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Regular article

Thermodynamics of Noncommutative Geometry Inspired Regular Black Holes Coupled with Nonlinear Electrodynamics

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Abstract. In this paper, we introduce an exact solution for a Hayward black hole (BH) by incorporating anisotropic perfect fluid influenced by nonlinear electrodynamics and non commutative geometry. The solution obtained resembles de Sitter spacetime at a small value of r ($r \to 0$) and at a large distance ($r \to \infty$) resembles the regular Schwarzschild geometry . In the absence of non commutative geometry the solution obtained interpolates with the Hayward BH and as non commutative geometry inspired BH in the absence of magnetic monopole charge. Non commutative geometry modifies thermodynamic properties of the BH. The calculation of Hawking temperature and its graphical analysis indicate that the temperature reaches its peak at the point of heat capacity divergence.

Keywords: Noncommutative Geometry; Black Hole; Nonlinear Electrodynamics.

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1 Introduction

Theoretically, the finding of radiating BH [1] was the first physical way to look into the secrets of quantum gravity. A lot of studies have been done in the area of BH for the last thirty years, but the various parts of the problem are still being discussed. In this case, there is still not a complete and acceptable explanation of BH late stage evaporation. In this very strong state, stringy effects can't be ignored according to the string/BH correspondence principle [2]. This is but one of numerous instances illustrating the diverse consequences of string theory's evolution. We emphasize that the chosen coordinates of spacetime transform into non commuting operators on a D-brane [3,4]. Consequently, string-brane coupling demonstrates the need for spacetime quantization. This hint provided further impetus to reevaluate earlier concepts of a similar nature, originally introduced in a largely overlooked study by Snyder [5].

Recent investigations have extensively examined the variations of quantum field theory. Initially, one might consider altering the $4D$ Einstein action to integrate non commutative effects. Research has demonstrated [6,7] that non commutativity eliminates point-like formations and replaces them with smeared distributions in flat spacetime. Theoretically, the smearing impact is represented as a universal replacement by a Gaussian distribution with a minimal width $\sqrt{\theta}$. Motivated by this outcome, we select them as the density of a spherically symmetric, static, particle-like, smeared gravitational source as referenced in [8,9].

$$\rho_\theta = \frac{M}{(4\pi\theta)^{3/2}} e^{r^2/4\theta}. \tag{1.1}$$

The particle mass M is not perfectly concentrated at the point; instead, it is spread out over an area of length $\sqrt{\theta}$. This arises from the inherent uncertainty encoded in commutator of coordinates [5]

$$[x^\mu, x^\nu] = i\theta^{\mu\nu}, \tag{1.2}$$

where $\theta^{\mu\nu}$ denotes an antisymmetric matrix that encodes the fundamental discretization of spacetime, analogous to the role of the Planck constant ($\hbar$) in the discretization of phase space.

In this paper, we have studied the Hayward BH in the presence of non commutative geometry (NCG). To find this solution, we have to evaluate the field equations and argue that it is not necessary to change the Einstein tensor part of the field equations, and that effects can be implemented by acting only on the matter source. The thermodynamics of BH have changed in the presence of a NCG and NLED. In addition, we also study the phase transition of the BH by studying the local and global stability of BH. This theory has led to extensive studies on various intriguing phenomena, including the van der Waals phase transition in BH thermodynamics, the weak cosmic supervision hypothesis, the repulsive interactions within BH micro structures, and consistency of the Smarr relation with the first law of thermodynamics, yielding numerous significant results [10–30], and conduct additional research on the thermodynamic phase transition of an area of charged BH [31–57].

The paper structure is as follows: we get a NCG inspired Bardeen BH solution in Sec. II, also get consistent Einstein field coupled to NLED, and study the horizon structure. The study of the thermodynamical properties of NCG inspired Bardeen BH solutions is the subject of Sec. III. Finally, the conclusion and results are presented in Sec. V. The signature of the metric is (-, +,+,+) with natural units 8G = c = 1.

2 NCG-inspired Hayward black hole

Based on these factors, we will try to find a spherically symmetric, static, Hayward BH solution in the presence of NCG. We use the non commutative method to figure out how to solve the NCG Hayward BH problem. To get used to the non commutative changes, a Gaussian distribution with a minimum width of $\sqrt{\theta}$ is applied instead of Dirac delta function. The BH mass is distributed throughout a region $\sqrt{\theta}$ rather than being concentrated at a point. The BH mass is determined by integrating (1.1) over a volume with radius r. Consequently, we acquire

$$M_\theta(r) = \int_0^r \rho_\theta 4\pi r^2 dr = \frac{2M(r)}{\sqrt{\pi}}\,\gamma\left(\frac{3}{2};\frac{r^2}{4\theta}\right), \tag{2.1}$$

where $\gamma(\frac{3}{2};\frac{r^2}{4\theta}) = \int_0^{r^2/4\theta} t^{1/2}e^{-t}dt$ is not the complete gamma function. This incomplete gamma function becomes the conventional gamma function in the limit $\theta \to 0$, and $M_\theta(r) \to m(r)$. The Hayward BH mass $M(r)$ is [22]

$$M(r) = \frac{2Mr^3}{(r^3+g^3)}, \tag{2.2}$$

substituting the mass $M(r)$ from Eq. (2.2) in Eq. (2.1), the mass of the NCG-inspired Hayward BH is

$$M_\theta(r) = \frac{2Mr^3}{(r^3+g^3)\sqrt{\pi}}\,\gamma\left(\frac{3}{2};\frac{r^2}{4\theta}\right). \tag{2.3}$$

Using the (2.3) in $f(r) = 1 - M_\theta(r)/r$, The NCG inspired Hayward BH becomes,

$$ds^2 = -\left(1 - \frac{2Mr^2}{(r^3+g^3)\sqrt{\pi}}\,\gamma\left(\frac{3}{2};\frac{r^2}{4\theta}\right)\right)dt^2 + \frac{1}{\left(1 - \frac{2Mr^2}{(r^3+g^3)\sqrt{\pi}}\,\gamma\left(\frac{3}{2};\frac{r^2}{4\theta}\right)\right)}dr^2 + r^2 d\Omega_2^2. \tag{2.4}$$

The metric (2.4) is the solution of the NCG inspired Hayward BH, which is characterized by the mass M, magnetic monopole charge, (g) and the minimal width $\sqrt{\theta}$. As $r/\sqrt{\theta} \to \infty$, the solution from (2.4) simplifies as a conventional Hayward BH [22] and in the absence of magnetic monopole charge as an NCG-inspired Schwarzschild BH [8]. The BH solution interpolates with Schwarzschild BH in the limits of $\sqrt{\theta}$ and g.

The NCG-inspired Hayward BH horizon can be found where $g^{rr}(r_+) = 0$. The equation $g^{rr} = 0$ can not be solved for r_+ analytically, however we evaluate it to obtain mass in terms of the radius of horizon r_+

$$M_+ = \frac{\sqrt{\pi}\theta(r_+^3+g^3)}{r_+^2\gamma\left(\frac{3}{2};\frac{r_+^2}{4\theta}\right)}, \tag{2.5}$$

The horizon of the NCG inspired Hayward BH for various entries of $g/\sqrt{\theta}$ illustrated in Fig. (2). The Fig. 2 demonstrates that the BH has 2 horizons for $g < 2.38\sqrt{\theta}$, no horizon for $g > 2.38\sqrt{\theta}$ an extremal BH when $g = g_c = 2.38\sqrt{\theta}$. Also, the horizon of the BH decreases as the value of the magnetic monopole charge (g) increases.

3 Thermodynamics

In this section, we study the thermodynamics of the NCG-inspired Hayward BH. The temperature of a BH is called the Hawking temperature. The temperature of the NGC-inspired

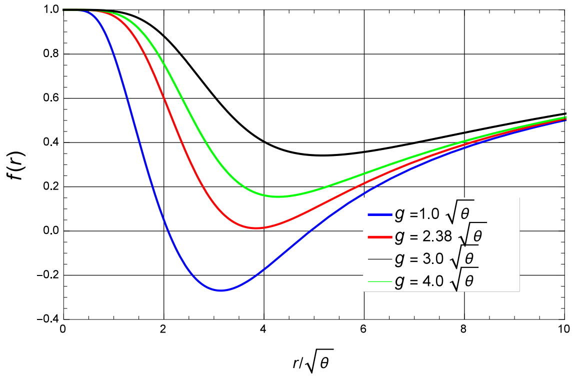

Figure 1: The metric $f(r)$ plot in terms of r for the changing values of magnetic monopole charge (g) with constant mass $M = 5\sqrt{\theta}$.

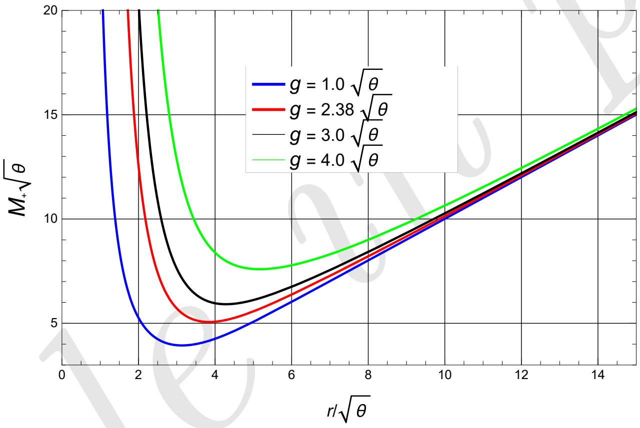

Figure 2: The plot of mass (M_+) vs. $r_+/\sqrt{\theta}$ for the various values of magnetic monopole charge $(g/\sqrt{\theta})$.

Hayward BH is calculated by this relation [58–61]

$$\kappa = \frac{1}{2\pi}\left(-\frac{1}{2}\nabla_\mu \xi_\nu \nabla^\mu \xi^\nu\right)^{1/2} = \frac{f'(r)}{4\pi}, \tag{3.1}$$

where κ is surface gravity. The temperature of the NCG-inspired Hayward BH is written as

$$T_+ = \frac{1}{4\pi r_+}\left(\frac{\sqrt{\pi}(r_+^3 - 2g^3)(4r_+\theta^{1/2}e^{r_+^2/4\theta} + r_+)}{\sqrt{\pi}(r_+^3 + g^3)\gamma\left(\frac{3}{2};\frac{r^2}{4\theta}\right)} - \frac{r_+(r_+^3 + g^3) + 2\theta(r_+^3 - 2g^3)}{2\theta^{3/2}(r_+^3 + g^3)\gamma\left(\frac{3}{2};\frac{r^2}{4\theta}\right)}\right). \tag{3.2}$$

The temperature as a function of the horizon radius of the NCG-inspired Hayward BH is illustrated in Figure 3. In the Fig. 3, we can see that the temperature of the NCG-inspired Hayward BH increases with a decrease in the magnetic monopole charge. It shows that the magnetic monopole charge (g) and temperature peak are inversely proportional to each other. For large r $(r/\sqrt{\theta} \to \infty)$, we recover the Hayward BH temperature

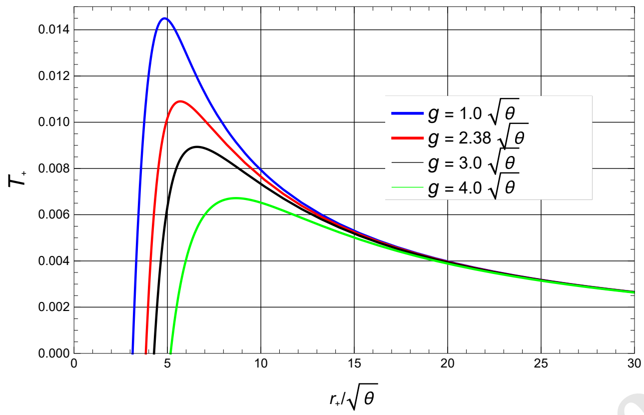

Figure 3: NCG inspired Hayward BH temperature $(T\sqrt{\theta})$ as a function of the horizon radius $(r_h/\sqrt{\theta})$ for varying values of $(g\sqrt{\theta})$.

$$T_+ = \frac{1}{4\pi} f'(r)|_{r=r_+} = \frac{1}{4\pi}\left(\frac{3r_+^2}{r_+^3+g^3} - \frac{2}{r_+}\right). \tag{3.3}$$

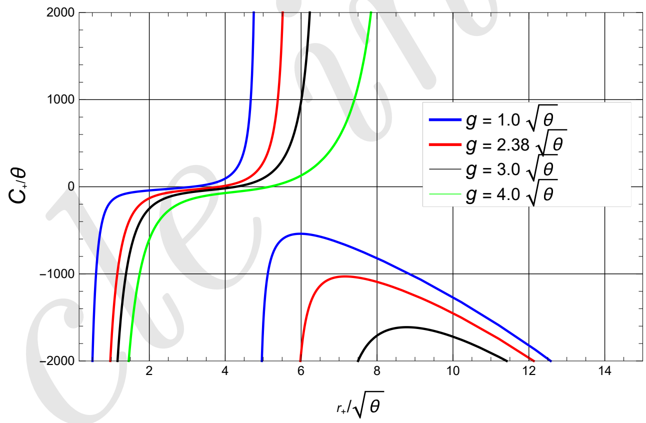

Figure 4: NCG inspired Hayward BH heat capacity $(C_+\sqrt{\theta})$ as a function of the horizon radius $(r_+/\sqrt{\theta})$ for the varying $(g\sqrt{\theta})$.

Now, we study the local stability of NCG-inspired Hayward BH by observing the behavior of heat capacity (C_+), when $(C_+ > 0)$, the BH is stable, and $(C_+ < 0)$, the BH is not stable. For the obtained BH solution the heat capacity is [62–64,66–69,69]

$$C_+ = \left(\frac{dM_+}{dT_+}\right) = \left(\frac{dM}{dr_+}\right)\left(\frac{dr_+}{dT}\right), \tag{3.4}$$

Substituting the values of temperature and Mass from Eq. (3.2) and Eq.(2.5) into Eq. (3.4), we obtain the heat capacity of NCG inspired Hayward BH. For large r $(r/\sqrt{\theta} \to \infty)$, we get heat capacity of Hayward BH

$$C_+ = \frac{2\pi(r_+^2 - 2g^3)(r_+^3+g^3)^2}{r_+^4(r_+^3 - 10g^3) - 2g^6 r_+}. \tag{3.5}$$

4 Conclusions

We examined the Hayward BH in context of a non commutative matter field. Notable aspect of this study is the formulation of the Hayward measure by incorporating non commutative effects. Thermodynamic quantities such as temperature and entropy are altered in the presence of non commutative materials. This outcome is examined comprehensively through graphical representations. Particular emphasis has been placed on the small scale behavior of BH temperature, where the results of both non commutative effects and back reactions are notably significant. In conclusion, while our research introduces the NCG-inspired Hayward BH, our framework is sufficiently robust to elucidate various varieties of non commutative BH.

Authors' Contributions

All authors have the same contribution.

Data Availability

The manuscript has no associated data or the data will not be deposited.

Conflicts of Interest

The author declares that there is no conflict of interest.

Ethical Considerations

The author has diligently addressed ethical concerns, such as informed consent, plagiarism, data fabrication, misconduct, falsification, double publication, redundancy, submission, and other related matters.

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