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Regular article

Regular Black Holes in Einstein-Gauss-Bonnet Gravity Coupled with a Cloud of Strings

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Abstract. In this paper, we construct regular black holes coupled with the Cloud of String, which becomes Maxwell's theory in the weak field limit, and we can compare new attributes against the standard Letelier black hole and Schwarzschild black hole. The thermodynamic quantities associated with the black hole are modified in the presence of CoS. We also study the global properties of the solutions and derive the corrected first law of thermodynamics. In addition, we also study the local and global stability of the black hole solution.

Keywords: Regular Black Holes; Cloud of String; Nonlinear Electrodynamics.

Contents

1 Introduction

The general theory of relativity encounters a substantial barrier due to the presence of singularities, especially gravitational singularities that arise in the context of black holes. Thus, identifying methods to circumvent singularities in general relativity has emerged as a critical challenge in contemporary theoretical physics. A promising method entails conventional black holes. Based on Gliner and Shakarov's model [1,2], Bardeen came up with the first regular black hole in 1968 [3]. Later, Ayon Beato and Garcia (ABG) found the black hole solution when general relativity was combined with nonlinear electrodynamics (NLED) [4–7]. NLED adds strong electromagnetic fields to Maxwell's theory. This is very helpful when studying charged black holes, since both gravitational and electromagnetic fields are very important [8].

In the past few years, many new regular black holes have been discovered, which has led some researchers to study them. Based on Bardeen's idea, other researchers have built a class of regular black holes [9–19]. Combining Einstein's gravity with NLED, different black hole metrics are created for EGB gravity [20–24], massive gravity [25], dark matter [26,27], and rotating black holes [28–30]. What happens to a black hole's thermodynamics when there isn't a singularity? The entropy doesn't follow the area law [31,32]. Many years of work have gone into studying how the thermodynamic parameters of this regular black hole change in reference [33–38] and black hole [39–46].

This study looks at the thermodynamics of a regular black hole when a Cloud of String (CoS) is present [47–50]. This answer finds the middle ground between the ABG BH, the Letelier black hole, the Schwarzschild BH when there is no magnetic monopole charge, and those three things together. We look into the horizon structure of the obtained black hole solution, and the answer that we got. We also talk about this black hole solution's thermal qualities and discover that it follows the changed first law of black hole thermodynamics. Also, we will look at thermodynamic factors like temperature, entropy, heat capacity, and Gibbs free energy as a function of horizon radius to sort the black hole's phase transition into different classes. Bekenstein [51,52] and Hawking [53] were the first people to study the thermodynamics of black holes by finding a link between entropy and the area of the horizon black hole's. There are changes to the black hole's thermal parameters when CoS and NLED are present. Hawking and Page made a big step forward in the field of thermodynamics by being the first people to study the thermodynamics of black holes in anti-de Sitter (AdS) space-time. Since then, many important studies have added to what we know about the physics of black holes.

This is how the paper is put together. In Section 2, we take a quick look at how a black hole is put together with NLED, and a CoS. We also talk about the horizon structure of the black hole solution. Then, in Section 3, we talk about thermodynamics and the phase transition of black holes, with Section 5 findings and discussion.

2 Black Hole Solution coupled with Nonlinear Electrodynamics and Cloud of String

This work commences with the formulation of Einstein gravity in conjunction with NLED and CoS [54], expressed as follows:

$$S = \frac{1}{2}\int_{\mathcal{M}} d^D x\sqrt{-g}\left[R + \mathcal{L}(F) + \mathcal{L}_{CoS}\right]. \tag{2.1}$$

The NLED is described by $\mathcal{L}(F)$ in the invariant $F = F_{\mu\nu}F^{\mu\nu}/4$, where $F_{\mu\nu}$ is the electromagnetic fied tensor associated with gauge potential A_ν via $F_{\mu\nu} = 2\nabla_{[\mu}A_{\nu]}$. The variation of the action with respect to the metric $g_{\mu\nu}$ gives the following equations of motion [55]

$$R_{\mu\nu} + \frac{1}{2}g_{\mu\nu}R = T^{NLED}_{\mu\nu} + T^{CoS}_{\mu\nu} \equiv 2\left[\frac{\partial\mathcal{L}(F)}{\partial F}F_{\mu\rho}F^{\rho}_{\nu} - g_{\mu\nu}\mathcal{L}(F)\right], \tag{2.2}$$

$$\nabla_\mu\left(\frac{\partial\mathcal{L}(\mathcal{F})}{\partial F}F_{\mu\nu}\right) = 0 \quad \text{and} \quad \nabla_\mu\left({}^*F_{\mu\nu}\right) = 0. \tag{2.3}$$

The Lagrangian density of the nonlinear source field is expressed as

$$\mathcal{L}(F) = Fe^{-\frac{s}{e}(2e^2F)^{\frac{1}{4}}}, \tag{2.4}$$

We use the following *anstaz* for the Maxwell field

$$F_{\mu\nu} \quad = \quad 2\delta^{\theta_1}_{[\mu}\delta^{\theta_2}_{\nu]}e(r)\sin\theta; \qquad D = 4, \tag{2.5}$$

Equation (2.5) says that $dF = 0$, so we get

$$e'(r)2\delta^{\theta_1}_{[\mu}\delta^{\theta_2}_{\nu]}e\sin\theta. \tag{2.6}$$

This means that $e(r) = e =$ is always true. It's interesting that $F_{\mu\nu}$'s other parts don't have much of an effect on $F_{\theta\phi}$. This is how you can write the energy-momentum tensor (EMT):

$$T^t_t \quad = \quad T^r_r = \frac{2Mke^{-\frac{k}{r^1}}}{r^4}. \tag{2.7}$$

Using the EMT from Eq. (2.7), we can obtain the 4-dimensional black hole. We obtain the EMT for a CoS as

$$T^{\mu\nu} = \frac{m\Sigma^{\mu\rho}\Sigma_\rho^{\ \nu}}{\sqrt{-\gamma}} = \frac{\rho\Sigma^{\mu\rho}\Sigma_\rho^{\ \nu}}{\sqrt{-\gamma}}. \tag{2.8}$$

The only surviving component of the bivector Σ is $\Sigma^{tr} = -\Sigma^{rt}$. Thus, $T^t_t = T^r_r = -\rho\Sigma^{tr}$, we obtain

$$T^t_t = T^r_r = \frac{a}{r^{D-2}}, \tag{2.9}$$

where a is a constant, which is associated with the global monopole [56].

3 Regular Black Hole Solutions with Cloud of strings

We want to derive static spherically symmetric solutions of Eq. (2.2) utilizing a CoS and NLED as sources, and examine their properties. We presume that the measure is structured as follows

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2d\Omega^2, \tag{3.1}$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta\, d\phi^2$ represents the metric of a two-dimensional sphere. Employing the metric ansatz (3.1) with $f(r) = 1 - \frac{2m(r)}{r}$, the Einstein field equations assume the following form

$$rf'(r) + f(r) - 1 = \frac{2Mk}{r^2}e^{-k/r} + a, \tag{3.2}$$

of substitutes the value of $f(r) = 1 - 2m(r)/r$ in (3.2), we get

$$m'(r) = \frac{e^2}{2r^2} e^{-k/r} + \frac{a}{2}, \tag{3.3}$$

where a prime signifies a derivative with regard to r. Upon integrating Eq. (3.4), one obtains

$$m(r) = me^{-e^2/2Mr} + ar + C_1 \tag{3.4}$$

where C_1 is a constant, then ($C_1 = \lim_{r\to\infty} m(r) - ar = M$) and substituting $m(r)$ into $f(r)$, the black hole solution (3.1) is transformed into

$$ds^2 = -\left[1 - \frac{2M}{r} e^{-k/r} - a\right] dt^2 + \frac{1}{\left[1 - \frac{2M}{r} e^{-k/r} - a\right]} dr^2 + r^2 d\Omega^2. \tag{3.5}$$

This represents a precise black hole solution incorporating a nonlinear source $e^{-k/r}$ and a CoS parameter (a). This black hole is defined by its mass (M), deviation parameter (k), and global monopole (a). By taking the limit as (k) approaches 0, it simplifies to the Letelier solution and the Schwarzschild black hole solution when ($a = 0, k = 0$). The solution exhibits characteristics akin to the Reissner-Nordström black hole when $r \gg k$. If $k < 0$, then $a = 0$, resulting in significant exponential growth for tiny r. For $k > 0$, the mass function $m(r)$ exhibits the following characteristics.

$$\lim_{r\to 0^+} e^{-k/r} + a = a, \qquad \lim_{r\to 0^-} e^{-k/r} + a = +\infty \tag{3.6}$$

The function is discontinous at $r = 0$ and $r >> k$ behaves as a Reissner Nordstrom black hole with CoS parameter a

$$ds^2 = 1 - \frac{2M}{r} + \frac{e^2}{r^2} - a + \mathcal{O}\left(\frac{k^2}{r^2}\right), \tag{3.7}$$

The charge e and mass M are connected by the equation $e^2 = 2Mk$. Determining the numerical range of mass (M), CoS parameter (a), and deviation parameter (k)for the black hole solution (3.5) is not challenging. The Cauchy and event horizon of the black hole (15) is

$$r_\pm = -\frac{k}{\mathrm{W}\left(-\frac{(1-a)k}{M}\right)} \qquad \Longrightarrow \qquad r_\pm = \frac{M}{(1-a)} e^{W\left(-\frac{(1-a)k}{M}\right)} \tag{3.8}$$

This black hole possesses two horizons, denoted as r_+ and r_-, representing the event horizon and inner horizon, respectively, in relation to the Lambert W function. The Lambert W function possesses two branches, W_0 and W_{-1}, which offer two possibilities.

$$\begin{aligned} W_0\left(-\frac{(1-a)k}{M}\right) < 0 \qquad &\Longrightarrow \qquad k \in \left(0, \frac{M}{(1-a)e}\right], \\ W_{-1}\left(-\frac{(1-a)k}{M}\right) < 0, \qquad &\Longrightarrow \qquad k \in \left[-\frac{M}{(1-a)e, 0}\right). \end{aligned} \tag{3.9}$$

The value of k resides inside this interval, since a well-defined coordinate point for a horizon is established when using the $(-(1-a)k/M)$ branch of the Lambert function. The range of the W_{-1} branch is totally negative and produces outputs solely within the interval $\left[-\frac{M}{(1-a)e}, 0\right)$.

Thus, all potential solutions will correspond to $r_+ > 0$. The inner and event horizons are situated at

$$r_- = \frac{M}{(1-a)} e^{W_{-1}\left(-\frac{(1-a)k}{M}\right)}, \qquad \text{and} \qquad r_+ = \frac{M}{(1-a)} e^{W_0\left(-\frac{(1-a)k}{M}\right)} \tag{3.10}$$

When $k = \frac{M}{(1-a)e}$, the inner and outer horizons coincide, resulting in an extremal black hole; when $k > \frac{M}{(1-a)e}$, the Lambert function becomes undefined.

We will now examine the energy conditions for the black hole (3.5). Examine the Einstein field equations for this spacetime, where the energy-momentum tensor is defined by $G_{\mu\nu} = 8\pi T_{\mu\nu}$. Refer to equations (2.7) and (2.9).

$$\rho = -P_1 = \frac{2Mk}{r^4} e^{-k/r} + \frac{a}{r^2}, \qquad \text{and} \qquad P_1 = P_2 = \frac{Mk}{r^5}(2r-k)e^{-k/r}. \tag{3.11}$$

Let us investigate the locus of maximum energy density within this spacetime

$$\frac{\partial \rho}{\partial r} = \frac{2Mk(k-4r)e^{-k/r}}{r^6} - \frac{2a}{r^3}, \tag{3.12}$$

We shall now examine the different energy conditions, namely the null energy condition ($\rho + P_1 \geq 0$ and $\rho + P_2 \geq 0$), the weak energy condition (WEC) ($\rho \geq 0$ and $\rho + P_i \geq 0$), and the strong energy condition (SEC) ($\rho + P_1 + P_2 + P_3 \geq 0$)

$$\begin{aligned}
&\rho = \frac{2Mk}{r^4} e^{-k/r} + \frac{a}{r^2} \\
&\rho + P_1 = 0 \\
&\rho + P_2 = \frac{Mk(k-4r)e^{-k/r}}{r^5} - \frac{a}{r^2} \\
&\rho + P_1 + 2P_2 = \frac{Mk}{r^5}(2r-k)e^{-k/r}.
\end{aligned} \tag{3.13}$$

The radial Null Energy Condition is fulfilled, however the transverse Null Energy Condition is breached. The Strong Energy Condition is violated when $r < a/2$, as indicated by the sign reversal.

4 Thermodynamics

We will now examine the thermodynamic quantities related to the black hole solution (3.5), which is defined by mass (M_+), deviation parameter (k), and CoS (a). The black hole mass can be calculated as $f(r) = 0$ in relation to the horizon radius (r_+).

$$M_+ = \frac{(1-a)}{2} r_+ e^{k/r_+}. \tag{4.1}$$

This is identical to the mass of a regular black hole; for $a = 0$, it simplifies to a Schwarzschild black hole when $k = 0$ and $a = 0$. The Hawking temperature of a black hole is described as proportional to the surface gravity κ by the equation $T = \kappa/2\pi$, where κ is specified by

$$\kappa = \frac{1}{2\pi}\left(-\frac{1}{2}\nabla_\mu \xi_\nu \nabla^\mu \xi^\nu\right)^{1/2} = \frac{1}{4\pi} f'(r_+), \tag{4.2}$$

Furthermore, $\xi^\mu = \partial/\partial t$ constitutes a Killing vector. According to the solution (3.5), the black holes possess a temperature

$$T_+ = \frac{(1-a)}{4\pi r_+}\left[1 - \frac{k}{r_+}\right]. \tag{4.3}$$

The temperature decreases to $T_+ = \frac{1}{4\pi r_+}$ for Schwarzschild black holes when $k = 0$ and

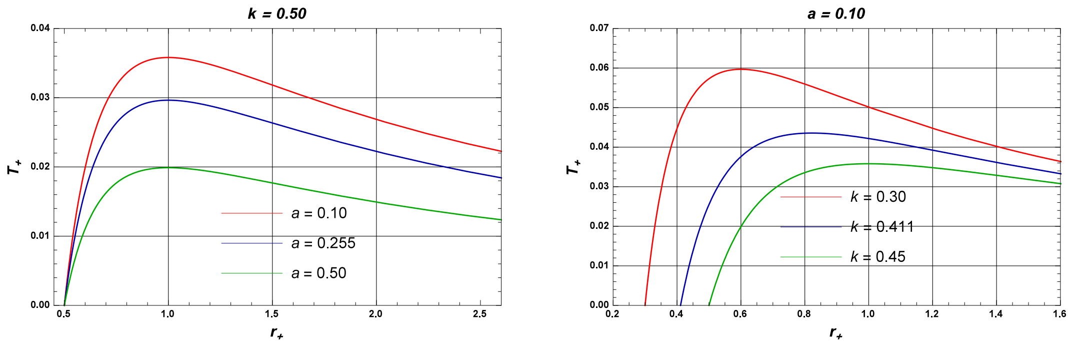

Figure 1: The plot temperature T_+ vs. horizon radius r_+ with various values of (k) and (a).

$a = 0$. Figure 1 illustrates the temperature behavior for various values of k and a. The temperature rises as the values of a and k diminish. Subsequently, we examine the entropy of black holes. The expression for entropy can be derived from the first rule of thermodynamics

$$dM_+ = T_+ \, dS_+, \tag{4.4}$$

The expression for black hole entropy is as follows, which results in

$$S_+ = \int \frac{1}{T_+}\frac{\partial M_+}{\partial r_+} dr_+ = 2\int r_+ e^{k/r_+}\, dr_+, \tag{4.5}$$

using the (4.1) and (4.3) into (4.5), we derive the entropy of the black hole as

$$S_+ = \frac{V_4}{2}\left[r_+(k + r_+)e^{k/r_+} - k^2 \mathrm{Ei}\left[\frac{k}{r_+}\right]\right]. \tag{4.6}$$

This entropy does not adhere to the area law. This is equivalent to the entropy of a Schwarzschild black hole when k is omitted from Eq. (4.6). The area law is evidently invalid for regular black holes, as demonstrated in (4.6). The temperature can now be derived from entropy via the first law of thermodynamics.

$$T_H = \frac{\partial M}{\partial S} = \frac{(1-a)}{4\pi r_+}\left[1 - \frac{k}{r_+}\right] e^{-k/r_+} \tag{4.7}$$

We have computed the temperature using Eq. (4.3) and Eq. (4.7) of the black holes by several methodologies [31]. The temperature derived from the Hawking temperature and tunneling approach yields identical results; nevertheless, according to the first law, the temperature differs. Ma *et al* [32] present the revised formulation of the first law for black holes. The divergence is contingent upon the overall configuration of the energy-momentum tensor of matter fields. When the black hole mass parameter M is incorporated into the energy-momentum tensor, the traditional formulation of the first law is altered by an additional factor. The revised first law is [31,32]

$$C(M, r_+)\, dM = T_+\, dS, \tag{4.8}$$

where T_+ is the Hawking temperature and $C(M, r_+)$ is

$$C(M, r_+) = 1 + 4\pi \int_{r_+}^{\infty} r^2 \frac{\partial T_0^0}{\partial M} dr = e^{-k/r}. \tag{4.9}$$

Substitute these value of $C(M, r_+)$, T_+ and M_T in Eq. (4.8) of $C(M, r_+)$,

$$S_+ = \pi r_+^2 = \frac{A}{4}. \tag{4.10}$$

Using the modified first law of thermodynamics, entropy follows the area law.

5 Local and Global Stability

We now investigate the thermodynamic stability of black holes by examining their heat capacity, denoted as

$$C_+ = \frac{\partial M_+}{\partial T_+} = -2V_4 r_+^2 e^{k/r_+} \frac{(1-k)}{2k - r_+}, \tag{5.1}$$

Where $V_4 = 2\pi$, it is noteworthy that the heat capacity (equation 5.1) is independent of a CoS parameter. The heat capacity is illustrated in Fig. 2 for various values of k and a. We identified two types of behavior, the first the black hole is stable, when $r_+ < r_c$ ($C_+ > 0$), whereas the second is negative heat capacity at $r_+ > r_c$, indicating instability of black holes. The discontinuity in heat capacity at $r_+ = r_c$ indicates the occurrence of a second-order phase transition

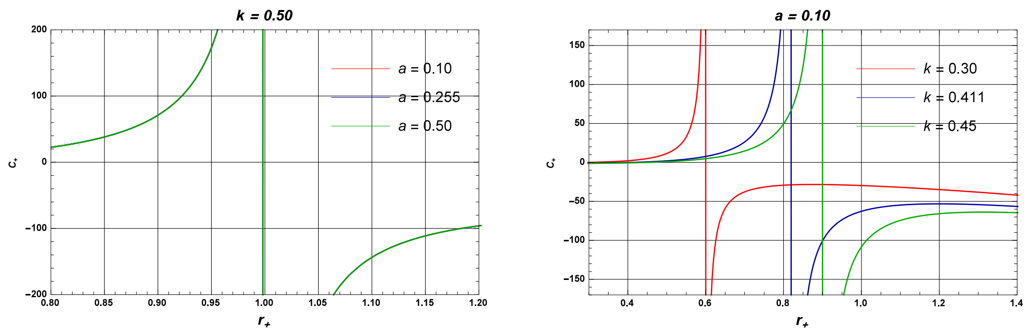

Figure 2: The plot of heat capacity vs. horizon radius r_+ with various values of deviation parameter k.

The global stability of black hole thermodynamics can be assessed by examining its free energy, which is described as

$$F_+ = M_+ - T_+ S_+ = \frac{(1-a)r_+}{2} \left[e^{k/r_+} - \frac{1}{2}\left(1 - \frac{k}{r_+}\right)\left((1 + \frac{k}{r_+})e^{k/r_+} - \frac{k^2}{r_+^2}\mathrm{Ei}\left[\frac{k}{r_+}\right]\right)\right] \tag{5.2}$$

The negative free energy indicates the stability of the black hole. Figure 3 clearly illustrates that the black hole exhibits stability within a limited radius.

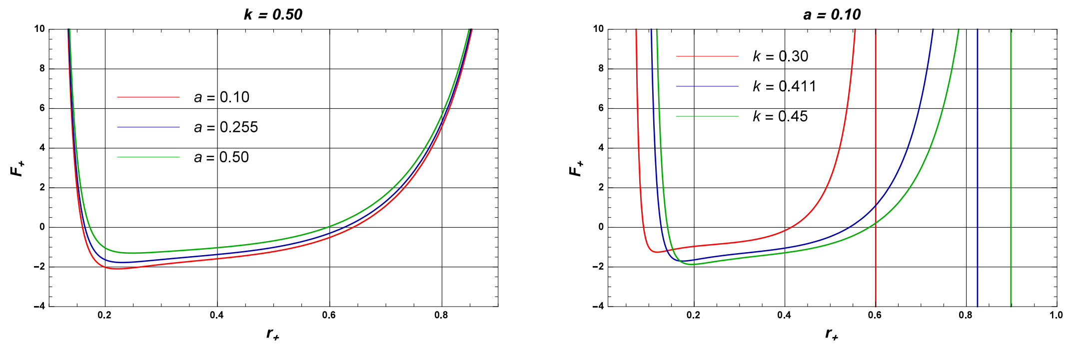

Figure 3: The plot of free energy F_+ vs horizon radius r_+ with different values of deviation parameter k and CS parameter a.

6 Conclusion

In this paper, we find the black hole solution in the presence of CoS and NLED field. This solution interpolates between a regular black hole in the absence of the CoS parameter, a Letelier black hole in the absence of magnetic monopole charge, and a Schwarzschild black hole in the absence of both of them, respectively. We explore the horizon structure of the obtained black hole solution. This black hole has two horizons, in contrast to the Schwarzschild black hole. In addition, we have also studied the energy condition of the obtained black hole solution. The thermodynamic quantities associated with the black hole solution are changed in the presence of the CoS and NLED field.

Authors' Contributions

All authors have the same contribution.

Data Availability

The manuscript has no associated data or the data will not be deposited.

Conflicts of Interest

The authors declare that there is no conflict of interest.

Ethical Considerations

The authors have diligently addressed ethical concerns, such as informed consent, plagiarism, data fabrication, misconduct, falsification, double publication, redundancy, submission, and other related matters.

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