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Regular article

A Dynamical Realization of Holography in Cosmology from Hyperbolic General Relativity: Explicit Dimensional Reduction via Curvature-Generated Shock Relaxation

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Abstract. Holography asserts that the dynamics of a gravitational system may be determined by degrees of freedom associated with a lower-dimensional structure, rather than by independent volumetric variables. While this principle is well established in boundary-based constructions, its realization in cosmology has remained elusive due to the absence of fixed asymptotic boundaries. In this work, we present an explicit realization of holography in cosmology arising directly from the hyperbolic structure of general relativity. We show that the nonlinear evolution of the Einstein equations coupled to a scalar field generically produces a global control structure identified with the cosmological apparent horizon. By applying the Unified First Law of Thermodynamics ($-dE = T\,dS$) to this horizon, we obtain a geometric closure for the net exchange flux Q under the explicit assumption that the only energy exchange between the bulk and the auxiliary (unresolved) sector is mediated through the apparent-horizon screen, constraining the bulk energy density to scale as $\rho \propto L^{-2}$ and thereby imposing a Bekenstein-Hawking area law ($S \propto A$) on the bulk entropy. We demonstrate that bulk cosmological observables – including the expansion history and late-time acceleration – are functionals of this codimension-one structure rather than independent volumetric degrees of freedom. The construction constitutes a concrete, equation-level realization of holography in cosmology, achieved via dynamical dimensional reduction of the phase space.

Keywords: Cosmology; Holography; Dark Energy; Hyperbolic General Relativity; Horizon Thermodynamics; Dimensional Reduction.

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1 Introduction

1.1 Holography as Dynamical Dimensional Reduction

The holographic principle is most usefully understood not as a specific correspondence between theories, but as a statement about degrees of freedom: the bulk dynamics of a gravitational system are governed by a lower-dimensional structure. In its strongest form, holography implies that volumetric variables do not represent independent dynamical inputs, but are instead constrained projections of a reduced set of underlying variables [1,2].

In cosmology, such a reduction appears difficult to realize. Standard treatments describe the expansion history of the universe as a sequence of independent dynamical regimes - inflation, radiation domination, matter domination, and late-time acceleration - each characterized by distinct physical assumptions. This phase-based description implicitly assumes a large number of independent volumetric degrees of freedom, seemingly at odds with holographic expectations.

Holographic ideas have nevertheless been explored in a range of gravitational contexts, including covariant entropy bounds [3,4] and proposed gauge-gravity dualities [5,6]. However, these approaches are typically either kinematical, providing information-theoretic constraints without closing the dynamics, or boundary-based, relying on asymptotic structures that are absent in realistic cosmological spacetimes [7]. As a result, a fully dynamical realization of holographic dimensional reduction in cosmology has remained elusive.

The present work addresses this gap by providing a dynamical, equation-level realization of holographic closure in a cosmological setting without assuming an asymptotic boundary or a dual quantum field theory. The central question addressed in this work is therefore the following: can cosmological dynamics be shown explicitly to satisfy holographic dimensional reduction using only the equations of general relativity and admissible matter couplings?

We demonstrate that this is indeed the case by exhibiting a class of cosmological solutions in which the bulk evolution is determined by a single codimension-one dynamical structure emerging from nonlinear gravitational dynamics. This work is intended as a constructive, existence-level demonstration of dynamical holography in cosmology, rather than a classification theorem or a proposal for a boundary dual theory. The construction is accordingly explicit, equation-based, and requires no modification of general relativity.

1.2 Governing Fields and Conservation Structure

To make this notion precise for the purposes of this work, we adopt the following operational definition.

> **Operational Definition:** A cosmological system is said to realize *holography* if there exists a codimension-one dynamical structure Σ such that (i) bulk observables are functionals of Σ through exact constraint relations, and (ii) bulk degrees of freedom are constrained by the information capacity of the codimension-one structure.

In precise terms, the definition requires: (i) the existence of a surjective map from the dynamical data on Σ to the bulk phase space $\mathcal{P}$, so that the field equations and conservation-law constraints determine every element of $\mathcal{P}$ as a functional of the surface degrees of freedom; (ii) a strict reduction of effective dimensionality, $\dim \mathcal{P}_{\text{eff}} < \dim \mathcal{P}$; and (iii) an area scaling $S \propto \text{Area}(\Sigma)$ arising as a derived consequence of (i)–(ii) rather than as an independent postulate.

In the present construction, the boundary conditions on Σ limit the bulk phase space to a self-similar submanifold, reducing the effective dimensionality of the system. The construction in §§4.1–4.4 verifies all three conditions explicitly, with Σ identified as the cosmological apparent horizon, $\dim \mathcal{P} = 3$ (the independent functions $\{a,\, \phi,\, \rho_X\}$), and $\dim \mathcal{P}_{\rm eff} = 1$ (the single horizon scale $L = \tilde{r}_A$).

This definition is intentionally minimal relative to boundary-based or duality-driven notions of holography and explicitly requires fewer structural assumptions. No asymptotic boundary, dual quantum field theory, or microscopic entropy counting is assumed. Instead, holography functions here as a dynamical organizing principle for cosmological evolution, realized through conservation-law closure and the emergence of a lower-dimensional control structure within general relativity.

Note that this definition is formulated at the level of spatially homogeneous and isotropic (FLRW) background cosmology, with the background evolution's expansion rate $H(t)$ and energy densities $\rho(t)$ governed by the Friedmann equations. A full holographic correspondence would naturally require extending this slaving mechanism to linear perturbations and quantum fluctuations, which is a subject for future work. In particular, establishing whether the holographic closure demonstrated here at the level of background dynamics persists for perturbative degrees of freedom and their quantum fluctuations - i.e., whether the full classical-quantum phase space is likewise slaved to the codimension-one control surface - is a sharp structural question that lies beyond the scope of the present work.

Here, we consider a spacetime governed by the Einstein equations

$$G_{\mu\nu} = 8\pi G \left(T^{(\phi)}_{\mu\nu} + T^{(X)}_{\mu\nu} \right), \tag{1.1}$$

where ϕ is a canonical scalar field [8] with stress-energy

$$T^{(\phi)}_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - g_{\mu\nu} \left(\frac{1}{2} g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi + V(\phi) \right), \tag{1.2}$$

and $T^{(X)}_{\mu\nu}$ denotes an auxiliary sector. The sector $T^{(X)}_{\mu\nu}$ does not represent an additional propagating degree of freedom. Rather, it is a covariant coarse-grained encoding of unresolved curvature structure permitted by admissible weak solutions of the Einstein equations [9,10], as developed in detail in §3.4. No independent equations of motion are introduced beyond those implied by total stress-energy conservation.

Total stress-energy conservation is imposed,

$$\nabla_\mu \left(T^{(\phi)\mu\nu} + T^{(X)\mu\nu} \right) = 0, \tag{1.3}$$

while allowing for internal redistribution of energy via a covariant exchange current J^ν,

$$\nabla_\mu T^{(\phi)\mu\nu} = J^\nu, \quad \nabla_\mu T^{(X)\mu\nu} = -J^\nu. \tag{1.4}$$

This structure preserves general covariance and introduces no new gravitational degrees of freedom. It will serve as the minimal dynamical setting for the analysis that follows.

1.3 Preview of the Holographic Mechanism

In this work, admissibility is used in the standard sense of hyperbolic conservation-law theory, ensuring compatibility with causal propagation and irreversible relaxation [11,12]. We do not attempt microscopic entropy counting or derive covariant entropy bounds. The key result of this paper is that the nonlinear evolution of this system, when treated as a hyperbolic partial differential equation admitting weak solutions subject to admissibility conditions, produces a global codimension-one structure associated with curvature steepening and shock formation [13,14]. The self-similar relaxation of this structure governs the redistribution of energy between curvature and scalar degrees of freedom, and thereby controls the bulk cosmological evolution.

To clarify the logical architecture of the construction, we summarize the chain of reasoning. The analysis proceeds through the following dependencies:

(i) The Einstein–scalar system is a nonlinear hyperbolic system admitting weak solutions subject to admissibility conditions (§3).

(ii) Nonlinear wave steepening generically produces curvature-generated discontinuities, which, when coarse-grained, define the auxiliary sector $T^{(X)}_{\mu\nu}$. This sector is not an independent postulate: it is the covariant encoding of unresolved curvature structure, and its conservation $\nabla_\mu T^{(X)\mu\nu} = -J^\nu$ follows from total stress-energy conservation once the scalar sector's exchange is specified (§3.4).

(iii) The coherent codimension-one structure arising from this process is identified with the cosmological apparent horizon Σ (§3.5).

(iv) The sole additional assumption - the *mediation closure* - is that the only net energy exchange between the bulk scalar sector and the auxiliary sector is the flux through Σ (§4.2).

(v) Applying the Clausius relation $-dE = T\,dS$ on Σ determines the exchange current Q as a functional of horizon variables (§4.2, Eq. (4.6)).

(vi) Substituting this Q into the Friedmann system reduces the independent dynamical variables to the single horizon scale $L = \tilde{r}_A(t)$, constituting dynamical closure (§4.4).

(vii) The energy density constraint $\rho \propto L^{-2}$ and the Bekenstein–Hawking area law $S \propto A$ then follow as thermodynamic consequences of steps (iv)–(vi), confirmed independently by the self-similar scaling of the shock mechanism (§4.7).

This chain makes explicit which elements are geometric consequences, which are dynamical consequences, which are thermodynamic identifications, and which are postulates. In particular, only step (iv) - that the apparent horizon is the causal boundary of the bulk, and the auxiliary sector, which is defined as the coarse-grained curvature structure, exchanges energy with the bulk only through that causal boundary - constitutes an independent assumption beyond the standard Einstein-scalar system and admissible weak-solution theory.

In the sections that follow, we demonstrate explicitly how:

1. hyperbolicity leads to curvature-generated shock structure [15];
2. admissibility enforces entropy-consistent relaxation [11,16];
3. coarse-graining yields a single global control object; and,
4. all bulk observables become functionals of this object.

This sequence constitutes a constructive, explicit realization of holography in cosmology at the level of background evolution.

2 Governing Cosmological Equations and Exchange-Driven Dynamics

2.1 Covariant Field Content and Conservation

We now restate the covariant field content of §1.2 in the form needed for the cosmological specialization that follows. We begin with the Einstein equations coupled to two sectors: a canonical scalar field and an auxiliary conserved sector. The gravitational field equations take the standard form given in Eq. (1.1), where the scalar stress-energy tensor is given by Eq. (1.2). The tensor $T^{(X)}_{\mu\nu}$ represents an auxiliary sector whose microscopic content is left unspecified. Its effective interpretation as coarse-grained curvature structure is developed in detail in §3.4.

General covariance requires total stress-energy conservation,

$$\nabla_\mu \left(T^{(\phi)\mu\nu} + T^{(X)\mu\nu} \right) = 0, \tag{1.3}$$

but does not forbid internal redistribution of energy between the two sectors. We therefore introduce a covariant exchange current J^ν such that

$$\nabla_\mu T^{(\phi)\mu\nu} = J^\nu, \qquad \nabla_\mu T^{(X)\mu\nu} = -J^\nu. \tag{1.4}$$

This construction preserves the Einstein equations exactly and introduces no new gravitational degrees of freedom. All departures from minimally coupled scalar dynamics arise through the internal redistribution of energy encoded in J^ν.

2.2 Specialization to Homogeneous and Isotropic Cosmology

We now specialize to a spatially flat Friedmann-Lemaître-Robertson-Walker spacetime with metric

$$ds^2 = -dt^2 + a^2(t)\, d\vec{x}^2, \tag{2.1}$$

where $a(t)$ is the scale factor and $H \equiv \dot{a}/a$ is the Hubble parameter. On this background, the scalar energy density and pressure are

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi), \qquad p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi), \tag{2.2}$$

while the auxiliary sector is characterized by an energy density ρ_X and pressure p_X. Projecting the exchange current along the cosmological four-velocity u^μ defines the background energy transfer rate

$$Q \equiv -u_\nu J^\nu. \tag{2.3}$$

The background conservation equations then take the form

$$\dot{\rho}_\phi + 3H(\rho_\phi + p_\phi) = Q, \tag{2.4}$$

$$\dot{\rho}_X + 3H(\rho_X + p_X) = -Q. \tag{2.5}$$

By construction, the total energy density obeys the standard conservation law,

$$\dot{\rho}_{\rm tot} + 3H(\rho_{\rm tot} + p_{\rm tot}) = 0, \tag{2.6}$$

with $\rho_{\rm tot} = \rho_\phi + \rho_X$. The Friedmann equations retain their usual form,

$$H^2 = \frac{8\pi G}{3}(\rho_\phi + \rho_X), \tag{2.7}$$

$$\dot{H} = -4\pi G \left(\rho_\phi + p_\phi + \rho_X + p_X\right). \tag{2.8}$$

2.3 Scalar Dynamics and Effective Friction

Combining the scalar continuity equation with the definitions above yields the scalar equation of motion,

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = \frac{Q}{\dot{\phi}}. \tag{2.9}$$

The exchange term therefore enters the scalar dynamics as a modification of the effective friction acting on ϕ, without altering the canonical kinetic structure of the scalar field or introducing phantom degrees of freedom. Throughout this work, we restrict attention to exchange functions satisfying a regularity condition,

$$Q = \mathcal{O}(\dot{\phi}) \quad \text{as} \quad \dot{\phi} \to 0, \tag{2.10}$$

ensuring that the ratio $Q/\dot{\phi}$ remains finite along the cosmological evolution. This condition guarantees regular background dynamics and admits fixed-point solutions in which $\dot{\phi}$ asymptotically vanishes. While mathematically convenient, this condition effectively acts as a stability criterion for the late-time attractor, selecting solutions where the exchange current acts as a friction or drive term proportional to velocity. No specific functional form for Q is assumed at this stage; the analysis that follows depends only on covariance, total conservation, and regularity, not on model-dependent choices.

2.4 Late-Time Fixed Points and Asymptotic Control

The structure of the scalar equation implies the existence of asymptotic solutions characterized by

$$\dot{\phi} \to 0, \qquad \rho_\phi \to V(\phi_*), \qquad \rho_X \text{ subdominant}, \tag{2.11}$$

where ϕ_* denotes a fixed-point value determined dynamically. At such a configuration, the Friedmann equation approaches

$$H^2 \to \frac{8\pi G}{3} V(\phi_*), \tag{2.12}$$

corresponding to accelerated expansion with an effective equation of state approaching

$$w_{\text{eff}} \to -1. \tag{2.13}$$

This behavior arises from energy redistribution rather than from the introduction of a cosmological constant or exotic matter [17]. The approach to the fixed point is governed by the exchange-modified friction term and does not require fine-tuning of initial conditions within the admissible class.

2.5 Interpretation: Exchange as a Holographic Flux

At the level of background cosmology, the exchange current Q may be interpreted as a *holographic transfer rate*. It represents the rate at which the boundary structure Σ updates the state of the bulk geometry. In the language of the correspondence, Q is the flux of constraints flowing from the codimension-one screen into the bulk volume. Although introduced here at the level of an effective description, this flux plays a structural role: it controls the relaxation of the system toward its asymptotic state and determines the expansion history within this framework through the Friedmann equations.

Physically, this exchange structure corresponds to a specific geometric mechanism. As derived in §3, the flux Q is the coarse-grained energy transfer from curvature-generated shocks relaxing on the background. Furthermore, as detailed in §4, the "boundary structure" Σ is uniquely identified with the cosmological apparent horizon - the causal boundary generated by the self-similar shock flow.

Thus, Q is not a phenomenological parameter, but the thermodynamic transfer rate required to satisfy the holographic constraint on the horizon. The bulk cosmological evolution is thereby slaved to the relaxation of this structure, providing the dynamical mechanism underlying holographic dimensional reduction. In this view, the late-time acceleration identified in §2.4 is simply the observable consequence of the system equilibrating to this holographic bound.

We emphasize that this dimensional reduction holds for the attractor solution; off-attractor fluctuations represent transient deviations that decay via the shock-dissipation mechanism. Physically, this implies that holography functions here as a thermodynamic equilibrium state rather than a kinematic identity. If the bulk geometry deviates from the screen-imposed constraint (for example, via an arbitrary injection of scalar kinetic energy), the resulting mismatch between the bulk evolution and the horizon flux generates curvature steepening. This steepening activates the shock sector, which dissipates the excess bulk information through the exchange current Q. This process acts as a dynamical restoring force, driving the system back toward the holographic manifold where the bulk phase space is fully slaved to the boundary.

2.6 Summary

In this section, we have introduced the full set of governing cosmological equations underlying the analysis. The system consists of general relativity coupled to a canonical scalar field and an auxiliary conserved sector with covariant energy exchange. The resulting background dynamics are regular, non-phantom, and admit late-time accelerating fixed points.

These equations already exhibit the key feature required for holography in cosmology: bulk evolution is controlled by a conserved flux rather than by independent volumetric degrees of freedom.

Note that the scalar-plus-auxiliary decomposition is the minimal two-sector system exhibiting this structure. Multi-component extensions - including explicit radiation and matter sectors - inherit the same total conservation constraint and mediation closure, with the exchange current generalized to a multi-sector form. The present analysis focuses on the minimal case sufficient to demonstrate holographic closure; the distinct cosmological epochs conventionally associated with radiation, matter, and dark energy domination emerge within this framework as asymptotic regimes of the single relaxation process (§4.4), rather than as phases requiring independent dynamical inputs.

In the next section, we show how the hyperbolic structure of the Einstein equations promotes this exchange dynamics to a geometric, codimension-one control structure through curvature steepening and admissible shock formation.

3 Hyperbolic Structure, Weak Solutions, and Curvature-Generated Shock Formation

3.1 Hyperbolic Formulation of the Einstein-Scalar System

Once appropriate gauge conditions are imposed, the Einstein equations coupled to matter fields may be written as a system of quasilinear hyperbolic partial differential equations [18–

21]. In particular, under harmonic or generalized harmonic gauge, the metric components satisfy coupled wave equations of the schematic form

$$\Box_g g_{\mu\nu} = \mathcal{N}_{\mu\nu}(g, \partial g) + 8\pi G \left(T^{(\phi)}_{\mu\nu} + T^{(X)}_{\mu\nu} \right), \tag{3.1}$$

where $\Box_g$ denotes the covariant wave operator associated with the spacetime metric and $\mathcal{N}_{\mu\nu}$ collects nonlinear terms quadratic in first derivatives of the metric. [1]

The scalar field obeys a second-order hyperbolic equation,

$$\Box_g \phi - V'(\phi) = -\frac{Q}{\dot{\phi}}. \tag{3.2}$$

This equation is fully covariant despite its apparent dependence on the cosmological four-velocity through Q and $\dot{\phi}$. To see this, note that for a canonical scalar field the divergence of the stress-energy tensor satisfies the identity

$$\nabla^\mu T^{(\phi)}_{\mu\nu} \;=\; \left(\Box_g \phi - V'(\phi)\right) \nabla_\nu \phi \,. \tag{3.3}$$

Combined with the exchange relation $\nabla^\mu T^{(\phi)}_{\mu\nu} = J_\nu$ from Eq. (1.4), this gives $J_\nu = (\Box_g \phi - V')\, \nabla_\nu \phi$: the exchange current for a canonical scalar is kinematically parallel to the scalar gradient. Contracting with u^ν and dividing by $\dot{\phi} \equiv u^\mu \nabla_\mu \phi$ yields $Q/\dot{\phi} = -(\Box_g \phi - V')$, in which the four-velocity cancels identically. The ratio $Q/\dot{\phi}$ is therefore an observer-independent scalar, and the exchange term enters Eq. (3.2) as a lower-order contribution that preserves the principal symbol and hence the hyperbolicity of the system.

Consequently, the coupled Einstein-scalar-exchange system constitutes a nonlinear hyperbolic system governed by local conservation laws and admits the full range of solution behavior familiar from hyperbolic PDE theory, including limited regularity and the formation of discontinuities [13].

3.2 Weak Formulations and Admissible Solutions

In nonlinear hyperbolic systems, classical smooth solutions represent only a subset of admissible solutions. More generally, one may consider weak formulations in which derivatives are interpreted in the sense of distributions and solutions are defined through integral identities [22]. For the Einstein equations, weak formulations require that the metric admit a distributional Levi-Civita connection and curvature tensor, ensuring that the Einstein tensor remains well defined as a distribution. Provided these conditions are met, the contracted Bianchi identities continue to hold weakly,

$$\nabla_\mu G^{\mu\nu} = 0, \tag{3.4}$$

implying local conservation of the total stress-energy tensor even in the presence of limited regularity. Admissibility of weak solutions is enforced by compatibility with causal propagation and conservation laws. In systems derived from conservation laws, this typically requires additional entropy conditions that select physically meaningful solutions and exclude unphysical discontinuities [11]. These admissibility conditions arise directly from the mathematical structure of hyperbolic systems and do not depend on a specific microscopic interpretation.

[1] We note that while the regularization of these shocks implies higher-derivative operators (e.g., curvature-squared terms), we treat these terms within the standard Effective Field Theory (EFT) framework. The associated massive modes remain non-dynamical at cosmological energy scales, thereby avoiding Ostrogradsky instabilities or ghost propagation in the background evolution.

The admissibility framework serves two distinct roles in this construction. First, it ensures causal propagation and entropy-consistent shock relaxation in the hyperbolic Einstein-scalar system, selecting physically meaningful weak solutions and excluding unphysical discontinuities. Second, it provides the mathematical justification for coarse-grained energy accounting across curvature discontinuities: by requiring that conservation laws hold in the weak sense across surfaces of limited regularity, admissibility defines the effective exchange current J_μ and legitimizes the thermodynamic treatment of the control surface developed in §4.

3.3 Nonlinear Wave Steepening and Curvature Concentration

A defining feature of nonlinear hyperbolic equations is wave steepening: initially smooth solutions can develop increasingly sharp gradients under evolution. In many systems, this process culminates in the formation of shock-like discontinuities in finite time [23]. In the gravitational setting, the nonlinearity of the Einstein equations implies that curvature itself can act as a source of wave steepening. Regions of high curvature or rapidly varying geometry amplify propagating modes, leading to a breakdown of classical differentiability while remaining within the class of admissible weak solutions [14]. These curvature-generated discontinuities are not singularities in the sense of divergent invariants. Rather, they correspond to localized drops in differentiability consistent with hyperbolic evolution. When treated in a weak formulation, they carry conserved fluxes and obey admissibility conditions analogous to entropy inequalities in classical shock theory [24].

We note that on an exact FLRW background, spatial homogeneity precludes gradient steepening in the background fields themselves. The shock formation invoked here operates at the level of subhorizon perturbations: the Einstein-scalar system governing inhomogeneous modes is nonlinear hyperbolic, and generic perturbations of sufficient amplitude satisfy the conditions for finite-time gradient blowup established in [13,14]. The auxiliary sector $T^{(X)}_{\mu\nu}$ in the homogeneous equations is then the coarse-grained imprint of this unresolved subhorizon structure on the background energy budget, following the averaging framework of Buchert [9]. A detailed perturbative analysis demonstrating curvature shock formation in near-FLRW Einstein-scalar evolution - and quantifying the conditions on initial perturbation spectra under which the self-similar attractor is reached - is a natural extension of the present work.

3.4 Coarse-Graining and Effective Shock Sector

When curvature-generated discontinuities are present on scales below observational or numerical resolution, it is natural to adopt a coarse-grained description. In such a description, unresolved high-gradient curvature structure is encoded in an effective sector contributing to the stress-energy budget. Within the framework introduced in §1.2, the auxiliary sector $T^{(X)}_{\mu\nu}$ may be interpreted in this way: as an effective description of coarse-grained curvature dynamics arising from admissible weak solutions [9]. The exchange current J^ν then represents the net flux of energy between resolved scalar degrees of freedom and unresolved curvature structure.

This interpretation introduces no new propagating degrees of freedom and no independent Cauchy data beyond that of the Einstein-scalar system. The auxiliary sector $T^{(X)}_{\mu\nu}$ is defined as the residual of total stress-energy conservation once the scalar sector's exchange is specified: given $\nabla_\mu(T^{(\phi)\mu\nu} + T^{(X)\mu\nu}) = 0$ as a fundamental consequence of general covariance, the decomposition $\nabla_\mu T^{(\phi)\mu\nu} = J^\nu$, $\nabla_\mu T^{(X)\mu\nu} = -J^\nu$ is not an independent postulate but a definitional consequence of the coarse-graining procedure. The exchange current J^ν is then not arbitrary but constrained by total conservation and the admissibility conditions of §3.2. Its functional form is ultimately fixed by the mediation closure introduced in §4.2.

The auxiliary sector is conserved up to exchange with the scalar field, and total stress-energy conservation is preserved exactly. The exchange dynamics therefore encode the influence of curvature-generated shock structure on the large-scale evolution without modifying the underlying gravitational equations.

3.5 Emergence of a Global Codimension-One Structure

The key dynamical object underlying the holographic formulation developed in this work is a global codimension-one structure that governs cosmological evolution through conservation-law closure. In a spatially flat FLRW background, a strictly planar global shock is topologically forbidden. However, we identify the relevant codimension-one structure not as a literal planar discontinuity, but as the cosmological apparent horizon (or marginally trapped surface), which acts as the statistical envelope of local curvature collapse events. The combination of expansion, large-scale symmetry, and conservation laws favors the organization of local discontinuities into this coherent global structure. Following the averaging formalism of Buchert [9], we treat this effective hypersurface as the bounding surface of the causal domain. This structure organizes regions of differing curvature behavior and carries conserved fluxes associated with the exchange current. Its evolution is governed by conservation laws, admissibility conditions, and scaling relations rather than by arbitrary local dynamics. The existence of such a structure is not an additional assumption. It follows from the hyperbolic nature of the equations and the requirement that admissible weak solutions respect conservation and causality.

On the spatially flat FLRW background, the identification of the shock envelope with the marginally trapped surface follows from uniqueness rather than from a separate calculation. Establishing this identification beyond the homogeneous and isotropic setting - in particular, demonstrating that the shock envelope of a perturbed Einstein-scalar evolution coincides with the perturbed marginally trapped surface - is a well-posed geometric question that lies beyond the scope of the present background-level analysis. In the background-level analysis, the self-similar attractor structure forces a single macroscopic dynamical scale L (§3.6). The unique spherically symmetric codimension-one surface at this scale is the apparent horizon $\tilde{r}_A = 1/H$; no competing codimension-one structure exists at the same scale on this background.

Once present, this codimension-one structure becomes the primary carrier of information about unresolved curvature dynamics. The identification is therefore not an independent modeling choice but a consequence of the symmetry of the background geometry and the absence of additional scales.

3.6 Self-Similarity and Absence of Intrinsic Scales

The gravitational sector, when treated classically, contains no intrinsic length or time scales beyond those introduced by initial conditions and matter content. In the absence of preferred microphysical scales, conservation-law-driven systems generically admit self-similar solutions [25]. The self-similarity invoked here does not refer to a strict homothetic or kinematic self-similarity (i.e., the existence of a homothetic Killing vector), but rather to the absence of intrinsic scale in the relaxation regime, so that dimensional analysis enforces scaling laws for the conserved fluxes [29]. In the language of dynamical systems, self-similarity functions as a scaling attractor: generic initial conditions within the admissible class evolve toward the self-similar profile under the combined action of expansion and entropy-consistent dissipation [30,31]. It is this attractor behavior, rather than an exact symmetry, that is required for the holographic reduction developed in §4.

The effective codimension-one structure identified above therefore evolves toward a self-similar profile governed by scaling relations. Its relaxation is determined by conservation of energy and momentum fluxes, with expansion acting to rescale physical quantities continuously. This self-similar behavior is a dynamical attractor, independent of the detailed microstructure of the underlying curvature discontinuities. As a result, the evolution of the codimension-one structure becomes a scale-free process controlling the large-scale dynamics of the spacetime.

3.7 Summary

In this section, we have shown that the Einstein-scalar-exchange system introduced in §2 constitutes a nonlinear hyperbolic system admitting weak solutions subject to admissibility conditions. Nonlinear wave steepening leads generically to curvature-generated discontinuities, which may be treated consistently through weak formulations and coarse-graining.

Here "generic" is used in the precise sense appropriate to nonlinear hyperbolic systems: within the class of admissible weak solutions of the Einstein equations satisfying (i) hyperbolicity, (ii) conservation-law structure, and (iii) entropy admissibility, curvature steepening and the emergence of codimension-one structures occur for open sets of initial data in function space. The present analysis does not claim that all smooth cosmological initial data evolve into such configurations, but rather that once admissible weak evolution is reached, the subsequent organization into a global control surface is structurally enforced by conservation and scaling, independent of microphysical details.

In a cosmological setting, these discontinuities organize into a coherent codimension-one structure identified with the apparent horizon, whose evolution is governed by conservation laws and admissibility conditions. The absence of intrinsic scales drives this structure toward self-similar relaxation. In the following section, we demonstrate that the bulk cosmological evolution is slaved to this structure, yielding an explicit realization of holographic dimensional reduction.

4 Emergence of a Holographic Control Surface and Closure of Cosmological Dynamics

4.1 From Coarse-Grained Shocks to a Holographic Surface

In Sections 2 and 3, we established three facts:

1. The Einstein-scalar-exchange system constitutes a nonlinear hyperbolic system admitting weak solutions subject to admissibility conditions.

2. Nonlinear wave steepening generically produces curvature-generated discontinuities.

3. When coarse-grained consistently, these discontinuities organize into a coherent codimension-one structure governed by conservation laws and admissibility.

We now show that this codimension-one structure functions as a holographic control surface for cosmological dynamics: once its evolution is specified, the bulk evolution is determined, within the admissible solution class considered here. By a holographic control surface, we mean a lower-dimensional dynamical structure whose degrees of freedom determine the evolution of bulk observables through constraint and conservation relations, rather than through independent volumetric dynamics.

4.2 Definition of the Control Surface and Its Degrees of Freedom

We define the holographic control surface Σ as the cosmological apparent horizon, the marginally trapped surface defined by the vanishing of the ingoing null expansion $\theta_{in} = 0$ and strictly positive outgoing expansion $\theta_{out} > 0$.

This identification of Σ with the apparent horizon is not an arbitrary choice of boundary condition, but a structural consequence of the underlying hyperbolic dynamics. In classical self-similar systems, the energy distribution is constrained by the physical boundaries of the domain - the 'container walls.' In the cosmological setting, where no static spatial boundaries exist, these walls become comoving horizons. The apparent horizon thus emerges as the natural causal boundary enclosing the bulk dynamics. By identifying this dynamical horizon as the holographic screen Σ, we ensure that the information content of the bulk is constrained by the same geometric surface that defines its causal extent.

In a flat FLRW geometry, the radius of this surface is simply the Hubble radius:

$$\tilde{r} = \frac{1}{H}. \tag{4.1}$$

To determine the exchange flux Q (and thus $\mathcal{F}_\Sigma$), we apply the Unified First Law of Thermodynamics to this surface. Following the approach of Cai & Kim [26] and Akbar & Padmanabhan [27], the change in energy dE within the horizon must balance the thermodynamic flux across it:

$$-dE = TdS, \tag{4.2}$$

where E is the Misner-Sharp mass enclosed by Σ, $T = 1/(2\pi\tilde{r}_A)$ is the horizon temperature, and $S = A/4G$ is the horizon entropy.

Explicit screen-flux derivation and the induced closure for Q. In a spatially homogeneous FRW spacetime, the Misner–Sharp energy inside the apparent horizon is

$$E \;=\; \rho_{\rm tot}\, V, \qquad V \;=\; \frac{4\pi}{3}\,\tilde{r}_A^{\,3}, \qquad A \;=\; 4\pi\,\tilde{r}_A^{\,2}, \tag{4.3}$$

with $\rho_{\rm tot} = \rho_\phi + \rho_X$. The energy crossing the horizon in a proper time interval dt is the standard Cai–Kim energy flux

$$\frac{dQ_\Sigma}{dt} \;=\; A\,(\rho_{\rm tot} + p_{\rm tot})\, H\, \tilde{r}_A, \tag{4.4}$$

which follows from contracting the perfect-fluid stress tensor with the approximate null generator of the horizon and integrating over the horizon two-sphere (see [26] for the canonical derivation). The Clausius relation on the screen is then

$$-\frac{dE}{dt} \;=\; \frac{dQ_\Sigma}{dt} \;=\; T\,\frac{dS}{dt}, \qquad T = \frac{1}{2\pi\tilde{r}_A}, \qquad S = \frac{A}{4G}. \tag{4.5}$$

Equations (4.4)–(4.5) reproduce the usual Friedmann dynamics for the *total* fluid (equivalently, they encode the total continuity equation together with the geometric relation defining $\tilde{r}_A$). To connect this screen flux to the *exchange* current used in the bulk effective description, we impose the following closure. This represents the sole additional assumption in the framework and is necessary to ensure that the auxiliary sector functions strictly as a boundary-slaved reservoir rather than an independent bulk fluid:

The auxiliary sector encodes degrees of freedom living on (or slaved to) the screen Σ, and the only net energy transfer between the bulk scalar sector and the auxiliary sector is the energy flux through Σ.

Under this closure, the bulk source term Q in (2.4)–(2.5) must equal the horizon flux per unit proper bulk volume, i.e.,

$$Q\,V \;=\; -\frac{dQ_\Sigma}{dt} \qquad\Longrightarrow\qquad Q \;=\; -\frac{A}{V}\,(\rho_{\rm tot}+p_{\rm tot})\,H\,\tilde r_A \;=\; -\,3H\,(\rho_{\rm tot}+p_{\rm tot}). \tag{4.6}$$

The sign convention here matches (2.4)–(2.5): $Q > 0$ corresponds to net energy transfer into the scalar sector, whereas a positive outward screen flux $dQ_\Sigma/dt > 0$ depletes the bulk scalar sector when the auxiliary sector is identified with the screen reservoir. Equation (4.6) is the sense in which the exchange term is no longer arbitrary: once one chooses the screen-mediated exchange closure, the functional dependence of Q is fixed by horizon thermodynamics.

It is worth stating here precisely what is meant by "closing the dynamics" in this holographic context. In kinematical holographic frameworks — such as covariant entropy bounds — one obtains inequality constraints ($S_{\rm bulk} \le A/4G$) that restrict the solution space but do not supply evolution equations. By contrast, the mediation closure and horizon thermodynamics developed above fix Q as a functional of $(\rho_{\rm tot}, p_{\rm tot}, H, \tilde r_A)$ via Eq. (4.6). Substituting this Q into the continuity equations (2.4)–(2.5) and the Friedmann system (2.7)–(2.8), the independent dynamical variables reduce from $\{a, \phi, \rho_X\}$ to the single horizon scale $L = \tilde r_A(t)$, with all remaining quantities determined by constraint. Dynamical closure therefore means: the horizon law and mediation constraint reduce the independent bulk variables so that the Friedmann system becomes dynamically determined by the codimension-one data alone.

The mediation closure is independently motivated by the self-similar attractor structure established in §3.6. In the self-similar regime, the absence of intrinsic scales forces the auxiliary sector energy density to be a functional of the single macroscopic scale, $\rho_X = \rho_X(L)$ with $L = \tilde r_A$. Total stress-energy conservation then determines Q as a functional of the horizon data $(L, \dot L)$ via Eq. (2.4):

$$Q \;=\; -\frac{d\rho_X}{dL}\,\dot L \;-\; 3H\big(1+w_X(L)\big)\,\rho_X(L)\,, \tag{4.7}$$

with no free functions remaining. The mediation closure is therefore not an arbitrary restriction on the exchange dynamics, but the unique closure consistent with the self-similar attractor and conservation-law structure of the admissible solution class. The Clausius relation on Σ provides an independent thermodynamic derivation of the same result, confirming the internal consistency of the construction (§4.7).

With the screen-mediated exchange closure made explicit, the exchange term Q is no longer an arbitrary phenomenological ansatz: it is fixed by the horizon flux required by the Clausius relation on Σ, thereby linking the bulk effective dynamics to the codimension-one control surface Σ.

4.3 Slaving Relations for Bulk Cosmological Variables

We now demonstrate that bulk cosmological observables are slaved to the evolution of Σ. The background Friedmann equation, Eq. (2.7) combined with the continuity equations, Eq. (2.4) and (2.5), shows that the expansion rate $H(t)$ depends on the cumulative effect of the exchange flux $Q(t)$. Since $Q(t)$ is now determined by the horizon thermodynamics of Σ, the expansion history within this framework $a(t)$ becomes a functional of the control surface dynamics,

$$H(t) = \mathcal{H}[\mathcal{F}_\Sigma(t)]. \tag{4.8}$$

Similarly, the scalar equation of motion, Eq. (2.9), shows that the scalar kinetic energy is dynamically regulated by the same flux. The approach to fixed points with $\dot{\phi} \to 0$ is therefore governed by the relaxation of Σ, not by independent volumetric tuning. Thus, the bulk variables $\{H, \rho_\phi, \rho_X, \phi\}$ do not constitute independent degrees of freedom. They are constrained projections of the reduced surface dynamics encoded in $\mathcal{F}_\Sigma$.

4.4 Holographic Closure of the Cosmological System

We now state the central result: once the evolution of the control surface Σ is specified, the bulk cosmological dynamics are determined, within the admissible class of initial conditions, by conservation-law closure. This constitutes holographic closure.

Definition (Admissible class). The holographic closure demonstrated above holds for solutions of the Einstein–scalar system satisfying:

(a) global hyperbolicity of the spacetime;

(b) existence of a coherent cosmological apparent horizon Σ;

(c) $L = \tilde{r}_A$ as the unique macroscopic dynamical scale (self-similar attractor regime);

(d) entropy production consistent with weak-solution admissibility in the sense of §3.2.

These conditions are not restrictive for the class of spatially homogeneous and isotropic cosmologies coupled to a canonical scalar field with regular potential, which generically satisfy (a)–(d) once admissible weak evolution is reached. Conversely, cosmological solutions that may fall outside this class include: strongly anisotropic spacetimes (e.g., Bianchi types with no isotropization mechanism) where no coherent apparent horizon forms; spacetimes with multiple competing horizons where the identification of a unique control surface becomes ambiguous; and matter configurations that violate the regularity condition $Q = \mathcal{O}(\dot{\phi})$ of Eq. (2.10). These exclusions delimit the scope of the present construction.

Within the present construction there are no remaining free functions governing the background cosmology beyond those already fixed by the surface dynamics and conservation laws. Apparent freedom in choosing independent cosmological phases is revealed to be an artifact of redundant parameterization. In particular:

1. Early-time accelerated expansion arises from the high-curvature, high-flux regime of Σ.
2. Intermediate radiation- and matter-like behavior arises from the self-similar relaxation tail.
3. Late-time acceleration arises from the approach to a low-flux fixed point. This regime is entered when the exchange flux Q reverses sign, a transition determined by the relative scaling of the shock reservoir ρ_{shock} compared to the scalar kinetic energy. As the shock cascade thermalizes and dilutes, the gradient flow naturally reverses, driving the system toward the attractor.

These are not independent epochs, but asymptotic regimes of the same surface-controlled evolution. We emphasize that the present analysis establishes the *existence* of a holographic closure mechanism within a broad admissible class of cosmological solutions. It is important to note that this closure is dynamical rather than ontological: the holographic surface determines the evolution of the geometry and energy distribution, but the underlying matter content (the scalar potential and auxiliary equation of state) remains an external input to the theory. Questions of full classification, similarity universality, and exhaustiveness lie beyond the scope of the present work.

4.5 Explicit Holographic Dictionary

The holographic nature of the construction may be summarized by the following correspondence:

Table 1: Operational dictionary mapping bulk evolution history to the relaxation state of the holographic control surface Σ, enforced by conservation-law closure.

Bulk Cosmological Quantity	**Encoded by Control Surface Σ**
Expansion history $a(t)$	Relaxation profile of $\mathcal{F}_\Sigma$
Effective equation of state w_{eff}	Scaling regime of surface flux
Inflationary impulse	Early-time high-flux scaling
Radiation/matter continuity	Self-similar decay tail
Late-time acceleration	Low-flux fixed point
Entropy production	Admissibility of surface dynamics
Degrees of freedom	Codimension-one surface variables

This dictionary is not postulated. It follows directly from the governing equations and the admissibility of weak solutions. This correspondence makes clear that the reduced description is dynamical and internal to the cosmological evolution, rather than imposed externally.

4.6 Distinction from Boundary-Based Holography

Unlike boundary-based holographic constructions, the control surface Σ is not imposed at spatial or conformal infinity, nor is it associated with a fixed asymptotic geometry. It is a dynamical, internal structure generated by the evolution of the gravitational field itself. As a result, the dimensional reduction demonstrated here is both local and dynamical. It does not rely on external dualities, asymptotic symmetries, or microscopic assumptions. Instead, it emerges from classical gravitational dynamics treated as a nonlinear hyperbolic system. While the location of the apparent horizon depends on the choice of foliation, the thermodynamic relation $-dE = T\,dS$ and the resulting flux Q transform consistently as scalar densities, ensuring that the background dynamics remain covariant.

4.7 Holographic Scaling from Dimensional Reduction

A defining feature of the holographic control surface identified here is that its dynamics enforce an area law for the bulk entropy [32]. To make the logical status of this result precise, we distinguish two independent routes to the same conclusion and clarify which elements are derived and which are postulated.

Route 1: Thermodynamic derivation from mediation closure. The mediation assumption (§4.2) requires that all net bulk–auxiliary exchange passes through Σ. The Clausius relation $-dE = T\,dS$ on Σ, with $T = 1/(2\pi\tilde{r}_A)$ and $E \sim \rho_{\text{tot}}\,\tilde{r}_A^3$, yields $d(\rho\,L^3) \sim L\,dS$.

In the self-similar regime where L is the unique macroscopic scale, dimensional consistency enforces $\rho \propto L^{-2}$, whence $S \propto L^2 \propto A$. This derivation does not require a specific scalar potential $V(\phi)$ or equation of state. It requires: (i) the existence of a trapping/apparent horizon, (ii) the mediation closure, and (iii) the assignment of horizon temperature $T \propto L^{-1}$. It is independent of the scalar equation of state except insofar as that equation admits an apparent horizon solution.

Route 2: Self-similar shock scaling (independent confirmation). Consider a region of characteristic length scale L. The shock mechanism ensures that the system maintains the dimensionless invariant

$$\Pi \equiv G\rho L^2 \sim \mathcal{O}(1), \tag{4.9}$$

which implies that the bulk energy density must scale as $\rho \propto L^{-2}$. Crucially, this scaling arises independently from the self-similarity of the hydrodynamic shock mechanism. We observe that this hydrodynamic necessity *coincides* exactly with the thermodynamic area law. Thus, the shock dynamics naturally satisfy the holographic bound without it being imposed as an external axiom.

In a standard volumetric system, the total energy E within a volume $V \sim L^3$ would scale as $E \propto L^3$. However, the shock-imposed constraint $\rho \propto L^{-2}$ forces the total energy to scale as

$$E \sim \rho L^3 \propto L^{-2} \cdot L^3 = L. \tag{4.10}$$

In gravitational thermodynamics, the bulk entropy S is proportional to the energy times the characteristic length, $S \sim E \cdot L$. Substituting the scaling for E, we find

$$S \sim (L) \cdot L \propto L^2 \sim A. \tag{4.11}$$

The two routes arrive at the same scaling independently: the first from horizon thermodynamics under mediation, the second from the self-similar conservation-law structure of the shock mechanism. The area law is therefore a derived consequence of the construction with the self-similar conservation-law structure providing an independent, purely hydrodynamic confirmation. Note further that the scaling $\rho_{\rm tot} \propto L^{-2}$ also follows algebraically from the Friedmann constraint $H^2 = 8\pi G\rho_{\rm tot}/3$ and the definition $L = \tilde{r}_A = 1/H$, which gives $\rho_{\rm tot} = 3/(8\pi G L^2)$ identically. The two routes above therefore demonstrate that the thermodynamic and self-similar structures are each independently consistent with this constraint, rather than deriving it from independent premises.

The shock mechanism acts as the dynamical realization of the holographic constraint, limiting the information content of the bulk to scale with the area of the codimension-one structure rather than the volume. This convergence of the thermodynamic and hydrodynamic arguments provides the physical basis for the holographic closure asserted in this work.

4.8 Summary

In this section, we have shown that the Einstein-scalar-exchange system admits an explicit holographic formulation within a broad admissible class of solutions in which cosmological dynamics are governed by a codimension-one control surface arising from curvature-generated discontinuities. Bulk observables are slaved to the evolution of this surface through conservation-law closure, yielding an explicit realization of holographic dimensional reduction in cosmology.

5 Discussion and Conclusions

5.1 Holography as a Dynamical Principle in Cosmology

The analysis presented in this work demonstrates that holography in cosmology need not be postulated, imposed through boundary conditions, or inferred indirectly through analogies with black hole thermodynamics or gauge-gravity duality. Instead, holographic dimensional reduction can arise dynamically and explicitly from the classical equations of general relativity when they are treated as nonlinear hyperbolic systems admitting weak solutions subject to admissibility conditions. In the present construction, holography is realized in its operational sense: bulk cosmological dynamics are determined, within the admissible class considered here, by degrees of freedom associated with a lower-dimensional structure. This structure is not an abstract boundary at infinity, but a codimension-one control surface generated by curvature steepening and constrained by conservation laws, admissibility, and self-similar scaling.

Once this surface is identified, the apparent multiplicity of independent cosmological degrees of freedom collapses. The expansion history, effective equation of state, and transitions between cosmological regimes are revealed to be projections of a single reduced dynamical process. Thus, this work establishes a concrete realization of holography in cosmology.

5.2 Comparison with Existing Holographic Frameworks

The holographic mechanism identified here belongs to a class of non-AdS, dynamical holographic constructions in which dimensional reduction is enforced by constraint relations and conservation laws, rather than by an explicit boundary duality. Most established realizations of holography rely on fixed asymptotic boundaries [4,28], special spacetime geometries, or dual quantum field theories [5]. While these approaches have yielded deep insights, they are poorly suited to realistic cosmological spacetimes, which lack static boundaries and evolve dynamically across wide curvature regimes.

The construction presented here differs in several essential respects: no asymptotic boundary is required; no special geometry is assumed; and no microscopic dual theory is invoked. The holographic control surface is generated internally by the gravitational dynamics, and the reduction arises within classical general relativity supplemented only by admissible matter couplings and conservation. In this sense, the holography exhibited here applies directly to cosmology without requiring extrapolation from special limits or idealized settings. The novelty is therefore not the presence of an exchange term, but the emergence of a lower-dimensional control structure that closes a class of cosmological dynamics.

It is instructive to further compare the present construction explicitly with the covariant entropy bound of Bousso [3,4]. The Bousso bound provides an inequality constraint - $S_{\mathrm{matter}} \leq A/4G$ for entropy flux through any light-sheet - that applies universally but does not supply dynamical evolution equations. By contrast, the present work derives an equality: under the mediation closure and within the admissible class, the cosmological system evolves dynamically toward area scaling $S \propto A$. The construction may therefore be interpreted as a dynamical saturation of an entropy bound within a specific class of spacetimes, rather than a derivation of the bound itself. We do not claim equivalence with the Bousso bound; rather, we show that under mediation and hyperbolic admissibility, the cosmological system evolves toward area scaling through a concrete dynamical mechanism. Whether this dynamical saturation can be shown to imply or be implied by the covariant entropy bound in general remains an open question.

5.3 Comparison with Existing Generic Interacting Dark Energy Frameworks

A number of cosmological models introduce phenomenological energy exchange between matter, radiation, or dark energy components in order to modify the late-time expansion history [33–35]. Such interacting dark energy frameworks are typically formulated at the level of background evolution and are designed to reproduce specific observational features.

While the background equations in the present construction take a superficially similar form, the underlying mechanism and interpretation differ in a fundamental way. The comparison is therefore not intended to contrast specific models or parameterizations, but to clarify the structural distinction between phenomenological exchange descriptions and a dynamical holographic closure mechanism.

In particular, the present construction does not treat the exchange term as an independent modeling choice, but interprets it as the coarse-grained manifestation of curvature-generated structure constrained by admissibility and conservation. As a result, the exchange dynamics enforce a reduction in effective degrees of freedom rather than introducing additional freedom at the level of cosmological phases. Table 2 summarizes the distinction between background exchange models and the dynamical holographic closure presented here.

Table 2: Distinction between background exchange models and dynamical holographic closure.

Generic interacting dark energy	**Present construction**
Exchange postulated phenomenologically	Exchange interpreted as coarse-grained curvature flux
No dimensional reduction enforced	Bulk observables slaved to codimension-one structure Σ
Independent epoch-by-epoch freedom	Regimes arise as asymptotic sectors of one relaxation
No closure requirement	Conservation-law closure enforces holography

5.4 Resolution of Redundancy in Cosmological Modeling

A persistent feature of cosmological modeling has been the treatment of different epochs - inflation, radiation domination, matter domination, and late-time acceleration - as dynamically independent phases requiring distinct physical explanations. This phase-based approach implicitly assumes a large number of independent volumetric degrees of freedom. The holographic formulation developed here shows that this assumption is unnecessary within the admissible class considered: these regimes arise as asymptotic projections of a single reduced dynamical system governed by the evolution of the control surface. The apparent complexity of cosmological history is therefore a manifestation of redundancy in the bulk description, not of fundamental multiplicity in the underlying dynamics. From this perspective, holography provides not merely a reinterpretation of cosmology, but a mechanism for explaining why cosmological evolution can exhibit reduced effective degrees of freedom relative to naïve volumetric counting.

5.5 Significance and Scope

Several aspects of the result are worth emphasizing. First, the construction is explicit: the governing equations are written down directly and the reduction follows from their structure. Second, the construction is conservative: no modification of the Einstein equations is required, and no exotic matter or phantom behavior is introduced. Third, the construction is dynamical: the holographic surface is not imposed, but generated by evolution. Fourth, the construction is cosmological: it applies to expanding spacetimes and directly governs observable quantities. While the analysis here has focused on homogeneous and isotropic background cosmology, the mechanism is more general. Extensions to perturbations, gravitational waves, and structure formation are natural directions for future work and may provide additional signatures of holographic control in cosmology.

We emphasize that while hyperbolic systems generically admit shock formation, we have not provided a rigorous proof that smooth cosmological initial data must evolve into the specific curvature-front configurations utilized here. What we have established is that if the system settles into such an admissible weak solution, holographic closure naturally follows. Conversely, the only way to avoid the emergence of a codimension-one control structure within the present framework is to enforce global smoothness and suppress admissible weak evolution by assumption. Such a restriction is non-generic for nonlinear hyperbolic systems and is unstable under perturbations. In this sense, the holographic closure identified here is not an optional feature of the dynamics, but the natural outcome once weak solutions consistent with conservation and admissibility are allowed.

Finally, we comment on the relationship between the present classical construction and quantum approaches to cosmological holography. The analysis above identifies a specific set of classical geometric ingredients - a dynamical codimension-one control surface, a thermodynamic closure linking bulk evolution to horizon flux, and an area-law scaling for the bulk entropy - that together constitute a candidate semiclassical structure for holography in cosmology.

The distinguishing feature of this construction relative to existing classical holographic frameworks is structural: holographic dark energy models [36] postulate both the holographic surface (as an IR cutoff) and the area-law scaling $\rho \propto L^{-2}$ from an entropy bound; emergent gravity programs [37,38] assume both a holographic screen and its thermodynamic properties, then derive gravitational dynamics.

In the present work, neither the holographic surface nor its scaling is assumed. The surface emerges from the hyperbolic structure of the Einstein equations, and the area-law scaling follows from a single assumption about the causal structure of energy exchange. Only the relationship between the emergent surface and the bulk - that net energy transfer is mediated through the apparent horizon - is postulated.

This construction may therefore be of practical use for quantum approaches to cosmological holography - including those inspired by dS/CFT, holographic entanglement entropy, or covariant entropy bounds - which require an explicit classical holographic structure to recover in an appropriate semiclassical limit. We do not claim that the present construction is the unique or necessary classical limit of quantum cosmological holography; other classical mechanisms achieving holographic dimensional reduction may exist. However, the specific combination identified here - emergent surface, derived scaling, single thermodynamic postulate - provides a concrete benchmark against which the semiclassical limits of such proposals can be checked. To our knowledge, no previous construction has offered this combination in a cosmological setting. We do not speculate further on the quantum completion.

5.6 Conclusions

We have presented an explicit, equation-level realization of holography in cosmology arising from hyperbolic general relativity. By coupling general relativity to a canonical scalar field and a conserved auxiliary sector with covariant energy exchange, and by treating the resulting system within admissible weak formulations, we have shown that nonlinear evolution generically produces a global codimension-one control surface. The self-similar relaxation of this surface governs cosmological evolution within this class, rendering bulk observables slaved, within this admissible class, to a reduced set of surface degrees of freedom. Distinct cosmological regimes emerge as asymptotic projections of the same underlying dynamics rather than as independent phases. By providing a constructive, non-AdS realization of holography in a cosmological setting, this work aligns naturally with holography applications beyond traditional gauge-gravity dualities. In contrast to boundary-imposed or conjectural implementations of holography in cosmology [4,5], the construction presented here applies holographic reasoning directly to cosmological dynamics by identifying a lower-dimensional control structure that emerges from hyperbolic gravitational evolution and determines observable expansion histories. In this sense, holography in cosmology is not merely a conjectural principle or an analogy imported from other gravitational systems, but a constructive consequence of classical general relativity – supplemented by a single thermodynamic closure on the causal structure of energy exchange – when its nonlinear dynamical structure and admissible weak solutions are treated consistently.

Authors' Contributions

The sole author is responsible for all aspects of this work.

Data Availability

The manuscript has no associated data or the data will not be deposited.

Conflicts of Interest

The author declares that there is no conflict of interest.

Ethical Considerations

The author has diligently addressed ethical concerns, such as informed consent, plagiarism, data fabrication, misconduct, falsification, double publication, redundancy, submission, and other related matters.

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