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Regular article

Analytic Correspondence between Barrow Holographic Dark Energy and $f(Q)$ Gravity

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Abstract. We investigate Barrow holographic dark energy within the framework of symmetric teleparallel $f(Q)$ gravity at the homogeneous background level. Adopting a reconstruction viewpoint, we require the effective geometric energy density of $f(Q)$ gravity to reproduce the Barrow holographic scaling when the Hubble radius is chosen as the infrared cutoff. This condition uniquely determines a simple analytic power-law form for the nonmetricity scalar in the gravitational Lagrangian, with the Barrow deformation parameter directly fixing the exponent. The reconstructed action smoothly reduces to the symmetric teleparallel equivalent of general relativity in the limit of vanishing Barrow correction Δ. We analyze the background cosmological behavior in the presence of pressureless matter and show that, for $0 < \Delta < 1$, the modified scaling admits an asymptotic de Sitter solution, while the standard $\Delta = 0$ case does not yield self-acceleration with the Hubble cutoff. Our results establish a minimal analytic embedding of Barrow holographic dark energy into nonmetricity-based modified gravity and provide a transparent geometric interpretation of the Barrow deformation parameter.

Keywords: Barrow Holographic Dark Energy; $f(Q)$ Gravity; Symmetric Teleparallel Gravity; Modified Gravity.

Contents

1 Introduction

The holographic principle, originally proposed in the context of black hole thermodynamics and later motivated by developments in string theory, suggests that the number of physical degrees of freedom within a region of space scales with the area of its boundary rather than its volume [1,2]. In cosmology, this idea led to the formulation of holographic dark energy (HDE), where the vacuum energy density of the Universe is related to an infrared (IR) cutoff scale associated with the size of the cosmic horizon [3]. The standard HDE model is constructed using the Bekenstein–Hawking entropy and has been extensively studied within general relativity and various modified gravity frameworks.

More recently, Barrow proposed a modification of the Bekenstein–Hawking entropy by allowing the black hole horizon to acquire a fractal structure due to quantum gravitational effects [4]. The thermodynamic and gravitational implications of this proposal have been further examined in various contexts, e.g., [5]. This leads to the so-called Barrow entropy, characterized by a deformation parameter Δ, and gives rise to Barrow holographic dark energy (BHDE) [6]. The corresponding energy density scales as $\rho_{\text{BHDE}} \propto L^{\Delta-2}$, where L is the cosmological IR cutoff. The standard HDE model is recovered in the limit $\Delta = 0$, while nonzero values of Δ introduce new phenomenological possibilities for late-time cosmic acceleration. It is worth noting that Barrow entropy shares a structural similarity with Tsallis non-extensive entropy [7] at the microcanonical level, although the interpretation of the deformation parameter and its admissible range differ. This relationship has been explored in various cosmological contexts, including comparative studies of Barrow and Tsallis holographic dark energy models [8–10]. Moreover, Barrow holographic dark energy can be viewed as a particular realization within the generalized holographic dark energy family introduced by Nojiri and Odintsov [11], placing it within a broader entropic cosmology framework.

In parallel, symmetric teleparallel gravity, also known as $f(Q)$ gravity, has emerged as a simple and geometrically appealing modification of general relativity. In this approach, gravitation is entirely encoded in the nonmetricity scalar Q, while curvature and torsion vanish identically, providing a formulation that is equivalent to general relativity within a flat and torsionless geometry [12,13]. The resulting field equations are of second order, and the theory admits a particularly simple formulation in the coincident gauge [14]. Cosmological applications of $f(Q)$ gravity have been actively investigated in recent years [15–17].

The combination of holographic dark-energy ideas with modified gravity has been explored in a wide range of contexts. These include HDE and BHDE models formulated within $f(R)$ gravity, teleparallel $f(T)$ gravity, Gauss–Bonnet and $f(R, G)$ theories, $f(R, T)$ and $f(G, T)$ gravity with matter couplings, scalar–tensor models, and other generalized gravitational frameworks [18–24]. Such studies aim to understand how holographic arguments may be modified by geometric extensions of general relativity and whether these modifications can provide viable explanations for late-time acceleration.

Several works have also studied BHDE within nonmetricity-based gravity and its extensions, including reconstructions of $f(Q)$ or related models [25–27]. In particular, Ref. [25] reconstructed BHDE-inspired $f(Q)$ gravity models involving nontrivial functional structures such as Lambert functions. The present work does not introduce a new reconstruction scheme at the conceptual level; rather, it focuses on obtaining a minimal and fully analytic closed-form realization in which the gravitational Lagrangian reduces to a simple power law with a direct one-to-one relation between the Barrow deformation parameter and the nonmetricity exponent. This analytic minimality and transparency constitute the main distinction of our approach.

The purpose of this paper is to establish a simple analytic correspondence between Barrow holographic dark energy and a minimal $f(Q)$ gravity model at the level of homogeneous FRW cosmology. Rather than deriving BHDE dynamically from the nonmetricity sector, we adopt a reconstruction viewpoint: the effective geometric energy density arising in $f(Q)$ gravity is required to reproduce the prescribed BHDE scaling. Working in the coincident gauge, we show that this requirement uniquely selects a power-law form of $f(Q)$, with the exponent directly determined by the Barrow deformation parameter.

Among the various infrared cutoffs used in holographic constructions, we focus on the Hubble radius $L = H^{-1}$. This choice is motivated primarily by analytic tractability, as it allows the reconstruction problem to be solved in closed form and yields a transparent power-law structure for the gravitational Lagrangian. It is well known that in standard holographic dark energy within general relativity the Hubble cutoff does not generically lead to late-time acceleration without introducing additional assumptions or interactions. In the present framework, however, we show that for nonvanishing Barrow deformation parameter $0 < \Delta < 1$, the modified scaling $\rho_{\mathrm{BHDE}} \propto H^{2-\Delta}$ alters the algebraic structure of the Friedmann equation and admits an asymptotic de Sitter solution once matter becomes subdominant. The present analysis should therefore be understood as an analytic background-level embedding of BHDE into symmetric teleparallel gravity, clarifying the geometric role of the Barrow parameter and providing a transparent starting point for more complete dynamical and phenomenological investigations.

The paper is organized as follows. In Section 2 we briefly review $f(Q)$ gravity and the BHDE energy density. In Section 3 we perform the analytic reconstruction of $f(Q)$. In Section 4 we discuss the physical implications and limitations of the model, and summarize our conclusions.

2 $f(Q)$ gravity and Barrow holographic dark energy

In this section we review the basic structure of $f(Q)$ gravity and summarize the definition of Barrow holographic dark energy. We restrict the discussion to the elements needed for the analytic reconstruction presented in the next section.

2.1 Basics of $f(Q)$ gravity

In symmetric teleparallel gravity, spacetime is characterized by vanishing curvature and torsion, while nonmetricity is nonzero. The nonmetricity tensor is defined as

$$Q_{\lambda\mu\nu} \equiv \nabla_\lambda g_{\mu\nu}. \tag{2.1}$$

From this tensor one defines the traces

$$Q_\lambda \equiv Q_\lambda{}^\mu{}_\mu, \qquad \tilde{Q}_\lambda \equiv Q^\mu{}_{\lambda\mu}, \tag{2.2}$$

and the nonmetricity scalar Q is constructed as the quadratic combination [12]

$$Q = -\frac{1}{4}Q_{\lambda\mu\nu}Q^{\lambda\mu\nu} + \frac{1}{2}Q_{\lambda\mu\nu}Q^{\nu\mu\lambda} + \frac{1}{4}Q_\lambda Q^\lambda - \frac{1}{2}Q_\lambda\tilde{Q}^\lambda. \tag{2.3}$$

The action of $f(Q)$ gravity is given by

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}f(Q) + \mathcal{L}_m\right], \tag{2.4}$$

where $\mathcal{L}_m$ denotes the matter Lagrangian.

We consider a spatially flat Friedmann–Robertson–Walker (FRW) metric in the following form

$$ds^2 = -dt^2 + a^2(t)\left(dx^2 + dy^2 + dz^2\right), \tag{2.5}$$

where $a(t)$ is the scale factor and the Hubble parameter is defined as

$$H \equiv \frac{\dot{a}}{a}. \tag{2.6}$$

Due to the absence of curvature and torsion, one may choose the coincident gauge, in which the affine connection vanishes globally,

$$\Gamma^{\lambda}_{\mu\nu} = 0. \tag{2.7}$$

In this gauge, the nonmetricity tensor in Eq. (2.1) reduces to

$$Q_{\lambda\mu\nu} = \partial_\lambda g_{\mu\nu}. \tag{2.8}$$

Then, using Eqs. (2.3), (2.5), and (2.8), one finds that the nonmetricity scalar reduces to the well-known result

$$Q = 6H^2. \tag{2.9}$$

Varying the action (2.4) with respect to the metric and specializing to the FRW background yields a Friedmann-like equation

$$3H^2 = \rho_m + \rho_{\text{eff}}, \tag{2.10}$$

where the effective energy density associated with the $f(Q)$ sector is given by

$$\rho_{\text{eff}} = \frac{f}{2} - 6H^2 f_Q, \tag{2.11}$$

in which we have used $f_Q \equiv \frac{df}{dQ}$. Moreover, ρ_m denotes the standard pressureless (dust) matter component of the cosmic fluid, representing baryonic and cold dark matter. Throughout this work we assume minimal coupling between matter and the nonmetricity sector, so that ρ_m satisfies the usual conservation equation

$$\dot{\rho}_m + 3H\rho_m = 0, \tag{2.12}$$

which implies $\rho_m = \rho_{m0}a^{-3}$. In the reconstruction performed in the next section, the effective geometric contribution ρ_{eff} is required to reproduce the Barrow holographic dark energy density at the homogeneous background level, whereas the matter sector follows its standard evolution.

2.2 Barrow holographic dark energy

Barrow holographic dark energy is based on the modified entropy–area relation [4]

$$S \propto A^{1+\Delta/2}, \tag{2.13}$$

where Δ is the Barrow deformation parameter. The corresponding BHDE energy density takes the form

$$\rho_{\rm BHDE} = C\, L^{\Delta-2}, \tag{2.14}$$

with C a positive constant and L the IR cutoff.

Various choices for the infrared cutoff L have been considered in the literature, including the future event horizon, particle horizon, and generalized dynamical cutoffs. In this work we adopt the Hubble radius as the IR cutoff,

$$L = H^{-1}, \tag{2.15}$$

which leads to

$$\rho_{\rm BHDE} = C\, H^{2-\Delta}. \tag{2.16}$$

In the next section, we equate Eqs. (2.11) and (2.16) and reconstruct the functional form of $f(Q)$ analytically.

3 Analytic reconstruction of $f(Q)$ from BHDE

In this section we adopt a reconstruction approach at the homogeneous background level. Rather than solving for the cosmological dynamics from a given gravitational action, we impose the BHDE scaling on the effective geometric energy density and determine the functional form of $f(Q)$ that reproduces this behavior in a spatially flat FRW Universe. This procedure establishes a background-level correspondence between BHDE and a minimal $f(Q)$ theory.

As shown in Section 2, the effective energy density in $f(Q)$ gravity is given by Eq. (2.11), while the BHDE density for the Hubble cutoff is given by Eq. (2.16). Imposing the correspondence

$$\rho_{\rm eff} = \rho_{\rm BHDE}, \tag{3.1}$$

and using Eqs. (2.11) and (2.16), we obtain

$$\frac{f}{2} - 6H^2 f_Q = C\, H^{2-\Delta}. \tag{3.2}$$

Using the relation between the nonmetricity scalar and the Hubble parameter given in Eq. (2.9), Eq. (3.2) can be expressed entirely in terms of Q as

$$\frac{f(Q)}{2} - Q f_Q(Q) = C \left(\frac{Q}{6}\right)^{1-\Delta/2}. \tag{3.3}$$

Equation (3.3) is a first-order linear differential equation of Euler type. Rewriting it in the form,

$$\frac{d}{dQ}\Big(Q^{-1/2} f(Q)\Big) = -\,C\, 6^{\Delta/2-1}\, Q^{-(\Delta+1)/2}, \tag{3.4}$$

and integrating with respect to Q, the general solution reads

$$f(Q) = \alpha\, Q^{1/2} + \frac{2C}{(\Delta-1)\, 6^{\,1-\Delta/2}}\, Q^{\,1-\Delta/2}, \tag{3.5}$$

where α is an integration constant corresponding to the homogeneous solution of Eq. (3.3).

Equation (3.5) provides a closed-form analytic expression for the function $f(Q)$ that reproduces the Barrow holographic dark energy density when the Hubble radius is used as the infrared cutoff. In the limit $\Delta \to 0$, the exponent in the second term approaches unity, yielding a linear dependence on Q, as required for the symmetric teleparallel equivalent of general relativity (STEGR). However, The homogeneous contribution $f_h(Q) = \alpha Q^{1/2}$ does not reduce to a linear Q term in the limit $\Delta \to 0$. Since our aim is to construct a deformation of GR that smoothly recovers STEGR in the limit of vanishing Barrow parameter, we impose the boundary condition $\alpha = 0$, which eliminates the non-GR branch of the solution.

The reconstructed Lagrangian thus takes the closed analytic form

$$f(Q) = \frac{2C}{(\Delta - 1)\, 6^{\,1-\Delta/2}}\, Q^{\,1-\Delta/2}. \tag{3.6}$$

In the limit $\Delta \to 0$, Eq. (3.6) reduces to

$$f(Q) \longrightarrow 2C\, Q. \tag{3.7}$$

Such a constant factor can be absorbed into a redefinition of the effective gravitational coupling,

$$G_{\text{eff}} = \frac{G}{2C}, \tag{3.8}$$

which is a standard feature in modified-gravity reconstructions. Therefore, the model consistently recovers GR dynamics (up to a normalization of the coupling) in the $\Delta \to 0$ limit.

It is important to clarify the domain of validity of the reconstructed solution (3.6). The coefficient of the particular solution contains the factor $(\Delta - 1)^{-1}$, which becomes singular at $\Delta = 1$. This behavior is not accidental but follows from the structure of the Euler-type differential equation (3.3). At $\Delta = 1$, the exponent of the source term becomes $1/2$ which coincides with the exponent of the homogeneous solution $Q^{1/2}$, leading to degeneracy of the particular solution. In such cases the differential equation must be treated separately, and the solution generally acquires logarithmic corrections rather than a simple power-law form. Therefore, the closed analytic expression (3.6) is mathematically well-defined only for

$$0 \leq \Delta < 1. \tag{3.9}$$

In the following analysis we therefore restrict ourselves to $0 \leq \Delta < 1$, where the reconstructed $f(Q)$ model remains regular and smoothly connected to the symmetric teleparallel equivalent of general relativity in the limit $\Delta \to 0$.

From a physical perspective, Barrow's original entropy proposal allows $0 \leq \Delta \leq 1$, where $\Delta = 1$ corresponds to maximal fractal deformation of the horizon entropy [4]. However, recent observational analyses of Barrow holographic dark energy consistently find best-fit values of Δ close to zero and significantly below unity [28,29]. Thus, the restriction $0 \leq \Delta < 1$ emerging in the present reconstruction is fully consistent with current phenomenological studies.

3.1 Stability and effective equation of state

Having reconstructed the function $f(Q)$ in Eq. (3.6), we now examine the background cosmological behavior of the model in the presence of pressureless matter. The expansion dynamics is governed by the Friedmann equation (2.10), where the total energy budget consists of the standard dust component ρ_m and the effective geometric contribution ρ_{eff} defined in Eq. (2.11). These two components evolve according to

$$\rho_m = \rho_{m0} a^{-3}, \tag{3.10}$$

while, due to the reconstruction condition imposed in Eq. (3.1), the geometric contribution reproduces the Barrow holographic dark energy scaling

$$\rho_{\text{eff}} = C\, H^{2-\Delta}. \tag{3.11}$$

The cosmological evolution then follows from the interplay between the standard matter dilution $\rho_m \propto a^{-3}$ and the geometric contribution above. At early times, when $a \ll 1$, the matter term dominates the Friedmann equation (2.10) and the expansion reduces to the usual matter-dominated behavior $H^2 \propto a^{-3}$. Therefore, the reconstructed model naturally admits the conventional decelerating phase required for structure formation.

At late times, as the Universe expands and ρ_m becomes negligible compared to ρ_{eff}, the Friedmann equation is effectively controlled by the geometric sector. In the limit $\rho_m \to 0$, Eq. (2.10) reduces to

$$3H^2 = C\, H^{2-\Delta}. \tag{3.12}$$

For $0 < \Delta < 1$, this relation admits a constant solution,

$$H^{\Delta} = \frac{C}{3}, \tag{3.13}$$

which corresponds to an asymptotic de Sitter phase with $H = H_{\text{dS}} = \text{constant}$. Thus, at the homogeneous background level, the reconstructed model admits a late-time accelerating solution.

It is instructive to comment on the special case $\Delta = 0$, which corresponds to the standard holographic dark energy scaling with the Hubble cutoff. In this limit the reconstructed Lagrangian reduces to a linear function of Q, and the model becomes dynamically equivalent (up to a rescaling of the gravitational coupling) to general relativity with $\rho_{\text{eff}} \propto H^2$. The late-time Friedmann equation then reduces to $3H^2 = CH^2$, which does not generically admit a self-accelerating solution unless the parameters are fine-tuned. Therefore, as in the standard HDE scenario with Hubble cutoff, the case $\Delta = 0$ does not naturally produce late-time acceleration.

In contrast, for any nonvanishing Barrow deformation $0 < \Delta < 1$, the modified scaling $\rho_{\text{eff}} \propto H^{2-\Delta}$ alters the algebraic structure of the Friedmann equation and yields a constant Hubble solution at late times without the need for an interaction term or additional dark-sector ingredients. In this sense, the Barrow deformation parameter controls the departure from the standard HDE behavior and is responsible for the emergence of a self-accelerating de Sitter attractor in the reconstructed $f(Q)$ framework.

By interpreting the geometric modifications as an effective dark energy component, the nonmetricity sector can be treated as a cosmic fluid with energy density ρ_{eff} and pressure p_{eff}. At the background level, the evolution of this effective fluid is governed by the standard continuity equation,

$$\dot{\rho}_{\text{eff}} + 3H(1 + w_{\text{eff}})\rho_{\text{eff}} = 0, \tag{3.14}$$

where $w_{\text{eff}} \equiv p_{\text{eff}}/\rho_{\text{eff}}$ represents the effective EoS parameter characterizing the nonmetricity-driven dark energy.

Using $\rho_{\text{eff}} \propto H^{2-\Delta}$, one obtains

$$w_{\text{eff}} = -1 - \frac{1}{3}\frac{d\ln\rho_{\text{eff}}}{d\ln a} = -1 - \frac{2-\Delta}{3}\frac{d\ln H}{d\ln a}. \tag{3.15}$$

In the asymptotic de Sitter regime derived above, H approaches a constant and therefore $d\ln H/d\ln a \to 0$, yielding $w_{\text{eff}} \to -1$. Thus, the effective equation of state converges dynamically to the cosmological-constant value at late times, while deviations at intermediate epochs are controlled by the Barrow parameter Δ. In the limit $\Delta \to 0$, the standard ΛCDM background behavior is consistently recovered.

Beyond the EoS behavior, a fundamental requirement for the viability of modified gravity theories is the insurance of a well-defined and positive-valued gravitational coupling. In the context of $f(Q)$ gravity, this stability criterion necessitates the positivity of the first derivative of the Lagrangian with respect to the nonmetricity scalar, namely:

$$f_Q > 0. \tag{3.16}$$

For the reconstructed power-law form in Eq. (3.6), the derivative scales as $f_Q \propto Q^{-\Delta/2}$. Since the nonmetricity scalar $Q = 6H^2$ is strictly positive in an expanding Universe, and the reconstructed solution is well-defined for $0 \le \Delta < 1$, it follows that the condition $f_Q > 0$ is satisfied throughout the physically relevant parameter space. This ensures a positive effective gravitational coupling at the background level.

It is important to emphasize that the present analysis is restricted to the homogeneous cosmological background and to the asymptotic behavior of the expansion history. Although we have shown that the reconstructed model admits a standard matter-dominated era and evolves toward a late-time de Sitter attractor for $0 < \Delta < 1$, this does not exhaust the full dynamical content of the theory. A comprehensive dynamical-systems investigation of the coupled matter–geometry equations would be required to rigorously establish the global stability of the fixed points and to characterize the complete phase-space structure.

Concerning stability, the condition $f_Q > 0$ ensures a positive effective gravitational coupling at the background level, but it is not sufficient to guarantee the full consistency of the theory. Recent perturbative analyses of $f(Q)$ gravity formulated in the coincident gauge have shown that non-linear extensions beyond the STEGR limit (i.e., $f_{QQ} \neq 0$) can propagate additional scalar degrees of freedom relative to general relativity and, in certain branches, may exhibit strong-coupling or ghost-like instabilities [30].

For the reconstructed power-law model $f(Q) \propto Q^{1-\Delta/2}$ with $0 \le \Delta < 1$, one has $f_{QQ} \propto Q^{-\Delta/2-1}$, which vanishes only in the GR limit $\Delta = 0$. The model therefore belongs to the class of genuinely non-linear $f(Q)$ theories whose perturbative sector requires dedicated analysis. Although the reconstructed branch is continuously connected to STEGR as $\Delta \to 0$, establishing the absence of ghost modes, gradient instabilities, strong-coupling behavior, and pathological sound speeds demands a full study of the scalar and tensor perturbation equations. Such an investigation lies beyond the scope of the present analytic reconstruction and is left for future work.

4 Discussion and concluding remarks

In this work, we have constructed an explicit analytic embedding of Barrow holographic dark energy within symmetric teleparallel $f(Q)$ gravity at the homogeneous background level. By imposing the Barrow holographic scaling on the effective geometric energy density, we derived a simple power-law form of the gravitational Lagrangian that reproduces the prescribed behavior in a spatially flat FRW universe. The Barrow deformation parameter Δ directly fixes the exponent of the nonmetricity scalar in the action, establishing a transparent link between the entropic deformation and the geometric sector. The resulting model is fully analytic and involves no auxiliary functions beyond those already present in the BHDE framework. In this sense, the construction provides a minimal geometric realization of the Barrow scaling within symmetric teleparallel gravity.

It should be emphasized that this procedure is kinematic rather than dynamical: the BHDE behavior is not derived from the underlying field equations but imposed at the background level as a reconstruction condition. The physical insight gained from this approach lies in identifying the simplest $f(Q)$ structure compatible with a given holographic scaling. This clarifies how a deformation of horizon entropy can be consistently encoded in the nonmetricity sector, thereby offering a clean geometric interpretation of the Barrow parameter within modified gravity.

The reconstructed action exhibits a well-behaved and physically consistent limit as $\Delta \to 0$, where the Lagrangian reduces to a linear dependence on Q. This corresponds to the symmetric teleparallel equivalent of general relativity (STEGR), up to an overall normalization of the linear term that can be absorbed into a redefinition of the effective gravitational coupling. Thus, the standard holographic dark energy scenario is smoothly recovered in the absence of Barrow corrections, providing a nontrivial consistency check of the reconstruction scheme.

It is important to emphasize, however, that the analytic expression in Eq. (3.6) is mathematically well-defined only for $0 \leq \Delta < 1$. At $\Delta = 1$ the coefficient of the reconstructed solution diverges due to the degeneracy between the homogeneous and particular solutions of the Euler-type differential equation. This signals that the maximal Barrow deformation requires a separate treatment and cannot be represented within the present simple power-law correspondence. From a phenomenological perspective, existing observational analyses of Barrow holographic dark energy typically favor small deviations from the standard case ($\Delta \ll 1$), so the physically relevant parameter space lies safely within the regular domain $0 \leq \Delta < 1$.

It is also useful to place the present construction within the broader framework of entropic cosmology. The Barrow entropy modification shares a similar power-law dependence on the horizon area with the Tsallis non-extensive entropy at the microcanonical level, although the underlying physical motivations and admissible parameter ranges differ. While Barrow entropy originates from possible quantum-gravitational fractal deformations of the horizon surface [4], Tsallis entropy arises from generalized non-extensive statistical mechanics [7]. Consequently, the associated dark-energy densities may exhibit analogous functional structures, but the two approaches should not be regarded as strictly equivalent. For a discussion of their relationship in cosmological settings, see [10].

Barrow holographic dark energy can also be viewed as a specific realization within the broader class of generalized holographic dark energy models introduced by Nojiri and Odintsov [11], in which the dark-energy density is constructed as a function of the Hubble parameter and its derivatives. From this viewpoint, the present construction does not introduce a new holographic ansatz, but rather provides a concrete geometric embedding of a Barrow-type holographic scaling into symmetric teleparallel gravity. This clarifies how entropic deformations can be consistently translated into modifications of the nonmetricity sector within a simple analytic framework.

At the dynamical level, the restriction $0 \leq \Delta < 1$ has direct cosmological implications. It is well known that in standard holographic dark energy models with the Hubble radius as the infrared cutoff, the dark energy density scales as H^2 and cannot by itself generate late-time acceleration without introducing an interaction in the dark sector. In the present reconstruction, the geometric contribution scales as $H^{2-\Delta}$, and for $0 < \Delta < 1$ the modified Friedmann equation admits, asymptotically, a de Sitter solution once the matter component becomes negligible. In contrast, in the strict limit $\Delta = 0$, the geometric sector reduces to a pure H^2 scaling and does not yield a self-accelerating solution with the Hubble cutoff. Thus, within the reconstructed background framework, the Barrow deformation plays a key role in enabling the asymptotic de Sitter behavior that would be absent in the $\Delta = 0$ limit.

The primary advantage of the present approach lies in its mathematical tractability. By employing the Hubble radius as the infrared cutoff, the reconstruction can be carried out analytically, allowing the underlying physical mechanism to be clearly identified. The stability analysis at the background level indicates that the effective equation of state remains in the vicinity of the phantom divide ($w_{\text{eff}} \approx -1$) at late times and that the reconstructed model satisfies fundamental viability conditions, such as the positivity of the effective gravitational coupling. These background-level results indicate that the model satisfies minimal consistency conditions and provides a plausible effective description of dark energy at the homogeneous level, pending a full perturbative analysis. We emphasize that this stability assessment is limited to the homogeneous background; recent analyses of $f(Q)$ perturbations have revealed potential ghost and strong-coupling issues in certain formulations, and a dedicated perturbative study of the present model is required to fully establish its consistency.

While the present work focuses on the Hubble cutoff to enable an analytic treatment, it lays the groundwork for several extensions. Generalizing the reconstruction to more dynamical infrared cutoffs, such as the future event horizon, may reveal richer holographic dynamics within the nonmetricity framework. Furthermore, the explicit power-law form of the action makes this model particularly suitable for perturbative analyses, which are essential to characterize the evolution of cosmological perturbations and the growth of large-scale structure in holographic $f(Q)$ scenarios.

From a phenomenological perspective, the explicit dependence of the gravitational action on the Barrow parameter provides a natural setting for confronting holographic modifications with observational data. Future studies could examine whether the reconstructed model leaves detectable imprints on late-time cosmological observables or helps address existing tensions in standard cosmology. In this regard, the analytic solution provided here serves as a robust benchmark for more elaborate numerical and observational analyses.

Beyond the formal reconstruction presented here, it is important to situate the model within the existing observational and phenomenological literature on Barrow-inspired cosmology. For instance, observational constraints on Barrow-modified cosmology and holographic scenarios have been extracted using type Ia supernovae, baryon acoustic oscillations, and $H(z)$ measurements [31]. Inflationary and late-time phenomenology of Barrow entropy–based cosmology has also been investigated and confronted with Planck data [9], while generalized holographic realizations incorporating Barrow entropy have been analyzed within broader dark-energy frameworks [11]. Comparative analyses between Barrow and Tsallis holographic constructions further clarify the viable parameter ranges and thermodynamic consistency conditions [8,10]. In addition, recent studies of Barrow holographic dark energy in different modified-gravity settings have explored viable background evolution and thermodynamic consistency [32,33].

The analytic $f(Q)$ reconstruction presented here provides a minimal geometric template for incorporating these entropic scenarios within symmetric teleparallel gravity. Its explicit power-law structure makes it particularly suitable for future numerical integration of the full background equations and direct statistical comparison with late-time data sets. A dedicated likelihood analysis confronting the reconstructed model with SNIa, BAO, and cosmic chronometer data will be pursued in subsequent work.

It is useful to clarify the relation between the present construction and previous BHDE reconstructions in $f(Q)$ gravity, such as Ref. [25]. In those works, the reconstruction procedure leads to more intricate functional forms of $f(Q)$, including special functions and additional parameter dependencies. In contrast, the present analysis yields a minimal power-law Lagrangian in closed analytic form, where the Barrow deformation parameter uniquely fixes the exponent of the nonmetricity scalar. The novelty of the present result is therefore not conceptual but structural: it provides the simplest analytic embedding of BHDE into symmetric teleparallel gravity, with no auxiliary functions and a direct geometric interpretation of Δ.

In conclusion, the results presented here demonstrate that Barrow holographic dark energy admits a consistent embedding in symmetric teleparallel gravity through a simple power-law modification of the nonmetricity scalar. This correspondence highlights the deep interplay between holography and modified gravity, providing a clear framework for exploring the role of nonmetricity in the dark energy sector. This study provides a baseline for further investigations into holographic cosmology within $f(Q)$ gravity and its various geometric extensions.

Data Availability

The manuscript has no associated data or the data will not be deposited.

Conflicts of Interest

The author declares that there is no conflict of interest.

Ethical Considerations

The author has diligently addressed ethical concerns, such as informed consent, plagiarism, data fabrication, misconduct, falsification, double publication, redundancy, submission, and other related matters.

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