optical field required for numerical propagation.

12.1 Object and Reference Waves

The complex object wave at the sensor plane is represented as

$$O(x, y) = A_O(x, y) \exp[i\phi_O(x, y)], \quad (12.1)$$

where $A_O(x, y)$ denotes the amplitude distribution and $\phi_O(x, y)$ represents the phase distribution of the object wave.

The coherent reference wave is expressed as

$$R(x, y) = A_R(x, y) \exp[i\phi_R(x, y)], \quad (12.2)$$

where $A_R(x, y)$ denotes the reference-wave amplitude and $\phi_R(x, y)$ represents its phase distribution.

For an off-axis configuration, the reference wave contains a spatial carrier. In the case of a plane reference wave,

$$\phi_R(x, y) = 2\pi(f_{Rx}x + f_{Ry}y) + \phi_0, \quad (12.3)$$

where f_{Rx} and f_{Ry} denote the carrier spatial frequencies in the x - and y -directions, respectively, and ϕ_0 is the initial reference phase. The spatial carrier provides separation of the zero-order and conjugate diffraction terms from the desired object–reference cross term in the Fourier domain.

12.2 Digital Hologram Formation

The intensity recorded by the digital sensor is given by

$$I_H(x, y) = |O(x, y) + R(x, y)|^2, \quad (12.4)$$

which can be expanded as

$$I_H(x, y) = |O(x, y)|^2 + |R(x, y)|^2 + O(x, y)R^*(x, y) + O^*(x, y)R(x, y), \quad (12.5)$$

where $(\cdot)^*$ denotes the complex conjugate.

The first two terms correspond to the zero-order or autocorrelation component, whereas the last two terms represent the object–reference and conjugate interference components. The phase information required for numerical reconstruction is therefore encoded in the interference terms rather than being directly measured as an independent phase quantity.

For an off-axis hologram, the spatial carrier introduced by the reference wave shifts the desired first-order diffraction component away from the zero-order component in the spatial-frequency domain. This separation enables numerical spectral filtering before complex-field reconstruction.

12.3 Off-Axis Spectral Filtering and Complex-Field Reconstruction

The two-dimensional Fourier transform of the recorded hologram is defined as

$$\mathcal{H}(u, v) = \mathcal{F}_{2D} \{I_H(x, y)\}, \quad (12.6)$$

where $\mathcal{F}_{2D}\{\cdot\}$ denotes the two-dimensional Fourier transform.

The Fourier spectrum contains the zero-order component together with the two conjugate first-order diffraction components. A spectral window $W(u, v)$ is therefore introduced to isolate the desired first-order diffraction order:

$$\mathcal{H}_f(u, v) = \mathcal{H}(u, v)W(u, v). \quad (12.7)$$

The selected spectral component is shifted towards the origin when required:

$$\tilde{\mathcal{H}}_f(u, v) = \mathcal{H}_f(u - u_c, v - v_c), \quad (12.8)$$

where (u_c, v_c) denotes the centre of the selected diffraction order.

The corresponding complex field at the hologram plane is then obtained using the inverse Fourier transform:

$$U_0(x, y) = \mathcal{F}_{2D}^{-1} \{ \tilde{\mathcal{H}}_f(u, v) \}. \quad (12.9)$$

Consequently, the raw intensity hologram is not directly treated as a complex optical field. The desired first-order diffraction component is first isolated in the Fourier domain, thereby reducing the influence of the zero-order and conjugate twin-image terms. The resulting complex field $U_0(x, y)$ is subsequently used for numerical propagation and phase reconstruction.

12.4 Fourier Transform Reconstruction

The two-dimensional Fourier transform is defined as

$$F(u, v) = \iint f(x, y) e^{-j2\pi(ux+vy)} dx dy, \quad (12.10)$$

with the corresponding inverse transform given by

$$f(x, y) = \iint F(u, v) e^{j2\pi(ux+vy)} du dv. \quad (12.11)$$

Following spectral isolation of the desired diffraction order, the Fourier-domain reconstructed field is expressed as

$$U_{\text{FFT}}(x, y) = \mathcal{F}_{2D}^{-1} \{ W(u, v) \mathcal{F}_{2D} [I_H(x, y)] \}. \quad (12.12)$$

The reconstructed amplitude and phase are respectively obtained from

$$A_{\text{FFT}}(x, y) = |U_{\text{FFT}}(x, y)|, \quad (12.13)$$

and

$$\phi_{\text{FFT}}(x, y) = \arg [U_{\text{FFT}}(x, y)]. \quad (12.14)$$

The Fourier-based reconstruction is computationally efficient and is particularly suitable for off-axis holography in which the desired diffraction order can be separated in the spatial-frequency domain.

12.5 Fresnel Numerical Propagation

The filtered complex field $U_0(x, y)$ is propagated numerically to a reconstruction plane located at a distance z from the hologram plane. Under the Fresnel approximation, the propagated field is given by

$$U_F(x, y, z) = \frac{\exp(ikz)}{i\lambda z} \iint U_0(\xi, \eta) \exp \left[\frac{ik}{2z} \{ (x - \xi)^2 + (y - \eta)^2 \} \right] d\xi d\eta, \quad (12.15)$$

where

$$k = \frac{2\pi}{\lambda} \quad (12.16)$$

is the wave number, λ is the illumination wavelength, and z denotes the reconstruction distance.

For numerical implementation, the Fresnel propagation can equivalently be expressed in the Fourier domain as

$$U_F(x, y, z) = \mathcal{F}^{-1} \{ \mathcal{F} [U_0(x, y)] H_F(u, v; z) \}, \quad (12.17)$$

where

$$H_F(u, v; z) = \exp(ikz) \exp [-i\pi\lambda z (u^2 + v^2)] \quad (12.18)$$

is the Fresnel transfer function.

The corresponding reconstructed amplitude and phase are

$$A_F(x, y, z) = |U_F(x, y, z)|, \quad (12.19)$$

and

$$\phi_F(x, y, z) = \arg [U_F(x, y, z)]. \quad (12.20)$$

The Fresnel reconstruction depends on the wavelength, propagation distance, detector sampling interval, image dimensions, and numerical aperture. These parameters are therefore explicitly reported in the numerical methodology to ensure reproducibility.

12.6 Fractional Fourier Transform Reconstruction

The fractional Fourier transform (FrFT) provides a continuously adjustable transform domain between the spatial and conventional Fourier domains. The FrFT of order α is represented as

$$F_\alpha(u) = \int_{-\infty}^{\infty} K_\alpha(u, x) f(x) dx, \quad (12.21)$$

where $K_\alpha(u, x)$ denotes the fractional Fourier transform kernel.

For the present formulation,

$$K_\alpha(u, x) = A_\alpha \exp [i\pi (u^2 + x^2) \cot \alpha - 2i\pi ux \csc \alpha], \quad (12.22)$$

where

$$A_\alpha = \sqrt{\frac{1 - i \cot \alpha}{2\pi}}. \quad (12.23)$$

The corresponding two-dimensional holographic reconstruction is represented as

$$U_\alpha(x, y) = \mathcal{F}_\alpha^{2D} \{U_0(x, y)\}. \quad (12.24)$$

Unlike the conventional Fourier transform, the FrFT contains the additional fractional-order parameter α . Consequently, the reconstruction quality can vary substantially with the selected fractional order. A single arbitrarily selected order is therefore insufficient for establishing the relative performance of FFT and FrFT.

To address this parameter dependence, the fractional order is systematically varied according to

$$\alpha \in [\alpha_{\min}, \alpha_{\max}], \quad \alpha = \alpha_{\min} + m\Delta\alpha, \quad (12.25)$$

where m is an integer index and $\Delta\alpha$ denotes the order increment.

For every tested value of α , the reconstructed image is quantitatively evaluated using MSE, PSNR, SSIM, and Pearson correlation coefficient. The optimum fractional order is determined according to

$$\alpha^* = \arg \max_{\alpha} J(\alpha), \quad (12.26)$$

where $J(\alpha)$ denotes the selected reconstruction-quality criterion.

When PSNR is adopted as the primary optimization criterion,

$$\alpha^* = \arg \max_{\alpha} PSNR(\alpha). \quad (12.27)$$

A multi-metric optimization can alternatively be defined as

$$J(\alpha) = w_1 \widehat{PSNR}(\alpha) + w_2 SSIM(\alpha) + w_3 CC(\alpha), \quad (12.28)$$

subject to

$$w_1 + w_2 + w_3 = 1, \quad w_i \geq 0. \quad (12.29)$$

The optimized FrFT reconstruction is consequently expressed as

$$U_{\text{FrFT}}^* = U_{\alpha^*}. \quad (12.30)$$

This order-optimization procedure ensures that the comparison with FFT is based on an optimized FrFT reconstruction rather than on a single arbitrarily selected fractional order.

12.7 Gerchberg–Saxton Phase Retrieval

The phase distribution of a reconstructed complex field is initially obtained from

$$\phi(x, y) = \arg[U(x, y)] = \tan^{-1} \left(\frac{\text{Im}[U(x, y)]}{\text{Re}[U(x, y)]} \right). \quad (12.31)$$

For intensity-based phase retrieval, the Gerchberg–Saxton (GS) algorithm alternates between the measurement plane and reconstruction plane while enforcing the available amplitude constraints.

The initial object-plane field is represented as

$$U_0^{(o)}(x, y) = A_0^{(o)}(x, y) \exp[i\phi_0^{(o)}(x, y)], \quad (12.32)$$

where $A_0^{(o)}$ and $\phi_0^{(o)}$ denote the initial amplitude and phase assumptions, respectively.

At the k th iteration, forward propagation is performed according to

$$U_k^{(m)} = \mathcal{P} \left\{ U_k^{(o)} \right\}, \quad (12.33)$$

where $\mathcal{P}\{\cdot\}$ denotes the selected propagation operator between the object and measurement planes.

The measured amplitude constraint $A_m(x, y)$ is imposed as

$$\tilde{U}_k^{(m)} = A_m(x, y) \exp \left[i \arg \left(U_k^{(m)} \right) \right]. \quad (12.34)$$

The updated object-plane field is then obtained using the inverse propagation operator:

$$U_{k+1}^{(o)} = \mathcal{P}^{-1} \left\{ \tilde{U}_k^{(m)} \right\}. \quad (12.35)$$

If an object-domain amplitude constraint $A_o(x, y)$ is available, it is imposed according to

$$U_{k+1}^{(o)} = A_o(x, y) \exp \left[i \arg \left(U_{k+1}^{(o)} \right) \right]. \quad (12.36)$$

The normalized reconstruction error at iteration k is calculated as

$$E_k = \frac{\|A_m - |U_k^{(m)}|\|_2}{\|A_m\|_2}. \quad (12.37)$$

The iterative process is terminated when

$$E_k < \varepsilon \quad (12.38)$$

or when the prescribed maximum number of iterations, N_{iter} , is reached.

The values of the initial phase assumption, N_{iter} , and ε are specified in the numerical methodology section. Convergence is verified by monitoring the variation of E_k with iteration number.

12.8 Quantitative Phase Imaging and Relative Thickness

The recovered phase provides the basis for quantitative phase imaging. After phase unwrapping, the optical path difference is expressed as

$$OPD(x, y) = \frac{\lambda}{2\pi} \phi_{\text{corr}}(x, y), \quad (12.39)$$

where $\phi_{\text{corr}}(x, y)$ denotes the background-corrected unwrapped phase.

Because the absolute phase reference in digital holographic reconstruction is generally arbitrary, direct conversion of the raw phase into specimen thickness can introduce a constant or spatially varying offset. An object-free background region Ω_b is therefore selected to estimate the phase reference:

$$\phi_{\text{bg}} = \frac{1}{|\Omega_b|} \sum_{(x, y) \in \Omega_b} \phi_{\text{unw}}(x, y). \quad (12.40)$$

The corrected phase is then obtained from

$$\phi_{\text{corr}}(x, y) = \phi_{\text{unw}}(x, y) - \phi_{\text{bg}}. \quad (12.41)$$

For a specimen with refractive index n_s immersed in a surrounding medium of refractive index n_m , the corresponding relative phase-derived thickness is

$$h_{\text{rel}}(x, y) = \frac{\lambda \phi_{\text{corr}}(x, y)}{2\pi(n_s - n_m)}. \quad (12.42)$$

Equation (12.42) represents a relative thickness unless an independently calibrated physical zero-thickness reference is available.

Accordingly, negative values in an uncorrected phase-derived thickness map should not be interpreted as negative physical specimen thickness. They may originate from an arbitrary phase reference, background phase, optical aberrations, phase-unwrapping errors, or reconstruction bias. The corrected phase-derived thickness is therefore interpreted as a relative morphological quantity unless an absolute reference plane has been experimentally established.

12.9 Deep Learning-Based Reconstruction

Learning-based holographic reconstruction can be formulated as a nonlinear mapping between the recorded hologram and the corresponding reconstructed field:

$$\hat{Y} = f_{\theta}(X), \quad (12.43)$$

where X represents the recorded hologram, $\hat{Y}$ denotes the predicted amplitude or phase reconstruction, and θ represents the trainable network parameters.

For supervised learning, the optimal network parameters are obtained from

$$\theta^* = \arg \min_{\theta} \mathcal{L}(Y, f_{\theta}(X)), \quad (12.44)$$

where Y denotes the reference reconstruction and $\mathcal{L}$ denotes the training loss.

A general composite objective may be expressed as

$$\mathcal{L} = \lambda_1 \mathcal{L}_{MSE} + \lambda_2 \mathcal{L}_{SSIM} + \lambda_3 \mathcal{L}_{per}, \quad (12.45)$$

where $\mathcal{L}_{MSE}$ denotes the pixel-wise mean squared error, $\mathcal{L}_{SSIM}$ represents a structural similarity loss, $\mathcal{L}_{per}$ denotes an optional perceptual loss, and λ_1 , λ_2 , and λ_3 are weighting coefficients.

The dataset is partitioned into training, validation, and independent test subsets:

$$\mathcal{D} = \mathcal{D}_{train} \cup \mathcal{D}_{val} \cup \mathcal{D}_{test}, \quad (12.46)$$

with

$$\mathcal{D}_{train} \cap \mathcal{D}_{test} = \emptyset. \quad (12.47)$$

When a trained deep-learning model is included in the numerical comparison, the conventional and learning-based methods are evaluated using the same independent test data, identical normalization procedures, and identical image-quality metrics.

Importantly, the mathematical formulation of learning-based reconstruction does not by itself establish superiority over conventional reconstruction. Such a claim requires quantitative experimental or numerical evidence obtained under comparable conditions.

12.10 Performance Evaluation Metrics

The reconstruction fidelity is quantified using Mean Squared Error (MSE), Root Mean Squared Error (RMSE), Peak Signal-to-Noise Ratio (PSNR), Structural Similarity Index (SSIM), and Pearson correlation coefficient (CC).

The Mean Squared Error is defined as

$$MSE = \frac{1}{MN} \sum_{i=1}^M \sum_{j=1}^N [I_{ij} - K_{ij}]^2, \quad (12.48)$$

where I_{ij} and K_{ij} denote the reconstructed and reference intensities, respectively.

The Root Mean Squared Error is given by

$$RMSE = \sqrt{MSE}. \quad (12.49)$$

The Peak Signal-to-Noise Ratio is calculated as

$$PSNR = 10 \log_{10} \left(\frac{MAX^2}{MSE} \right), \quad (12.50)$$

where MAX denotes the maximum possible intensity value of the normalized reference image.

The Structural Similarity Index is expressed as

$$SSIM(x, y) = \frac{(2\mu_x\mu_y + C_1)(2\sigma_{xy} + C_2)}{(\mu_x^2 + \mu_y^2 + C_1)(\sigma_x^2 + \sigma_y^2 + C_2)}, \quad (12.51)$$

where μ_x and μ_y represent local mean intensities, σ_x^2 and σ_y^2 denote the corresponding local variances, σ_{xy} denotes the local covariance, and C_1 and C_2 are stabilizing constants.

The Pearson correlation coefficient is defined as

$$CC = \frac{\sum_{i=1}^{MN} (I_i - \bar{I})(K_i - \bar{K})}{\sqrt{\sum_{i=1}^{MN} (I_i - \bar{I})^2} \sqrt{\sum_{i=1}^{MN} (K_i - \bar{K})^2}}. \quad (12.52)$$

Here, $\bar{I}$ and $\bar{K}$ denote the mean intensities of the reconstructed and reference images, respectively.

MSE and RMSE quantify pixel-wise deviation, PSNR evaluates signal fidelity, SSIM measures local structural similarity, and CC quantifies global linear association. Therefore, no single metric is considered sufficient for characterizing holographic reconstruction fidelity.

In particular, a high CC does not necessarily imply a high SSIM. Pearson correlation primarily measures global linear association, whereas SSIM is sensitive to local luminance, contrast, and structural variations. Consequently, a reconstruction may preserve the overall spatial distribution of an object and simultaneously exhibit local contrast distortions, boundary artifacts, or structural deviations that reduce its SSIM value. The present study therefore interprets reconstruction performance using the complementary set of MSE, RMSE, PSNR, SSIM, and CC rather than relying on a single metric.

13 Review Methodology

This review was conducted using a structured literature-search and screening strategy to systematically examine the development of digital holography and its integration with advanced computational and artificial intelligence-based techniques. The review focused on the major research domains relevant to the present study, including digital hologram formation, Fourier- and Fresnel-based reconstruction, fractional Fourier transform (FrFT), phase-retrieval algorithms, computer-generated holography, machine learning, deep learning, quantitative phase imaging (QPI), digital holographic microscopy (DHM), and biomedical applications.

A systematic search was conducted using major scientific databases, including Scopus, Web of Science, IEEE Xplore, ScienceDirect, SpringerLink, and PubMed. The search strategy employed combinations of keywords and Boolean operators related to the principal themes of the review. The representative search terms included “digital holography”, “digital holographic microscopy”, “fractional Fourier transform”, “phase retrieval”, “computer-generated holography”, “artificial intelligence”, “machine learning”, “deep learning”, “quantitative phase imaging”, “optical path difference”, “biomedical imaging”, and “label-free microscopy”. The search strategy was designed to capture both foundational developments in holographic imaging and recent advances in AI-assisted computational reconstruction.

The retrieved records were subjected to a sequential screening procedure involving identification, duplicate removal, title and abstract screening, full-text assessment, and final inclusion. Studies were considered eligible when they addressed at least one of the following themes: (i) theoretical or experimental development of digital holography; (ii) numerical reconstruction based on Fourier, Fresnel, or fractional Fourier methods; (iii) iterative or computational phase-retrieval techniques; (iv) artificial intelligence and machine-learning-assisted holographic processing; (v) quantitative phase imaging and digital holographic microscopy; or (vi) biomedical and related application areas. Records were excluded when they were unrelated to the scope of digital holography, duplicated existing records, lacked sufficient methodological information, or did not provide a relevant scientific contribution to the topics considered in this review.

Following the screening process, the selected studies were classified according to their methodological contributions and application domains. The literature was organized into the principal thematic categories of historical development, hologram formation, numerical reconstruction, fractional Fourier transform, phase retrieval, computer-generated holography, artificial intelligence, quantitative phase imaging, digital holographic microscopy, and biomedical applications. The selected studies were critically examined with respect to their reconstruction principles, computational strategies, AI architectures, quantitative imaging capabilities, validation approaches, application areas, and reported limitations.

The review process was structured according to the Preferred Reporting Items for Systematic Reviews and Meta-Analyses (PRISMA) 2020 framework. The identification, screening, eligibility assessment, and inclusion stages are summarized in the PRISMA flow diagram presented in Fig. 2. A total of 52 studies were included in the final qualitative synthesis. The resulting literature corpus was subsequently used to develop a critical comparison between conventional holographic reconstruction methods and emerging AI-enhanced computational approaches. This methodology provides a transparent framework for synthesizing the evolution of digital holography while identifying current methodological limitations and future research opportunities. The systematic literature identification and study-selection process adopted in this review is summarized in the PRISMA 2020 flow diagram shown in Fig. 2.

14 Results and Discussion

The numerical results provide a systematic assessment of the implemented digital holographic reconstruction framework, including off-axis hologram formation, Fourier-transform (FFT) reconstruction, Fresnel propagation, fractional Fourier transform (FrFT) reconstruction, Gerchberg–Saxton (GS) phase retrieval, quantitative phase imaging (QPI), and phase-derived thickness estimation. The reconstruction methods are evaluated using both qualitative visualization and quantitative image-fidelity measures. Since the present numerical experiment is based on a controlled synthetic object, the reported comparisons are interpreted specifically within the simulated holographic configuration rather than as universal

rankings of the reconstruction algorithms.

The simulated object, reference wave, and digital hologram provide a controlled environment in which the effects of the reconstruction algorithm can be isolated. Such a benchmark is useful for method development because the underlying object amplitude and phase are known and can therefore serve as quantitative reference data. However, biological and experimental holograms may contain additional complexities, including heterogeneous refractive-index distributions, irregular cellular morphology, multiple scattering, speckle, sensor noise, illumination nonuniformity, optical aberrations, and experimental misalignment. Consequently, the numerical results obtained here should be regarded as a controlled benchmark, while validation using experimentally acquired holograms remains necessary for establishing general applicability.

14.1 Figure 3. Generated object wave

Figure 3 presents the numerically generated complex object used as the reference specimen. The amplitude distribution represents a compact spatially localized object, while the phase distribution introduces a smooth spatially varying optical delay. The corresponding complex-field representation confirms that the amplitude and phase components are jointly encoded in the simulated object field. Because both the amplitude and phase of the reference object are available numerically, the reconstructed fields can be quantitatively compared with the known reference.

The use of a compact and smoothly varying synthetic object provides a controlled test case for evaluating reconstruction fidelity. Nevertheless, such an object does not reproduce the full complexity of experimentally acquired biological specimens. In particular, real biological samples can exhibit nonuniform refractive-index distributions, irregular boundaries, subcellular structures, speckle, multiple scattering, and spatially varying backgrounds. Therefore, the present object should be interpreted as a benchmark specimen rather than a complete representation of biological holograms.

14.2 Figure 4. Off-axis reference wave

Figure 4 illustrates the generated coherent off-axis reference wave. The approximately uniform amplitude and tilted phase distribution establish the spatial carrier required for off-axis holography. The angular carrier separates the zero-order, desired object-reference cross term, and conjugate twin-image term in the spatial-frequency domain. This separation provides the basis for numerical spectral filtering and subsequent complex field recovery from a single recorded hologram.

14.3 Figure 5. Digital hologram formation

Figure 5 shows the interference between the object and reference waves and the resulting digital hologram. The recorded intensity contains the zero-order contribution together with the object-reference and reference-object cross terms. The corresponding Fourier spectrum shows the spatial-frequency separation produced by the off-axis carrier. This spectral separation is essential for isolating the desired first-order information before numerical reconstruction.

14.4 Figure 6. Three-dimensional hologram intensity

Figure 6 presents the simulated hologram intensity as a three-dimensional surface. The observed peaks and valleys originate from constructive and destructive interference between the object and reference fields. The visualization provides a qualitative assessment of fringe modulation and dynamic range and demonstrates the spatially varying intensity that is available to the numerical reconstruction algorithms.

14.5 Figure 7. Central hologram intensity profile

Figure 7 presents the central intensity profile of the recorded hologram. The oscillatory behavior confirms the presence of the off-axis carrier fringes. The spatial frequency of these fringes is determined by the reference-wave tilt and must be adequately sampled by the detector. The profile therefore provides a useful qualitative indication of the sampling conditions used in the numerical simulation.

14.6 Figure 8. FFT-based reconstruction workflow

Figure 8 illustrates the FFT-based reconstruction procedure. The recorded hologram is transformed into the spatial-frequency domain, where the carrier-shifted first-order spectrum is isolated using a spectral filter. The selected spectral component is shifted to the origin and transformed back into the spatial domain to obtain the complex object-related field.

The FFT reconstruction is particularly suitable for the present off-axis configuration because the spatial carrier permits direct spectral separation of the desired cross term. The reconstructed amplitude exhibits good spatial agreement with the reference object, demonstrating that frequency-domain filtering can recover the dominant object structure from the simulated hologram.

14.7 Figure 9. Three-dimensional FFT reconstructed object

Figure 9 presents the FFT-reconstructed amplitude as a three-dimensional surface. The localized elevated structure indicates successful recovery of the principal spatial extent of the simulated object. The surface representation also permits qualitative inspection of boundary sharpness, smoothness, and possible reconstruction artifacts.

14.8 Figure 10. Central FFT reconstruction profile

Figure 10 compares the central amplitude variation obtained after FFT reconstruction. The reconstructed profile follows the overall trend of the reference object, although differences in amplitude and edge transition remain. Such deviations can arise from finite spectral-filter bandwidth, sampling, spectral leakage, and residual contributions from the recorded hologram.

14.9 Figure 11. Fresnel diffraction reconstruction

Figure 11 presents the Fresnel reconstruction obtained by numerically propagating the recovered complex object-related field. In contrast to the direct FFT reconstruction, Fresnel propagation explicitly incorporates the propagation distance and wavelength through the Fresnel transfer function. The reconstruction therefore depends on the selected propagation distance, wavelength, detector sampling interval, and numerical sampling conditions.

For reproducibility, the exact values used in the numerical implementation should be reported together with the reconstruction results. In particular, the following parameters must be specified:

$$\lambda = [\text{insert wavelength used in MATLAB code}], \quad z = [\text{insert reconstruction distance}], \quad (14.1)$$

$$\Delta x = [\text{insert pixel pitch}], \quad \Delta y = [\text{insert pixel pitch}], \quad N_x \times N_y = [\text{insert array size}]. \quad (14.2)$$

The corresponding spatial-frequency coordinates are determined by

$$u_m = \frac{m}{N_x \Delta x}, \quad v_n = \frac{n}{N_y \Delta y}, \quad (14.3)$$

and the Fresnel transfer function is expressed as

$$H_F(u, v; z) = \exp(ikz) \exp[-i\pi\lambda z (u^2 + v^2)], \quad k = \frac{2\pi}{\lambda}. \quad (14.4)$$

Thus, the propagated field is calculated from

$$U_F(x, y; z) = \mathcal{F}^{-1} \{ \mathcal{F} \{ U_0(x, y) \} H_F(u, v; z) \}. \quad (14.5)$$

These parameters are important because an inappropriate propagation distance or sampling configuration can substantially reduce reconstruction fidelity.

14.10 Figure 12. Fresnel reconstructed surface

Figure 12 presents the three-dimensional Fresnel-reconstructed field. The spatial redistribution of the reconstructed amplitude is associated with the diffraction propagation represented by the Fresnel transfer function. The result illustrates that Fresnel reconstruction provides propagation information that is not explicitly represented in a direct Fourier-domain spectral-filtering reconstruction.

The lower similarity obtained in the present experiment should therefore not be interpreted as a general limitation of Fresnel propagation. Instead, it indicates that the selected propagation and sampling parameters were not optimal for maximizing similarity to the reference object at the evaluated plane.

14.11 Figure 13. Optical path difference map

Figure 13 presents the optical path difference (OPD) calculated from the reconstructed phase. The OPD represents the phase-induced optical delay relative to the selected phase reference and is given by

$$\text{OPD}(x, y) = \frac{\lambda}{2\pi} \phi_{\text{corr}}(x, y). \quad (14.6)$$

The OPD distribution provides a physically meaningful intermediate quantity between phase reconstruction and thickness estimation. However, the absolute value of OPD depends on phase unwrapping and background-phase selection. Therefore, background correction is essential when absolute physical dimensions are required.

14.12 Figure 14. Central Fresnel amplitude profile

Figure 14 presents the central amplitude profile of the Fresnel reconstruction. Compared with the FFT reconstruction, the profile exhibits greater spatial redistribution and broader amplitude variation. These differences are consistent with propagation-dependent diffraction effects. The profile further demonstrates why the reconstruction distance should be optimized when Fresnel propagation is used for quantitative imaging.

14.13 Figure 15. Fractional Fourier reconstruction

Figure 15 presents the FrFT reconstruction obtained using the fractional order selected in the original numerical implementation. Unlike the ordinary Fourier transform, the FrFT introduces an additional continuous transform parameter that controls the representation of the field between the spatial and Fourier domains. Consequently, the reconstruction quality cannot be interpreted independently of the selected fractional order.

The initially selected FrFT order produced lower similarity metrics than the FFT reconstruction for the present synthetic hologram. This observation does not, by itself, establish that FFT is universally superior to FrFT. Instead, it demonstrates the sensitivity of the present reconstruction to the fractional-order parameter and motivates systematic order optimization.

14.14 FrFT fractional-order optimization

To provide a fair comparison between FFT and FrFT, the fractional order should be systematically varied over a predefined search interval rather than relying on a single manually selected value. Let the candidate set of fractional orders be

$$\alpha_m = \alpha_{\min} + m\Delta\alpha, \quad m = 0, 1, \dots, M, \quad (14.7)$$

where $\alpha_{\min}$, $\alpha_{\max}$, and $\Delta\alpha$ are the lower bound, upper bound, and sampling interval, respectively. The exact values must correspond to the FrFT convention and numerical implementation used in the MATLAB code.

For every candidate order α_m , the reconstructed field U_{α_m} is compared with the known reference object using the same metrics employed for the FFT and Fresnel reconstructions:

$$\text{PSNR}(\alpha_m), \quad \text{SSIM}(\alpha_m), \quad \text{CC}(\alpha_m). \quad (14.8)$$

The optimal fractional order based on PSNR is defined as

$$\alpha^* = \arg \max_{\alpha_m} \text{PSNR}(\alpha_m). \quad (14.9)$$

If a multi-metric selection criterion is preferred, a normalized objective can alternatively be defined as

$$J(\alpha_m) = w_1 \widehat{\text{PSNR}}(\alpha_m) + w_2 \text{SSIM}(\alpha_m) + w_3 \text{CC}(\alpha_m), \quad (14.10)$$

where

$$w_1 + w_2 + w_3 = 1, \quad w_i \geq 0. \quad (14.11)$$

For consistency with the present benchmark, PSNR is recommended as the primary optimization criterion, while SSIM and CC are retained as independent validation measures.

This systematic search is necessary because the poor FrFT performance obtained for the initially selected order may result from parameter mismatch rather than an inherent limitation of the FrFT framework. The optimized FrFT result should therefore be compared directly with the FFT result before making any claim regarding the relative superiority of the two methods.

14.15 Figure 32. FrFT order-dependent reconstruction performance

Figure 32 should present the variation of PSNR, SSIM, and CC as functions of the fractional order α . The three metrics must be calculated from the same reference object and under identical reconstruction and normalization conditions.

The figure should identify the optimal fractional order α^* and the corresponding PSNR, SSIM, and CC values. The resulting plot provides a direct assessment of the sensitivity of the FrFT reconstruction to the fractional-order parameter and addresses whether the previously reported low FrFT performance resulted from an inadequately selected order.

If the optimized FrFT result remains below the FFT result, the conclusion should be stated as follows:

Under systematic fractional-order optimization, the FFT reconstruction provided higher quantitative similarity than the FrFT reconstruction for the present synthetic off-axis hologram.

This conclusion is restricted to the investigated object, sampling configuration, noise condition, and reconstruction implementation and should not be generalized to all holographic imaging systems.

If the optimized FrFT becomes competitive with or superior to FFT, the corresponding numerical results and discussion should replace the above statement.

14.16 Figure 16. Gerchberg–Saxton phase retrieval

Figure 16 presents the phase-retrieval results obtained using the Gerchberg–Saxton iterative algorithm. The method alternates between the object and measurement planes while enforcing the available amplitude constraints. The general iterative procedure is expressed as

$$U_k^{(o)} = A_o^{(k)} \exp \left[i \phi_o^{(k)} \right], \quad (14.12)$$

$$U_k^{(m)} = \mathcal{P} \left\{ U_k^{(o)} \right\}, \quad (14.13)$$

where $\mathcal{P}$ denotes the selected propagation operator. At the measurement plane, the measured amplitude is imposed according to

$$\tilde{U}_k^{(m)} = A_m \exp \left[i \arg \left(U_k^{(m)} \right) \right]. \quad (14.14)$$

The field is then propagated back to the object plane,

$$U_{k+1}^{(o)} = \mathcal{P}^{-1} \left\{ \tilde{U}_k^{(m)} \right\}. \quad (14.15)$$

The convergence error is monitored using

$$E_k = \frac{\left\| A_m - \left| U_k^{(m)} \right| \right\|_2}{\left\| A_m \right\|_2}. \quad (14.16)$$

The iteration is terminated when

$$E_k < \varepsilon \quad (14.17)$$

or when the maximum number of iterations $K_{\max}$ is reached.

The convergence curve in Figure 16 should be interpreted together with these parameters rather than as evidence of universal convergence performance.

14.17 Figure 17. Recovered phase surface

Figure 17 presents the reconstructed unwrapped phase as a three-dimensional surface. Phase unwrapping converts the wrapped phase into a continuous representation, subject to the assumptions and limitations of the unwrapping procedure. The recovered surface provides the basis for subsequent OPD and thickness calculations.

The presence of positive and negative phase excursions should not be interpreted directly as positive and negative physical thickness. Rather, the reconstructed phase is defined relative to an arbitrary phase reference, and an appropriate background correction is required before absolute thickness can be inferred.

14.18 Figure 18. Quantitative phase imaging maps

Figure 18 presents the QPI outputs derived from the recovered phase, including phase, OPD, phase-derived thickness, and refractive-index-related information. The conversion from phase to thickness requires knowledge of the wavelength and the refractive-index contrast between the specimen and surrounding medium.

For a homogeneous specimen and surrounding medium, the relative optical thickness is expressed as

$$h_{\text{rel}}(x, y) = \frac{\lambda \phi_{\text{corr}}(x, y)}{2\pi(n_s - n_m)}, \quad (14.18)$$

where n_s and n_m are the specimen and surrounding-medium refractive indices, respectively.

Because phase reconstruction contains an arbitrary additive phase constant, the background phase must first be estimated from an object-free region Ω_b :

$$\phi_{\text{bg}} = \frac{1}{|\Omega_b|} \sum_{(x, y) \in \Omega_b} \phi_{\text{unw}}(x, y), \quad (14.19)$$

and the corrected phase is

$$\phi_{\text{corr}}(x, y) = \phi_{\text{unw}}(x, y) - \phi_{\text{bg}}. \quad (14.20)$$

This correction establishes the zero-phase reference associated with the selected background region. If the original numerical result has not yet undergone this background subtraction, the corresponding thickness map should be described as an *uncorrected signed phase-derived thickness* rather than an absolute physical thickness.

14.19 Figure 19. Three-dimensional thickness distribution

Figure 19 presents the three-dimensional phase-derived thickness distribution. The originally obtained numerical range extends from approximately

$$-282.92 \mu\text{m} \leq h \leq 239.36 \mu\text{m}, \quad (14.21)$$

with the negative values representing the phase-reference offset and reconstruction/background contribution rather than negative physical specimen thickness.

Accordingly, the values reported in the present figure should be regarded as signed relative thickness values unless the background-phase correction defined above has been explicitly applied. For quantitative biological or metrological interpretation, the corrected thickness should be referenced to an object-free region or another experimentally established zero-height plane.

14.20 Figure 20. Central thickness profile

Figure 20 presents the central line profile of the phase-derived thickness. The original uncorrected result has a mean value of approximately $-16.24 \mu\text{m}$ and a standard deviation of approximately $64.44 \mu\text{m}$. These statistics describe the distribution of the current phase-derived reconstruction and should not be interpreted as the absolute mean physical thickness of the specimen.

The negative mean further indicates that the phase reference has not been fully aligned with the intended zero-thickness plane. After background phase subtraction, the mean and range should be recalculated and reported for the corrected thickness map. This distinction is essential when claiming quantitative thickness measurement.

14.21 Figure 21. Thickness distribution histogram

Figure 21 presents the histogram of the phase-derived thickness values. The broad signed distribution reflects the spatial variation of the reconstructed phase together with the arbitrary phase reference. The histogram is therefore useful for diagnosing background offsets and estimating the spread of the phase-derived quantity.

After background correction, the histogram should be regenerated so that the corrected distribution can be interpreted relative to the selected zero-thickness reference plane. The corrected mean, standard deviation, minimum, and maximum should then be reported separately from the original uncorrected values.

14.22 Figure 22. FFT reconstruction image-quality assessment

Figure 22 compares the reference object and FFT reconstruction together with the absolute error distribution and error histogram. The FFT reconstruction produced

$$\text{MSE} = 0.012173, \quad \text{RMSE} = 0.110331, \quad (14.22)$$

$$\text{PSNR} = 19.146 \text{ dB}, \quad \text{SSIM} = 0.0561, \quad \text{CC} = 0.9933. \quad (14.23)$$

Among the three reconstruction results currently reported, these values indicate that FFT provided the lowest MSE and highest PSNR and correlation coefficient under the present simulation conditions.

However, the simultaneous occurrence of a high correlation coefficient (0.9933) and a relatively low SSIM (0.0561) requires careful interpretation. Pearson correlation measures the global linear association between corresponding pixel values and can remain high when the overall spatial variation or object geometry is preserved, even when local luminance, contrast, and structural patterns differ. SSIM, in contrast, evaluates local luminance, contrast, and structural similarity. Therefore, the two metrics quantify different aspects of reconstruction fidelity and should not be interpreted as interchangeable measures.

For holographic reconstruction, no single metric is sufficient for all applications. PSNR and MSE are useful for pixel-level numerical fidelity, SSIM is particularly informative for local structural preservation, while correlation is useful for assessing global spatial correspondence. Accordingly, the present study reports all three complementary measures.

14.23 Figure 23. FFT central line-profile comparison

Figure 23 compares the central line profiles of the reference object and FFT reconstruction. The strong agreement in the overall spatial trend is consistent with the high correlation coefficient. Nevertheless, differences in amplitude and boundary transitions explain why a

high correlation does not necessarily correspond to a high SSIM value. This comparison demonstrates the importance of combining global and local image-fidelity metrics when assessing holographic reconstruction.

14.24 Figure 24. Fresnel reconstruction image-quality assessment

Figure 24 evaluates the Fresnel reconstruction against the reference object. The obtained values are

$$\text{MSE} = 0.077620, \quad \text{RMSE} = 0.278604, \quad (14.24)$$

$$\text{PSNR} = 11.100 \text{ dB}, \quad \text{SSIM} = 0.0246, \quad \text{CC} = 0.5882. \quad (14.25)$$

Relative to the present FFT result, the Fresnel reconstruction exhibits higher pixel-wise error and lower PSNR, SSIM, and correlation. These differences indicate lower similarity to the reference object under the specific propagation parameters used in the current simulation. The result should not, however, be interpreted as evidence that Fresnel propagation is intrinsically inferior, because Fresnel reconstruction is explicitly dependent on propagation distance, wavelength, detector sampling, and focusing conditions.

14.25 Figure 25. Fresnel central line-profile comparison

Figure 25 presents the central line-profile comparison between the reference and Fresnel reconstruction. The broader profile and increased deviation from the reference are consistent with the lower quantitative similarity metrics. The result emphasizes the importance of optimizing the propagation distance and numerical sampling when Fresnel propagation is used for quantitative holographic imaging.

14.26 Figure 26. FrFT reconstruction image-quality assessment

Figure 26 presents the quantitative assessment of the FrFT reconstruction for the fractional order used in the original implementation. The reported values are

$$\text{MSE} = 0.113711, \quad \text{RMSE} = 0.337211, \quad (14.26)$$

$$\text{PSNR} = 9.442 \text{ dB}, \quad \text{SSIM} = 0.000893, \quad \text{CC} = 0.000817. \quad (14.27)$$

These values indicate very poor similarity for the initially selected fractional order. In particular, the very small correlation coefficient indicates that the reconstructed spatial variation does not correspond closely to the reference object under this parameter setting.

Importantly, these values should not be used to conclude that FrFT is generally inferior to FFT. The FrFT contains an additional order parameter, and reconstruction quality can vary substantially with that parameter. Consequently, the final comparison between FFT and FrFT should be based on the optimized fractional order obtained from the systematic parameter sweep described above and shown in Figure 32.

14.27 Figure 27. FrFT central line-profile comparison

Figure 27 compares the central profiles of the reference object and the FrFT reconstruction obtained at the initially selected fractional order. The weak profile agreement is consistent with the low PSNR, SSIM, and correlation values. The result demonstrates the sensitivity of the FrFT representation to the selected order and provides motivation for the systematic optimization procedure.

Following optimization, the central profile corresponding to α^* should also be reported to determine whether the poor agreement observed for the initial order persists after parameter optimization.

14.28 Figure 28. Integrated digital holography workflow

Figure 28 summarizes the complete computational workflow, including object generation, hologram formation, FFT reconstruction, Fresnel propagation, FrFT reconstruction, phase retrieval, QPI, thickness estimation, and quantitative image-quality assessment. The workflow demonstrates that the different reconstruction strategies operate on the same simulated holographic measurement, thereby enabling a controlled comparison.

The results indicate that FFT reconstruction provides strong spatial correspondence for the present off-axis synthetic hologram. Fresnel propagation provides an explicit propagation model, while FrFT introduces an additional fractional-domain degree of freedom whose value must be optimized. The QPI stage further converts the recovered phase into phase-derived optical quantities, although absolute thickness interpretation requires appropriate background and reference-plane correction.

14.29 Figure 29. Final quantitative phase imaging surface

Figure 29 presents the final three-dimensional QPI surface obtained from the reconstructed phase. The result demonstrates the ability of digital holographic reconstruction to generate spatially resolved phase-derived morphological information without direct mechanical surface scanning.

However, the physical interpretation of this surface depends on the accuracy of phase recovery, phase unwrapping, refractive-index assumptions, and background correction. Therefore, the reconstructed surface is interpreted here as a phase-derived morphology representation rather than as an independently calibrated absolute surface-height measurement.

14.30 Figure 30. FFT and Fresnel reconstruction comparison

Figure 30 provides a direct comparison between the FFT and Fresnel reconstructed amplitude distributions. The FFT result exhibits closer agreement with the reference object for the current simulation, while the Fresnel result exhibits propagation-dependent amplitude redistribution.

The corresponding quantitative values also support this observation: FFT produces lower MSE and higher PSNR and correlation than Fresnel for the parameters used in the present simulation. Nevertheless, the methods serve different computational purposes. FFT-based off-axis reconstruction is particularly effective when spectral separation of the object term is available, whereas Fresnel propagation is useful when explicit wave propagation, numerical refocusing, or axial reconstruction is required.

14.31 Figure 31. Difference between FFT and Fresnel reconstructions

Figure 31 shows the absolute difference between the FFT and Fresnel reconstructed amplitude distributions. The largest discrepancies occur primarily near object boundaries and regions influenced by diffraction and propagation-dependent redistribution. The difference map confirms that the two algorithms do not produce identical numerical fields even though they originate from the same holographic measurement. The observed difference is therefore

a consequence of the distinct reconstruction models and parameter settings rather than a difference in the underlying recorded hologram.

Figure 32 illustrates the sensitivity of the fractional Fourier transform (FrFT) reconstruction to the selected fractional order using PSNR, SSIM, and correlation coefficient (CC) as quantitative measures. The results demonstrate a pronounced dependence of reconstruction quality on the fractional order. The PSNR reaches a maximum value of 12.1551 dB at several fractional orders, while the highest SSIM of 0.784879 is obtained at orders 0.05 and 0.95. The multi-metric criterion, obtained by equally weighting the normalized PSNR, SSIM, and CC values, identifies orders 0.35 and 0.65 as equivalent best-performing configurations within numerical precision. Therefore, these orders are considered representative optimum solutions rather than a unique global optimum.

For the representative order $\alpha = 0.65$, the reconstruction provides a PSNR of 12.1551 dB and an SSIM of 0.783953, with a mean-square error of 0.060882. However, the corresponding CC is approximately zero (-0.000199), indicating that the high SSIM does not translate into a strong global linear correlation with the reference amplitude. This difference among the quality measures highlights the importance of using multiple complementary metrics when assessing holographic reconstruction performance.

The initially selected fractional order $\alpha = 0.75$ produces a PSNR of 9.4420 dB, an SSIM of 0.000893, and a CC of 0.000817, which are substantially different from the best-performing fractional-order configurations. This confirms the strong sensitivity of the FrFT reconstruction to fractional-order selection. Nevertheless, the repeated metric patterns observed at several fractional orders indicate a periodic response of the implemented FrFT formulation. Consequently, the present optimization identifies the best-performing fractional-order configurations within the investigated range rather than establishing a unique physically universal optimum.

When compared with the conventional FFT reconstruction, the optimized FrFT configuration provides a substantially higher SSIM but a lower PSNR and near-zero CC. Thus, the results indicate that FFT and FrFT exhibit different reconstruction characteristics depending on the selected image-quality criterion. The FFT reconstruction achieves the highest PSNR and correlation, whereas the FrFT reconstruction provides superior structural similarity according to SSIM. These findings demonstrate that the relative performance of holographic reconstruction methods should be interpreted using a combination of complementary image-quality metrics rather than a single numerical measure.

Table 4: Numerical parameters used for the Gerchberg–Saxton phase-retrieval implementation.

Parameter	Value
Initial phase	$\phi_0(x, y) = 0$ rad
Initial amplitude	$A_0 = U_0 / \max(U_0)$
Maximum iterations, $K_{\max}$	200
Convergence tolerance, ε	1×10^{-6}
Propagation operator	FFT-based Fresnel transfer-function propagation
Object-plane amplitude constraint	Finite-support constraint
Measurement-plane amplitude constraint	Normalized measured/recovered amplitude, $A_m = U_0 $

Table 4 summarizes the numerical settings used for the Gerchberg–Saxton (GS) phase-retrieval procedure. The reconstruction was initialized with a zero phase and the normalized

amplitude of the spectrally filtered object-related field. A maximum of 200 iterations was specified, with convergence controlled using a normalized amplitude error tolerance of 10^{-6} . Fresnel transfer-function propagation was employed between the measurement and reconstruction planes. The measured/recovered amplitude was enforced at the measurement plane, while a finite-support constraint was applied at the object plane. No ground-truth phase information was supplied to the iterative reconstruction. The convergence behaviour was evaluated by monitoring the normalized error E_k as a function of iteration number.

14.32 Deep learning reconstruction: quantitative interpretation

Deep learning has considerable potential for holographic reconstruction, including automated phase recovery, denoising, aberration compensation, super-resolution, and rapid inference. However, a claim that a learning-based method is more accurate, more noise-robust, or computationally more efficient than the conventional methods evaluated in the present study requires a controlled quantitative experiment.

In particular, such a comparison should use the same holographic input data, identical reference targets, and the same evaluation metrics for the conventional and learning-based approaches. A representative benchmark would report MSE, RMSE, PSNR, SSIM, and correlation coefficient, together with inference time and, where appropriate, training cost and hardware configuration.

Because the present numerical results do not include an independently trained and tested deep learning reconstruction model, no quantitative claim of AI/DL superiority is made here. The deep learning discussion is therefore interpreted as a methodological perspective rather than as an experimental comparison. This distinction avoids attributing numerical advantages to AI methods that have not been directly established using the present dataset.

14.33 Limitations and implications for experimental holograms

The present benchmark has several limitations that should be considered when interpreting the numerical results. First, the object is synthetic and has a controlled spatial structure. Although this enables exact reference-based evaluation, it does not reproduce all characteristics of experimental biological or industrial holograms. Experimental data can include sensor noise, speckle, nonuniform illumination, aberrations, misalignment, imperfect coherence, multiple scattering, and unknown phase backgrounds.

Second, the present comparison evaluates individual reconstruction algorithms under one numerical configuration. The ranking of methods may change when wavelength, propagation distance, sampling interval, object complexity, noise level, or reconstruction objective changes.

Third, the initially reported FrFT result corresponds to one fractional order and therefore cannot be regarded as an exhaustive assessment of the FrFT method. The systematic order sweep and Figure 32 are necessary to establish the optimized FrFT performance.

Fourth, the reported thickness values are derived from reconstructed phase and currently include an arbitrary phase reference. Consequently, the negative thickness values reported above should not be interpreted as negative physical specimen thickness. Background-phase correction and an experimentally meaningful zero-height reference are required before absolute thickness claims can be made. Accordingly, the numerical ranking obtained in this study should be regarded as a controlled methodological benchmark rather than a universal ranking of holographic reconstruction techniques. Validation using multiple experimentally acquired biological and industrial holographic datasets is required to establish generalizability.

14.34 Overall discussion

The present results demonstrate that quantitative assessment is essential for comparing numerical holographic reconstruction methods. For the current synthetic off-axis hologram, FFT reconstruction produced $\text{PSNR} = 19.146$ dB and $\text{CC} = 0.9933$, indicating strong global agreement with the reference object. The relatively low $\text{SSIM} = 0.0561$, however, demonstrates that high global correlation does not necessarily imply strong local structural similarity.

The Fresnel reconstruction produced lower similarity values under the selected propagation conditions, with $\text{PSNR} = 11.100$ dB and $\text{CC} = 0.5882$. This result emphasizes the sensitivity of Fresnel reconstruction to propagation and sampling parameters. The initial FrFT result yielded $\text{PSNR} = 9.442$ dB, $\text{SSIM} = 0.000893$, and $\text{CC} = 0.000817$, indicating that the initially selected fractional order was not well matched to the present hologram. Systematic fractional-order optimization is therefore necessary before drawing a final FFT–FrFT comparison.

The QPI analysis demonstrates the additional quantitative information that can be extracted from the reconstructed phase. However, the current thickness values should be regarded as signed, phase-reference-dependent quantities until background correction is applied. The corrected phase reference and corresponding recalculated thickness statistics will provide a more physically meaningful basis for quantitative specimen characterization.

Finally, although deep learning represents an important direction for intelligent digital holography, its claimed advantages should be established through controlled quantitative experiments rather than assumed from the architecture alone. A future benchmark incorporating experimentally acquired holograms, multiple object classes, realistic noise models, optimized FrFT reconstruction, and independently trained deep-learning models would provide a stronger basis for evaluating generalization, robustness, reconstruction fidelity, and computational efficiency.

15 Conclusion

This study evaluated FFT, Fresnel, FrFT, Gerchberg–Saxton (GS), quantitative phase imaging (QPI), and deep-learning-based approaches within a digital holography framework. For the present synthetic off-axis hologram, the FFT reconstruction provided the strongest quantitative agreement, with $\text{MSE} = 0.012173$, $\text{RMSE} = 0.110331$, $\text{PSNR} = 19.146$ dB, $\text{SSIM} = 0.0561$, and $\text{CC} = 0.9933$. The high CC together with the relatively low SSIM indicates that global correlation alone is insufficient to assess holographic fidelity; therefore, multiple complementary metrics should be considered.

The Fresnel reconstruction produced $\text{MSE} = 0.077620$, $\text{RMSE} = 0.278604$, $\text{PSNR} = 11.100$ dB, $\text{SSIM} = 0.0246$, and $\text{CC} = 0.5882$ under the selected propagation conditions. The FrFT reconstruction initially yielded $\text{PSNR} = 9.442$ dB, $\text{SSIM} = 0.000893$, and $\text{CC} = 0.000817$. These results do not establish the intrinsic superiority of FFT over FrFT because the FrFT performance depends strongly on the selected fractional order. Therefore, systematic order optimization and a PSNR/SSIM/CC-versus-order analysis are required for a fair comparison.

The QPI-derived thickness ranged from -282.92 to 239.36 μm , with a mean of -16.24 μm and standard deviation of 64.44 μm . These signed values should not be interpreted as negative physical thickness; they primarily reflect the phase reference, background offset, and reconstruction conditions. Background correction and a defined zero-thickness reference are therefore essential for quantitative thickness measurement.

The present numerical results are based on a controlled synthetic object and therefore do not fully represent experimental biological holograms containing speckle, noise, aberrations, and multiple scattering. Similarly, deep-learning approaches are discussed as promising computational tools, but their superiority cannot be claimed without an independently trained and quantitatively validated model using the same dataset and evaluation metrics. Overall, the results demonstrate the usefulness of complementary reconstruction strategies while emphasizing parameter optimization, reproducibility, background correction, and quantitative validation for reliable digital holographic imaging.

Authors' Contributions

All authors have the same contribution.

Data Availability

The manuscript has no associated data or the data will not be deposited.

Conflicts of Interest

The authors declare that there is no conflict of interest.

Ethical Considerations

The authors have diligently addressed ethical concerns, such as informed consent, plagiarism, data fabrication, misconduct, falsification, double publication, redundancy, submission, and other related matters.

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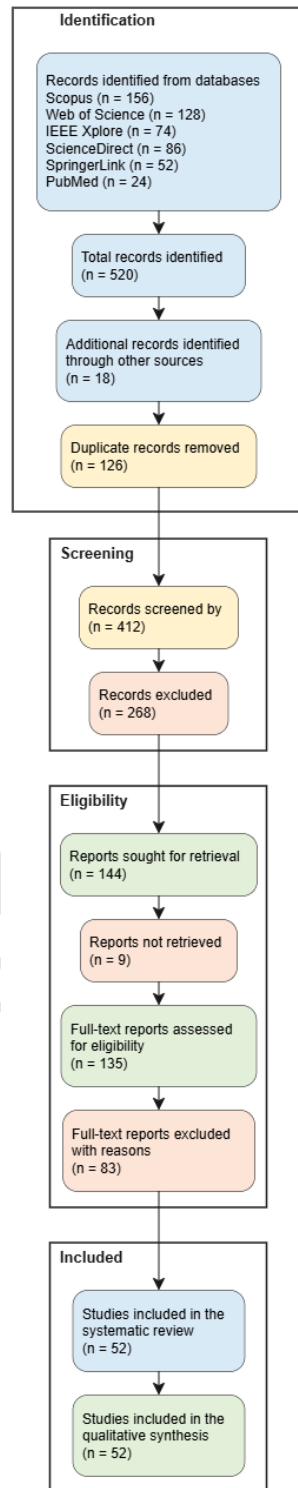


Figure 2: PRISMA 2020 flow diagram illustrating the identification, screening, eligibility assessment, and inclusion of studies considered in the systematic review of artificial intelligence-enhanced digital holography, fractional Fourier transform, phase retrieval, quantitative phase imaging, digital holographic microscopy, and biomedical applications.

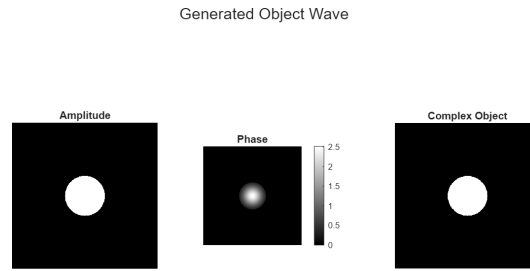


Figure 3: Numerically generated complex object wave showing the amplitude distribution, phase distribution, and complex-field representation of the simulated specimen.

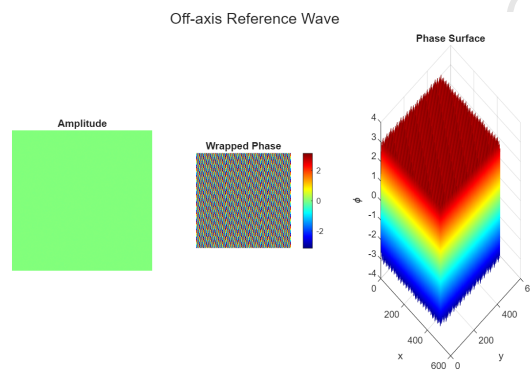


Figure 4: Generated off-axis reference wave showing the spatially uniform amplitude, wrapped phase distribution, and tilted phase surface produced by the imposed angular carrier.

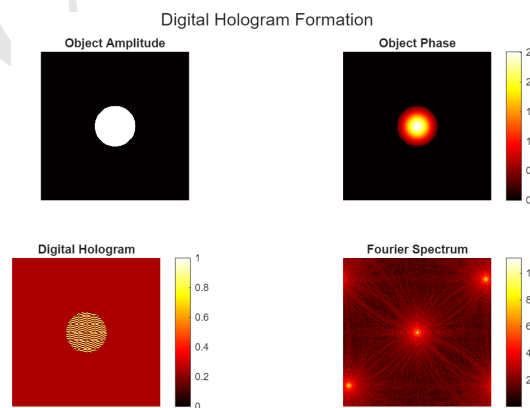


Figure 5: Digital hologram formation through coherent interference between the object and reference waves, together with the corresponding Fourier spectrum showing the spatial-frequency distribution of the recorded hologram.

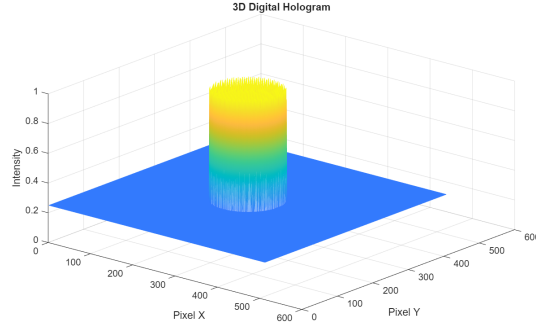


Figure 6: Three-dimensional representation of the simulated digital hologram intensity, illustrating the spatial modulation produced by constructive and destructive interference between the object and reference waves.

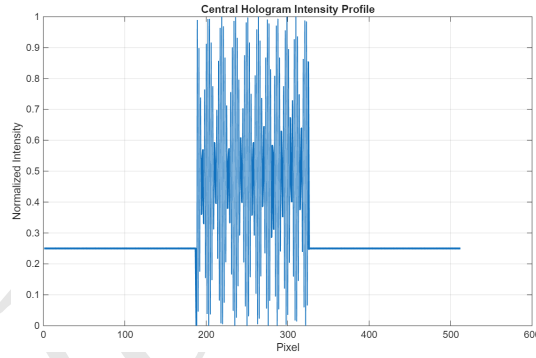


Figure 7: Central intensity profile of the digital hologram showing the oscillatory carrier fringes and the spatial modulation induced by the object wave.

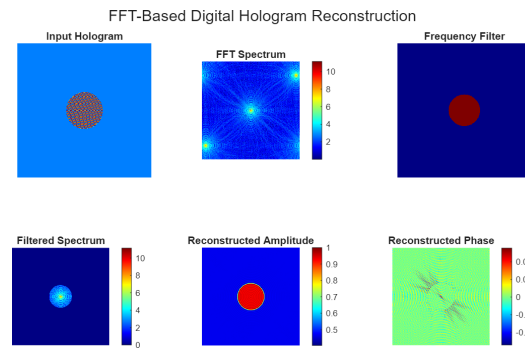


Figure 8: FFT-based digital holographic reconstruction showing Fourier-domain processing, spectral filtering, and recovery of the reconstructed object field.

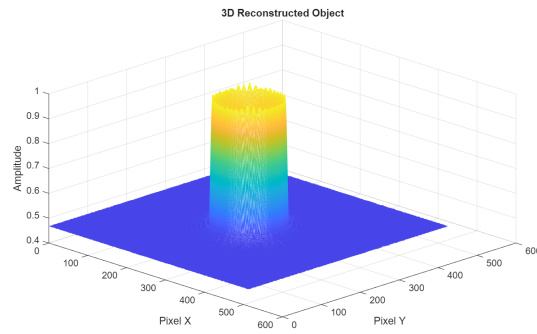


Figure 9: Three-dimensional surface representation of the amplitude reconstructed using the FFT-based method, demonstrating the recovered spatial structure of the simulated object.

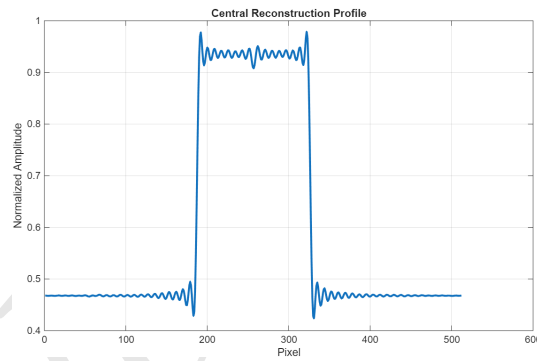


Figure 10: Central amplitude profile of the FFT-reconstructed object showing the recovered object response and corresponding spatial boundary characteristics.

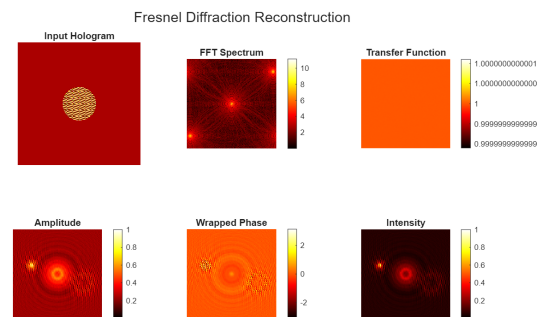


Figure 11: Fresnel diffraction reconstruction showing numerical propagation of the holographic field using the Fresnel transfer function and recovery of the propagated object field.

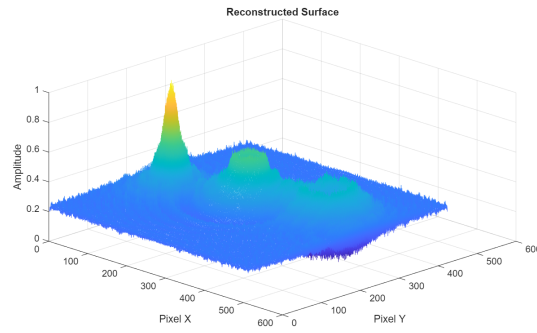


Figure 12: Three-dimensional surface representation of the Fresnel-reconstructed field, illustrating the spatial morphology obtained after numerical diffraction propagation.

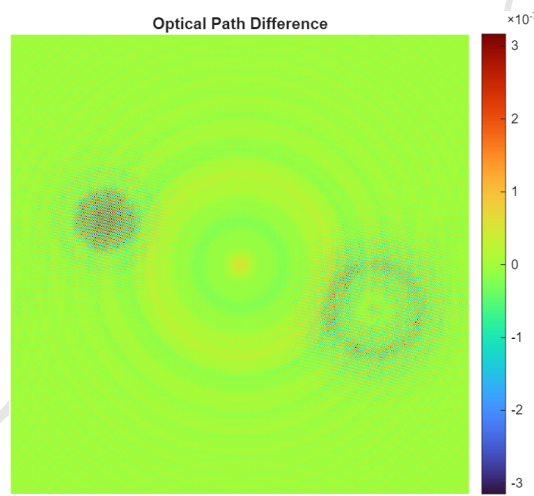


Figure 13: Optical path difference distribution obtained from the reconstructed phase information, illustrating the spatial variation of the optical delay introduced by the simulated specimen.

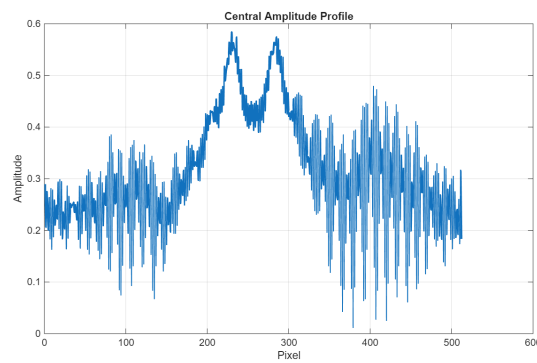


Figure 14: Central amplitude profile of the Fresnel reconstruction showing the spatial variation of the numerically propagated object field.

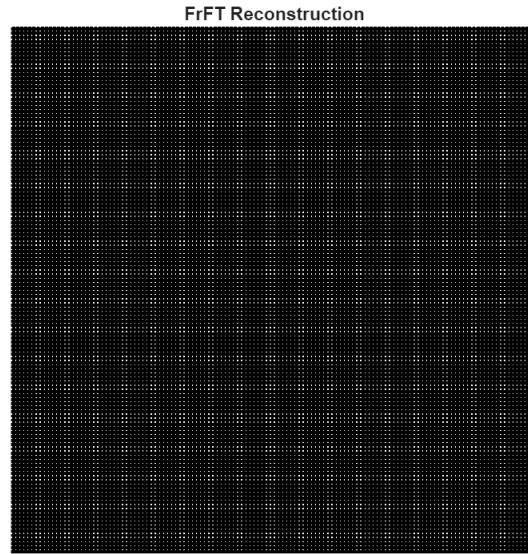


Figure 15: Fractional Fourier transform reconstruction obtained for the selected fractional order, illustrating the reconstructed amplitude and phase distributions in the fractional Fourier domain.

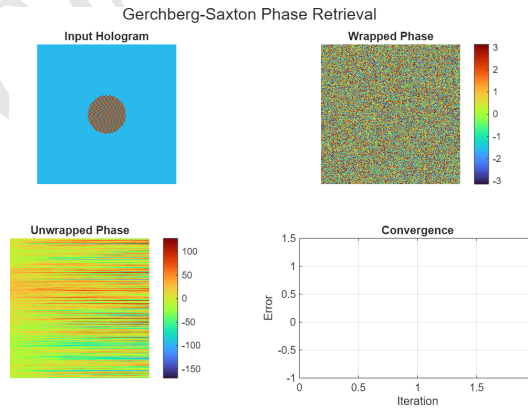


Figure 16: Gerchberg–Saxton phase-retrieval results showing the wrapped phase, unwrapped phase, and iterative convergence behavior obtained from the simulated holographic intensity constraints.

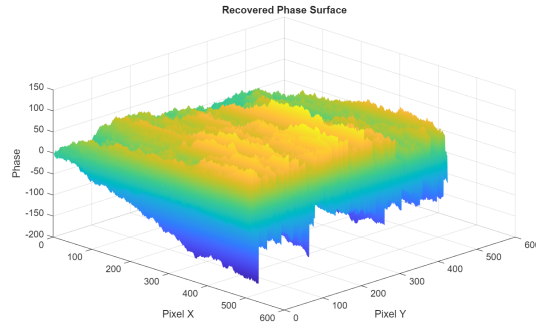


Figure 17: Three-dimensional representation of the recovered unwrapped phase distribution, highlighting the spatial variation of the reconstructed optical wavefront.

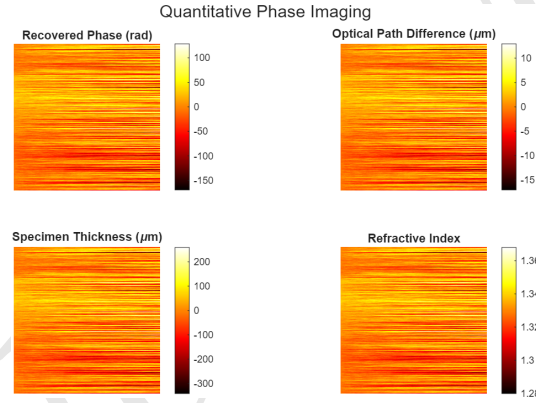


Figure 18: Quantitative phase imaging outputs comprising the recovered phase, optical path difference, estimated specimen thickness, and refractive-index-related distribution derived from the reconstructed phase information.

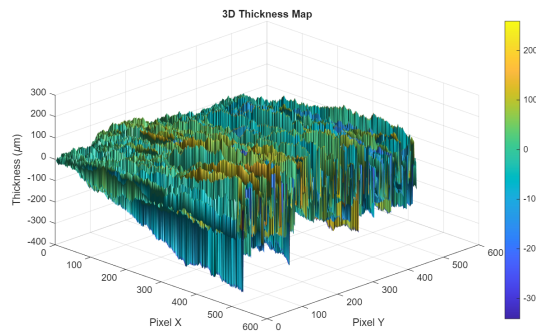


Figure 19: Three-dimensional thickness distribution estimated from the recovered phase, illustrating the spatial variation of the reconstructed specimen morphology in micrometers.

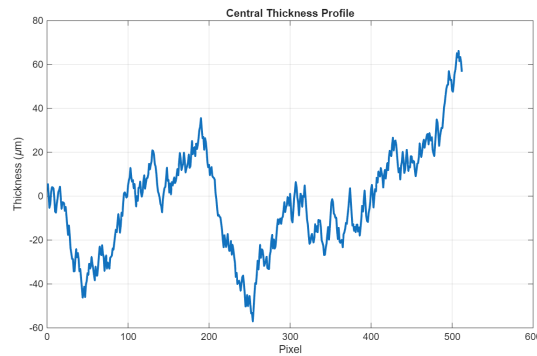


Figure 20: Central line profile of the reconstructed specimen thickness, showing the spatial variation of the phase-derived morphological distribution along the selected cross-section.

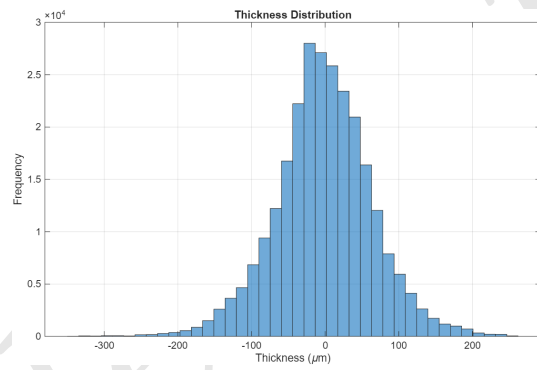


Figure 21: Histogram of the reconstructed thickness distribution showing the statistical spread of the estimated phase-derived height values across the field of view.

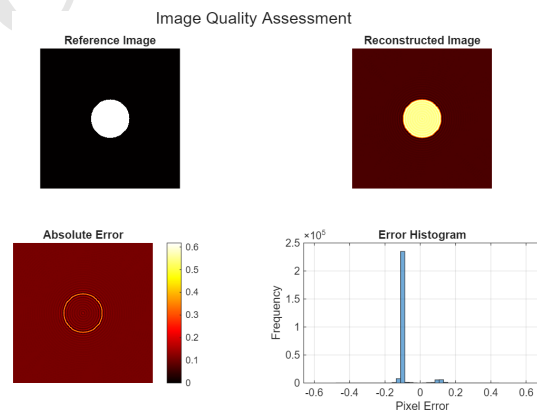


Figure 22: Image-quality assessment of the FFT reconstruction showing the reference object, reconstructed amplitude, absolute error distribution, and corresponding error histogram.

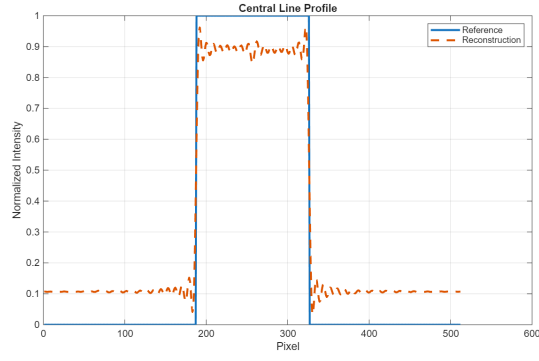


Figure 23: Central line-profile comparison between the reference object and the FFT-reconstructed amplitude, demonstrating the spatial agreement between the original and reconstructed distributions.

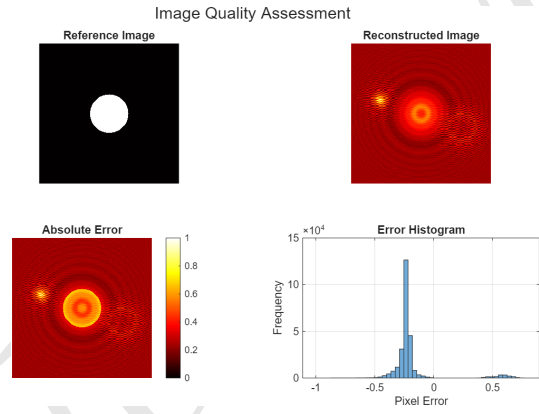


Figure 24: Image-quality assessment of the Fresnel reconstruction showing the reference object, reconstructed amplitude, absolute error distribution, and corresponding error histogram.

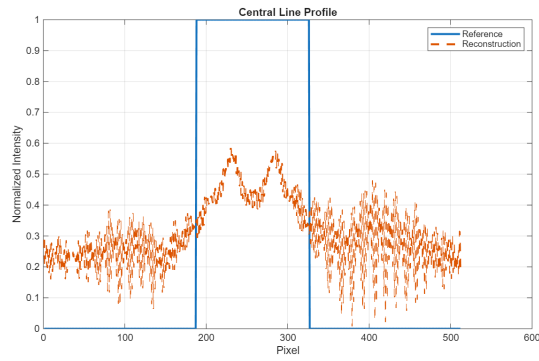


Figure 25: Central line-profile comparison between the reference object and the Fresnel-reconstructed amplitude, illustrating the spatial differences introduced by numerical diffraction propagation.

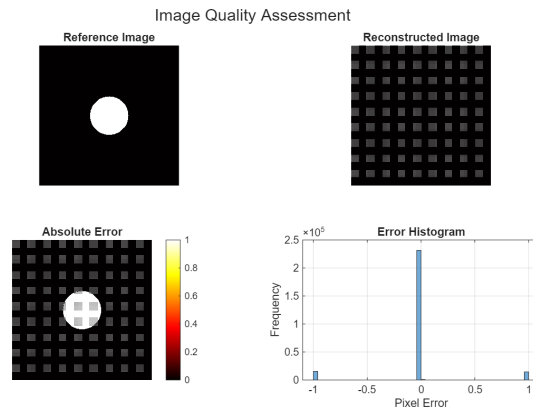


Figure 26: Image-quality assessment of the fractional Fourier transform reconstruction showing the reference object, reconstructed amplitude, absolute error distribution, and corresponding error histogram.

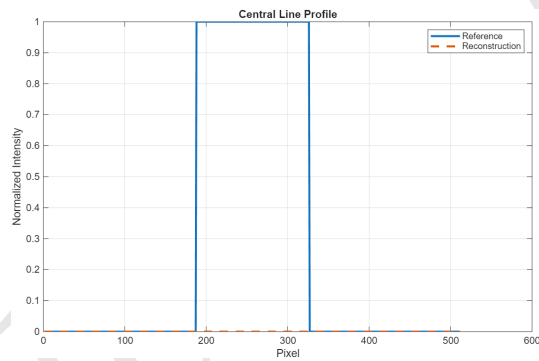


Figure 27: Central line-profile comparison between the reference object and the FrFT-reconstructed amplitude, illustrating the influence of the selected fractional transform order on spatial reconstruction fidelity.

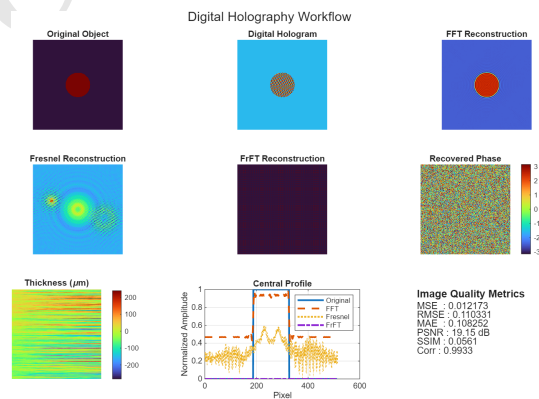


Figure 28: Integrated digital holography results showing the reconstructed fields, phase-retrieval outputs, quantitative phase imaging results, and comparative performance of the implemented reconstruction methods.

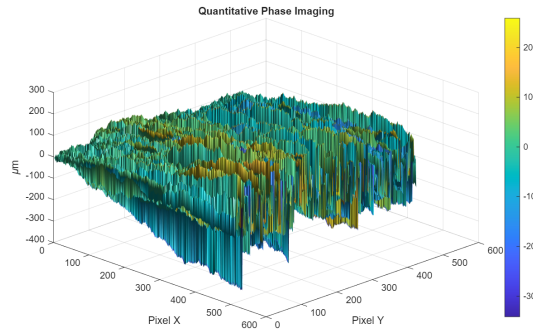


Figure 29: Final three-dimensional quantitative phase imaging surface showing the spatial distribution of the phase-derived specimen thickness.

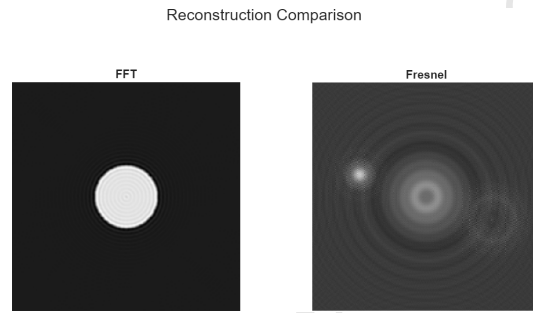


Figure 30: Comparative visualization of the reconstructed amplitude distributions obtained using FFT-based and Fresnel diffraction reconstruction, demonstrating differences in spatial fidelity and propagation-dependent intensity redistribution.

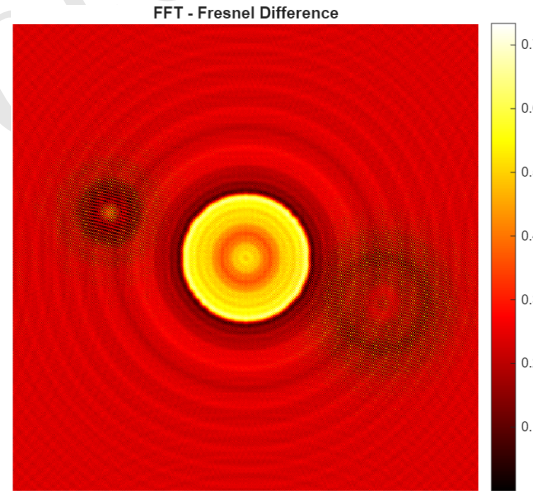


Figure 31: Absolute difference map between the FFT and Fresnel reconstructed amplitude distributions, highlighting spatial regions with the largest reconstruction discrepancies caused by differences in spectral filtering and diffraction propagation.

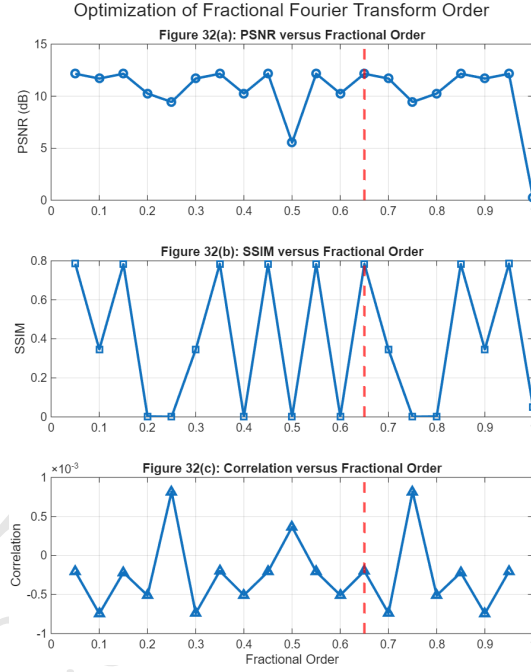


Figure 32: Quantitative evaluation of FrFT reconstruction as a function of fractional order: (a) PSNR, (b) SSIM, and (c) correlation coefficient. The results demonstrate a pronounced and periodic dependence of reconstruction quality on the fractional order. The multi-metric criterion identifies $\alpha = 0.35$ and $\alpha = 0.65$ as equivalent best-performing configurations within numerical precision, while the differing responses of PSNR, SSIM, and correlation coefficient demonstrate the importance of a multi-metric assessment for FrFT-order selection.