Letter

Imprints of Quantum Gravity on the Cooper-Frye Freeze-Out

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Abstract. This work shows that quantum-gravity-motivated generalized uncertainty principles (GUP) produce calculable and phenomenologically relevant modifications to the Cooper-Frye freeze-out prescription that maps hydrodynamic fields to hadronic momentum spectra in relativistic heavy-ion collisions. Using the linear Ali Das Vagenas GUP, which alters both the phase-space measure and the single-particle dispersion relation, the corresponding deformed particle current is constructed and its flux across a freeze-out hypersurface is evaluated. The resulting invariant spectrum acquires a momentum-dependent correction governed by a single dimensionless function that enhances high-momentum modes. For a static, homogeneous hypersurface the full expression can be written in closed analytic form, and the structure of the correction allows straightforward implementation in blast-wave-type models. The result is also directly relevant to holography-informed heavy-ion modeling, where gauge/gravity duality constrains the strongly coupled plasma dynamics but the conversion to hadron spectra is still performed through a Cooper-Frye freeze-out map. Our findings demonstrate that Planck-scale deformations of quantum mechanics can leave characteristic imprints on freeze-out observables, opening a novel avenue for constraining GUP scenarios with heavy-ion data.

Keywords: Quark–Gluon Plasma (QGP); Generalized Uncertainty Principle (GUP); Heavy-Ion Collisions; Quantum Gravity Phenomenology; Cooper–Frye Freeze-Out.

Understanding whether quantum gravity leaves observable signatures in relativistic many-body systems is a central open question in fundamental physics. Heavy-ion collisions provide a unique environment for such inquiries. They create the hottest, densest, and shortest-lived state of matter accessible in the laboratory, and generate gradients, flow fields, and freeze-out conditions that are sensitive to microscopic dynamics at sub-fermi scales. Although far below the Planck energy, the quark-gluon plasma (QGP) and its subsequent hadronic freeze-out involve momenta, temperatures, and space-time curvatures where quantum-gravity-motivated deformations of quantum mechanics can accumulate into measurable modifications of spectral shapes. This possibility has motivated a growing body of work exploring minimal-length effects in QGP thermodynamics and transport, including corrections to the equation of state, entropy, and speed of sound derived from generalized uncertainty principles (GUP) [1–4], and, in a different setting, characteristic linear-GUP signatures in mesoscopic Aharonov-Bohm rings [5].

Among these approaches, the linear GUP proposed by Ali, Das, and Vagenas stands out for its mathematical consistency and phenomenological tractability. It modifies both the single-particle dispersion relation and the phase-space measure in a controlled expansion, allowing systematic computation of deviations from standard relativistic kinetic theory [1,2]. Crucially, these deformations grow with momentum and temperature, directly overlapping with the regime where hadron spectra at freeze-out are most sensitive. Previous studies using this GUP framework have shown that it alters bulk thermodynamic quantities in ways that resemble lattice-QCD deviations from conformality [3,4]. Such results suggest that minimal-length effects can subtly reshape the microscopic distributions that feed into hydrodynamics and eventually determine particle emission. The Cooper-Frye freeze-out prescription is therefore a natural and essential place to search for quantum-gravity imprints. It is also useful to place this motivation alongside holography [6]. Gauge/gravity duality [7–10] and bottom-up holographic QCD models [11–14] have become standard tools to characterize strongly coupled plasma dynamics and transport [15], but the comparison to identified-hadron data still relies on a freeze-out mapping from macroscopic fields to particle spectra, most commonly the Cooper-Frye prescription [16–18]. The present analysis is complementary in that it targets this final interface: it provides a controlled, analytically tractable deformation of the Cooper-Frye map that can be applied on top of any hydrodynamic background, including those constrained or motivated by holographic calculations [11–13]. It is the final step connecting hydrodynamic fields to experimentally measured hadron spectra, and it weights the distribution function by the local flow velocity and geometry of the freeze-out hypersurface. Any quantum-gravity-driven deformation of phase space or on-shell energies feeds directly into this mapping. Because the Cooper-Frye formula is linear in the distribution function, even small GUP corrections translate into coherent, momentum-dependent distortions of the spectrum particularly at high transverse momentum, where experimental uncertainties are smallest and GUP-induced enhancements are strongest.

For these reasons, the GUP is ideally suited for probing quantum-gravity imprints in heavy-ion collisions. It offers a minimal, UV-agnostic, and analytically tractable extension of quantum mechanics whose corrections scale in precisely the region where freeze-out physics operates. By providing explicit, closed-form modifications to the Cooper-Frye spectrum, this framework opens a new phenomenological channel for testing Planck-scale physics using precision hadronic observables.

Relativistic heavy-ion collisions at RHIC and the LHC create hot and dense QCD matter which evolves hydrodynamically and ultimately decouples into a spectrum of hadrons. Recent hadron-resonance-gas analyses with finite baryon-size effects, thermodynamically consistent equations of state and excluded-volume corrections have further refined our un-

derstanding of hadron yields and freeze-out conditions in ultra-relativistic nuclear colli-
sions [19–21]. The standard bridge between hydrodynamics and hadron spectra is the
Cooper-Frye prescription [16,22,23] given a freeze-out hypersurface Σ and a local equilibrium
distribution function $f(x,p)$, the invariant single-particle spectrum is

$$E\frac{dN}{d^3p} = \frac{g}{(2\pi)^3}\int_\Sigma p^\mu d^3\sigma_\mu\, f(x,p), \tag{0.1}$$

where g is the degeneracy and $d^3\sigma_\mu$ is the directed surface element.

In parallel, the generalized uncertainty principle (GUP) has emerged as a well-motivated
phenomenological framework for quantum-gravity effects. In particular, the linear GUP
proposed by Ali, Das and Vagenas modifies the canonical commutator in such a way as
to introduce a minimal length scale, leading to corrections in the dispersion relation and
phase-space measure [1,2,24]. This GUP has been implemented in the thermodynamics of the
quark-gluon plasma (QGP) and has been shown to significantly affect the high-temperature
behavior of thermodynamic quantities [3,4].

Existing GUP-QGP studies focus on bulk equilibrium properties. To our knowledge, no
work has yet incorporated GUP effects into the freeze-out prescription that directly connects
hydrodynamics to observables. In this Letter we fill this gap by deriving a GUP-deformed
Cooper-Frye formula, starting from a modified phase-space measure and dispersion relation.
We keep terms linear in the GUP parameter and obtain explicit analytic correction factors.
For a simple static freeze-out hypersurface, we derive a closed form result which can be
readily implemented in blast-wave descriptions of hadron spectra [23].

Throughout, we use natural units $\hbar = c = k_B = 1$ and metric signature $(+,-,-,-)$.
We begin by recalling the derivation of Eq. (0.1) from relativistic kinetic theory [16,22].

The phase-space distribution function $f(x,p)$ is defined such that

$$dN = g\,f(x,p)\,\frac{d^3x\,d^3p}{(2\pi)^3}, \tag{0.2}$$

gives the number of particles in a small hypervolume $d^3x\,d^3p$ around (x,p) in a given frame.

The particle four-current is

$$N^\mu(x) = g\int \frac{d^3p}{(2\pi)^3 E_0}\,p^\mu f(x,p), \tag{0.3}$$

where $p^\mu = (E_0, \boldsymbol{p})$ and $E_0 = \sqrt{\boldsymbol{p}^2 + m^2}$ is the on-shell energy.

Let Σ be a three-dimensional freeze-out hypersurface in space-time with normal vector
$d^3\sigma_\mu$. The total number of particles that cross Σ is given by the flux of the current,

$$N = \int_\Sigma N^\mu(x)\,d^3\sigma_\mu. \tag{0.4}$$

Substituting Eq. (0.3) and exchanging the order of integrations,

$$N = g\int \frac{d^3p}{(2\pi)^3 E_0}\left[\int_\Sigma p^\mu d^3\sigma_\mu f(x,p)\right]. \tag{0.5}$$

The number of particles with momentum in d^3p is then

$$dN = g\frac{d^3p}{(2\pi)^3 E_0}\int_\Sigma p^\mu d^3\sigma_\mu f(x,p), \tag{0.6}$$

so that

$$E_0 \frac{dN}{d^3 p} = \frac{g}{(2\pi)^3} \int_\Sigma p^\mu d^3\sigma_\mu f(x, p). \tag{0.7}$$

In local thermal equilibrium, one takes

$$f(x, p) = \exp\left[-\frac{p \cdot u(x) - \mu(x)}{T(x)}\right], \tag{0.8}$$

in the Boltzmann approximation, where $T(x)$, $\mu(x)$ and $u^\mu(x)$ are the local temperature, chemical potential and flow velocity.

Equation (0.7) is the standard Cooper-Frye formula. We now introduce the generalized uncertainty principle and its consequences for the dispersion relation and phase-space measure. The linear GUP proposed in Refs. [1,2] is defined by the commutator

$$[\hat{X}_i, \hat{P}_j] = i\delta_{ij}\left(1 - \alpha\hat{P}\right), \qquad \hat{P} = \sqrt{\hat{P}_k \hat{P}_k}, \tag{0.9}$$

where $\alpha = \alpha_0/M_{\mathrm{Pl}}$ is a small parameter with dimension of inverse momentum and α_0 is dimensionless. To linear order in α, this deformation implies a minimal length scale and leads to corrections in the single-particle dynamics and the density of states [2,24].

Following Elmashad *et al.* [3], the corresponding classical phase-space measure for a free particle becomes

$$d^3x \, d^3p \longrightarrow d^3x \, d^3p \, \frac{1}{(1 - \alpha p)^4}, \qquad p = |\boldsymbol{p}|. \tag{0.10}$$

For $\alpha p \ll 1$ we expand

$$\frac{1}{(1 - \alpha p)^4} \simeq 1 + 4\alpha p + \mathcal{O}(\alpha^2). \tag{0.11}$$

The dispersion relation is also modified. For a particle of rest mass m, the GUP-corrected energy satisfies [3]

$$E^2(p) = p^2(1 - 2\alpha p) + m^2, \tag{0.12}$$

which, to first order in α, gives

$$E(p) = \sqrt{E_0^2 - 2\alpha p^3} \simeq E_0 - \alpha \frac{p^3}{E_0}, \qquad E_0 = \sqrt{p^2 + m^2}. \tag{0.13}$$

Equations (0.11) and (0.13) encode the leading GUP corrections relevant for our freeze-out analysis.

In local thermal equilibrium, we assume that the Boltzmann factor depends on the GUP-deformed energy,

$$f_{\mathrm{GUP}}(x, p) = \exp\left[-\frac{E(p, u) - \mu(x)}{T(x)}\right], \qquad E(p, u) = E(p{\cdot}u), \tag{0.14}$$

with E given by Eq. (0.13). To first order in α,

$$E(p, u) = E_0(p, u) - \alpha \frac{p^3}{E_0(p, u)}, \qquad E_0(p, u) = p{\cdot}u, \tag{0.15}$$

and we may write

$$f_{\mathrm{GUP}}(x, p) = \exp\left[-\frac{E_0(p, u) - \mu(x)}{T(x)}\right] \exp\left[\frac{\alpha p^3}{T(x) \, E_0(p, u)}\right]. \tag{0.16}$$

Expanding the second exponential,

$$f_{\text{GUP}}(x,p) \simeq f_0(x,p) \left[1 + \alpha \frac{p^3}{T(x)\, E_0(p,u)} \right] + \mathcal{O}(\alpha^2), \tag{0.17}$$

where

$$f_0(x,p) = \exp\left[-\frac{E_0(p,u) - \mu(x)}{T(x)} \right], \tag{0.18}$$

is the standard local equilibrium distribution.

We now derive the GUP-deformed analogue of Eq. (0.7). In the presence of a modified phase-space measure, the particle number in an element $d^3x\, d^3p$ is

$$dN = g\, f_{\text{GUP}}(x,p)\, \frac{d^3x\, d^3p}{(2\pi)^3}\, \frac{1}{(1-\alpha p)^4}. \tag{0.19}$$

Following the same reasoning as in the standard case, the four-current becomes

$$N_{\text{GUP}}^{\mu}(x) = g \int \frac{d^3p}{(2\pi)^3 E_0}\, \frac{1}{(1-\alpha p)^4}\, p^{\mu} f_{\text{GUP}}(x,p), \tag{0.20}$$

where we have kept p^{μ}/E_0 as the classical four-velocity and take into account GUP corrections only through the measure and f_{GUP}.

The total number of particles crossing Σ is

$$N = \int_{\Sigma} N_{\text{GUP}}^{\mu}(x)\, d^3\sigma_{\mu}. \tag{0.21}$$

As before, we exchange integrations and identify the differential spectrum. Substituting Eq. (0.20) into Eq. (0.21) we obtain

$$N = g \int \frac{d^3p}{(2\pi)^3 E_0} \left[\int_{\Sigma} d^3\sigma_{\mu}\, \frac{p^{\mu}}{(1-\alpha p)^4}\, f_{\text{GUP}}(x,p) \right]. \tag{0.22}$$

Hence the number of particles with momenta in d^3p is

$$dN = g \frac{d^3p}{(2\pi)^3 E_0} \int_{\Sigma} d^3\sigma_{\mu}\, \frac{p^{\mu}}{(1-\alpha p)^4}\, f_{\text{GUP}}(x,p), \tag{0.23}$$

and the invariant spectrum is

$$E_0 \frac{dN}{d^3p} = \frac{g}{(2\pi)^3} \int_{\Sigma} p^{\mu} d^3\sigma_{\mu}\, \frac{1}{(1-\alpha p)^4}\, f_{\text{GUP}}(x,p). \tag{0.24}$$

We now expand Eq. (0.24) to first order in α, using Eqs. (0.11) and (0.17). Up to $\mathcal{O}(\alpha)$,

$$\frac{1}{(1-\alpha p)^4} f_{\text{GUP}}(x,p) \simeq f_0(x,p) \left[1 + 4\alpha p + \alpha \frac{p^3}{T(x)\, E_0(p,u)} \right]. \tag{0.25}$$

Substituting into Eq. (0.24),

$$E_0 \frac{dN}{d^3p} = \frac{g}{(2\pi)^3} \int_{\Sigma} p^{\mu} d^3\sigma_{\mu}\, f_0(x,p) \left[1 + \alpha\, C(x,p) \right] + \mathcal{O}(\alpha^2), \tag{0.26}$$

where we have defined the dimensionless correction

$$C(x,p) \equiv 4p + \frac{p^3}{T(x)\,E_0(p,u)}. \tag{0.27}$$

Separating the zeroth-order and first-order pieces,

$$E_0 \frac{dN}{d^3p} = \left(E_0 \frac{dN}{d^3p} \right)_0 + \alpha\,\Delta\left(E_0 \frac{dN}{d^3p} \right) + \mathcal{O}(\alpha^2), \tag{0.28}$$

with

$$\left(E_0 \frac{dN}{d^3p} \right)_0 = \frac{g}{(2\pi)^3} \int_\Sigma p^\mu d^3\sigma_\mu\, f_0(x,p), \tag{0.29}$$

$$\Delta\left(E_0 \frac{dN}{d^3p} \right) = \frac{g}{(2\pi)^3} \int_\Sigma p^\mu d^3\sigma_\mu\, f_0(x,p)\,C(x,p). \tag{0.30}$$

Equivalently, whenever the space-time dependence of T and u^μ is mild over the freeze-out region relevant for a given momentum, one can write

$$E_0 \frac{dN}{d^3p} \simeq \left(E_0 \frac{dN}{d^3p} \right)_0 \left[1 + \alpha\,\overline{C}(p) \right], \tag{0.31}$$

where

$$\overline{C}(p) \equiv \frac{\displaystyle\int_\Sigma p^\mu d^3\sigma_\mu\, f_0(x,p)\,C(x,p)}{\displaystyle\int_\Sigma p^\mu d^3\sigma_\mu\, f_0(x,p)}. \tag{0.32}$$

Equation (0.26) is the desired GUP-deformed Cooper-Frye formula. To gain intuition and obtain a fully analytic result, we consider a simple but instructive case. A static, homogeneous fireball freezing out on a constant-time hypersurface. Let freeze-out occur at constant laboratory time $t = t_{\rm f}$ in a static volume V, with uniform temperature T and chemical potential μ, and zero collective flow. The surface element is

$$d^3\sigma_\mu = (d^3x, 0, 0, 0), \qquad \int_\Sigma d^3x = V, \tag{0.33}$$

and $u^\mu = (1, \mathbf{0})$. The local equilibrium distribution reduces to

$$f_0(p) = \exp\left[-\frac{E_0 - \mu}{T} \right]. \tag{0.34}$$

Equation (0.29) then gives

$$E_0 \frac{dN}{d^3p}\bigg|_0 = \frac{g}{(2\pi)^3} \int_\Sigma d^3x\, E_0 f_0(p) = \frac{gV}{(2\pi)^3} E_0\, e^{-(E_0-\mu)/T}. \tag{0.35}$$

In the same setup, $C(x,p)$ is independent of position and reduces to

$$C(p) = 4p + \frac{p^3}{TE_0}. \tag{0.36}$$

Substituting Eqs. (0.36) and (0.18) into Eq. (0.26),

$$E_0 \frac{dN}{d^3 p} = \frac{g}{(2\pi)^3} \int_{\Sigma} d^3 x \, E_0 e^{-(E_0 - \mu)/T} \left[1 + \alpha \left(4p + \frac{p^3}{TE_0} \right) \right]. \tag{0.37}$$

Performing the spatial integral gives

$$E_0 \frac{dN}{d^3 p} = \frac{gV}{(2\pi)^3} E_0 \, e^{-(E_0 - \mu)/T} \left[1 + \alpha \left(4p + \frac{p^3}{TE_0} \right) \right] + \mathcal{O}(\alpha^2). \tag{0.38}$$

Comparing with Eq. (0.35) we see explicitly that

$$\frac{\left(E_0 dN/d^3 p \right) - \left(E_0 dN/d^3 p \right)_0}{\left(E_0 dN/d^3 p \right)_0} \simeq \alpha \left(4p + \frac{p^3}{TE_0} \right), \tag{0.39}$$

so that the GUP enhances the high-momentum tail of the spectrum whenever $\alpha > 0$.

In realistic heavy-ion phenomenology, static fireballs are replaced by expanding blast-wave models that incorporate radial flow and realistic freeze-out surfaces [23]. The invariant spectrum of a hadron species is typically written as

$$E \frac{dN}{d^3 p} = \frac{g}{(2\pi)^3} \int_{\Sigma} p^{\mu} d^3 \sigma_{\mu} \, \exp\left[-\frac{p \cdot u(x) - \mu}{T} \right], \tag{0.40}$$

with $u^{\mu}(x)$ parametrized by a transverse flow profile. In the presence of GUP effects, Eq. (0.40) is modified to

$$E \frac{dN}{d^3 p} = \frac{g}{(2\pi)^3} \int_{\Sigma} p^{\mu} d^3 \sigma_{\mu} \, \exp\left[-\frac{p \cdot u(x) - \mu}{T} \right] \left[1 + \alpha \, C(x, p) \right], \tag{0.41}$$

with $C(x, p)$ given by Eq. (0.27). Thus, at fixed hydrodynamic fields (T, u^{μ}), the GUP modifies the weight of different momentum modes in a calculable manner. To leading order in α, one can continue to use standard blast-wave codes and incorporate the factor $1 + \alpha C(x, p)$ as a multiplicative correction to the integrand.

We have derived the leading corrections to the Cooper-Frye freeze-out formula induced by a linear generalized uncertainty principle. Starting from a GUP-modified phase-space measure and dispersion relation, we constructed the deformed particle current and integrated its flux across a freeze-out hypersurface. To first order in the GUP parameter α we obtained, a compact expression for the GUP-deformed Cooper-Frye formula, Eq. (0.26), which reduces to the standard result when $\alpha \to 0$. An explicit analytic correction factor $C(x, p)$, Eq. (0.27), which enhances the contributions of high-momentum modes relative to the standard Boltzmann weight. For a static, homogeneous freeze-out hypersurface, a fully analytic spectrum, Eq. (0.38), is suitable for use as a simple benchmark or as an exam-level derivation. A straightforward prescription, Eq. (0.41), for incorporating GUP corrections into blast-wave parameterizations used in current heavy-ion data analyses. From a conceptual perspective, our analysis shows how Planck-suppressed quantum-gravity effects encoded by a generalized uncertainty principle can be consistently mapped onto observable hadron spectra at freeze-out. From a phenomenological perspective, Eq. (0.39) suggests that the relative correction to the spectrum grows with momentum, roughly as $\alpha p^3/(TE_0)$, making the high-p_T region the most sensitive to GUP effects.

This work introduces a generalized uncertainty principle (GUP) deformation of the Cooper-Frye freeze-out procedure, providing a conceptually new and phenomenologically testable setting in which Planck-suppressed quantum-gravity effects can imprint themselves

on identified-hadron spectra. The construction is analytic and self-consistent within the stated assumptions, and it furnishes a practical correction factor that can be implemented as a post-processing step in standard particlization pipelines. Since the Cooper-Frye map is also the standard bridge used in holography-informed heavy-ion frameworks, the present result can be viewed as a universal, post-processing deformation of particlization that is independent of how the hydrodynamic fields were obtained. It would be interesting to explore whether the resulting momentum dependence admits an effective interpretation in terms of finite-coupling/higher-derivative corrections on the holographic side, thereby translating phenomenological bounds on the GUP parameter into constraints on such UV corrections. Finally, it may be worthwhile to sharpen the interpretation of such bounds by framing the inference problem in terms of identifiability and principled limits on model discrimination, in the spirit of undecidability and incompleteness considerations applied to quantum-gravity programmes [25–28], including extensions to third-quantized settings [29]. With only a finite set of freeze-out observables, distinct classes of UV deformations can in principle lead to indistinguishable spectral modifications. Establishing clear criteria for when a given dataset can (or cannot) decisively separate GUP-type effects from other microscopic corrections would strengthen the robustness of the phenomenological conclusions.

Several natural extensions remain. First, one can combine the present freeze-out corrections with GUP-deformed hydrodynamic evolution and equation of state,[3,4] yielding a fully GUP-consistent space-time picture of the QGP. Such a framework could be matched consistently to hadron-resonance-gas descriptions with finite baryon-size effects, thermodynamically consistent equations of state and excluded-volume corrections, thereby connecting GUP freeze-out signatures directly to precision hadronic yield analyses [19–21]. Second, it would be interesting to confront GUP-corrected blast-wave fits with precision spectra and harmonic flow data from RHIC and the LHC, thereby constraining the GUP parameter α. Finally, alternative GUP models, including linear-quadratic deformations and models with modified measure but unmodified dispersion, can be treated within the same framework and may yield qualitatively different signatures.

Data Availability

The manuscript has no associated data or the data will not be deposited.

Conflicts of Interest

The author declares that there is no conflict of interest.

Ethical Considerations

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