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Regular article

Thermodynamic Structure of Einstein-Gauss-Bonnet Regular Black Holes Coupled with Cloud of String

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Abstract. We construct several black holes coupled with the nonlinear electrodynamics (NLED) known as a regular black hole, which becomes Maxwell's theory in the weak field limit. We present an exact regular black hole solution in the presence of a cloud of strings (CoS), and Einstein-Gauss-Bonnet (EGB) gravity. We study the global properties of the solutions and derive the corrected first law of thermodynamics, because the first law of thermodynamics is modified in the presence of NLED. In addition, we have studied the thermodynamics properties associated with the EGB regular black hole solution, the thermodynamic quantities (Mass, Temperature, Entropy, Heat Capacity and Free Energy) is change in the presence of NLED, CoS, and EGB parameter.

Keywords: Einstein-Gauss-Bonnet Gravity; Nonlinear Electrodynamics.

Contents

1 Introduction

Lovelock theories [1–3], incorporating higher-order curvature factors, extend Einstein's general relativity to higher dimensions. In the second-order Lovelock theory, often known as Einstein-Gauss-Bonnet (EGB) gravity, the action is augmented with quadratic curvature terms. This specific instance of EGB gravity has garnered considerable attention since the EGB action inherently emerges in the low-energy regime of heterotic string theory [4]. The static spherically symmetric solution in the EGB theory was initially identified by Boulware and Deser [5], and then Wiltshire [6] gave the charged EGB black hole solution. A succession of notable studies examined black hole solutions in EGB gravity [7] for diverse sources [8–13], including those associated with nonlinear electrodynamics (NLED) fields [14,15].

Due to their significance in astrophysical observations, black holes with NLED as a matter source have been extensively studied [16–26]. The First regular black hole model was proposed by Bardeen [27]. It is demonstrated that a regular black hole merely explains how a black hole forms from an initial vacuum region in the setting of NLED. The only reason NLED is preferred over Maxwell's theory is that, in addition to conformal breaking, self-energy and an infinite electric field also manifest at the location of a point charge. Based on the Bardeen proposal, there are numerous black hole solutions based on Bardeen proposal [28–40], and the characteristics of ordinary black holes are found in [41–52]. Following Bekenstein and Hawking's propositions regarding black hole entropy [53,54] and black hole radiation [55], black holes are now seen as thermal systems. Bekenstein asserted that the sum of black hole entropy and matter entropy constitutes a non-decreasing function of time [56]. Hawking and Page [57,58] were the first to investigate the phase transition between the AdS black hole and thermal AdS space. Phase transition is crucial for examining the thermodynamic properties of entities at the critical point. This analysis has been examined for different black holes [59–74].

Motivated by the above arguments, we find the regular black hole solution in the presence of Cloud of String (CoS), and EGB gravity. The obtained black hole solution interpolates with the Letailier black hole [75] in the limit of NLED, and EGB parameter as well as the Schwarzschild black hole in the absence of NLED, CoS parameter, and EGB gravity. In turn, we analyze the thermodynamic properties of these models and study the thermodynamic stability (local and global stability) of this black hole. The black hole's thermodynamic quantities are modified by the presence of NLED and CoS.

The paper is organised as follows: we obtain a higher-dimensional regular (AdS) black hole solution in Sec. 2, and also give the consistent equations of gravity coupled to NLED and CoS. The study of the thermodynamical properties of higher-dimensional regular black hole solutions in Sec. 3. In Sec. 4, we discuss the thermodynamic properties of regular EGB black hole with CoS. Finally, the concluding remarks and results are presented in Sec. 5.

2 Einstein-Gauss-Bonnet gravity with Nonlinear Electrodynamics

This study commences with the formulation of EGB gravity with a CoS in conjunction with NLED, expressed as follows:

$$S = \frac{1}{2}\int_{\mathcal{M}} d^D x\sqrt{-g}\left[R + \alpha\mathcal{L}_{GB} + \mathcal{L}(F)\right], \tag{2.1}$$

where α denotes the Gauss-Bonnet (GB) coupling coefficient, with the condition $\alpha \geq 0$ [1–3,7]. NLED is characterized by the Lagrangian $\mathcal{L}(F)$ in the invariant $F = F_{\mu\nu}F^{\mu\nu}/4$, where $F_{\mu\nu}$ denotes the electromagnetic field tensor related to the gauge potential A_ν through the expression $F_{\mu\nu} = 2\nabla_{[\mu}A_{\nu]}$. The GB Lagrangian is expressed in the form [5,6]

$$\mathcal{L}_{GB} = R_{\mu\nu\gamma\delta}R^{\mu\nu\gamma\delta} - 4R_{\mu\nu}R^{\mu\nu} + R^2, \tag{2.2}$$

where, $R_{\mu\nu}$ denotes the Ricci tensor, $R_{\mu\nu\gamma\delta}$ signifies the Riemann tensor, and R represents the Ricci scalar. The variation of the action concerning the metric $g_{\mu\nu}$ yields the subsequent GB equations of motion [5,6]

$$G_{\mu\nu} + \alpha H_{\mu\nu} = T_{\mu\nu}^{NLED} + T_{\mu\nu}^{CoS} \equiv 2\left[\frac{\partial \mathcal{L}(F)}{\partial F}F_{\mu\rho}F^{\rho}_{\nu} - g_{\mu\nu}\mathcal{L}(F)\right], \tag{2.3}$$

$$\nabla_\mu\left(\frac{\partial \mathcal{L}(\mathcal{F})}{\partial F}F_{\mu\nu}\right) = 0 \quad \text{and} \quad \nabla_\mu\left({}^*F_{\mu\nu}\right) = 0. \tag{2.4}$$

where $G_{\mu\nu}$ denotes the Einstein tensor and $H_{\mu\nu}$ represents the Lanczos tensor [1,2]

$$H_{\mu\nu} = 2\Big(- R_{\mu\sigma\kappa\tau}R^{\kappa\tau\sigma}{}_{\nu} - 2R_{\mu\rho\nu\sigma}R^{\rho\sigma} - 2R_{\mu\sigma}R^{\sigma}{}_{\nu} + RR_{\mu\nu}\Big) - \frac{1}{2}\mathcal{L}_{GB}g_{\mu\nu}. \tag{2.5}$$

The Lagrangian density of the nonlinear matter field is

$$\mathcal{L}(F) = Fe^{-\frac{s}{e}(2e^2F)^{\frac{D-3}{2D-4}}}, \qquad \text{with} \qquad s = \frac{e^{D-3}}{(D-2)\mu^{D-3}}. \tag{2.6}$$

We consider the following *anstaz* for the Maxwell field [72,73]

$$\begin{aligned} F_{\mu\nu} &= 2\delta^{\theta_1}_{[\mu}\delta^{\theta_2}_{\nu]}g\sin\theta_1, && D = 4, \\ F_{\mu\nu} &= 2\delta^{\theta_{D-3}}_{[\mu}\delta^{\theta_{D-2}}_{\nu]}\frac{g^{D-3}}{r^{D-4}}\sin\theta_{D-3}\left[\prod_{j=1}^{D-4}\sin^2\theta_j\right], && D \geq 5, \end{aligned} \tag{2.7}$$

with F as

$$F = \frac{e^{2(D-3)}}{2r^{2(D-2)}}. \tag{2.8}$$

Equation (2.7) indicates that $dF = 0$, hence we derive

$$e'(r)2\delta^{\theta_{D-3}}_{[\mu}\delta^{\theta_{D-2}}_{\nu]}\frac{g^{D-3}}{r^{D-4}}\sin\theta_{D-3}\left[\prod_{j=1}^{D-4}\sin^2\theta_j\right]d\theta\wedge d\phi\wedge\ldots\wedge d\psi_{(D-2)}. \tag{2.9}$$

This results in $e(r) = e =$ constant. The other components of $F_{\mu\nu}$ exert minimal influence compared to $F_{\theta\phi}$ [72,73]. The energy-momentum tensor (EMT) can be expressed as

$$T_t^{tNLED} \quad = \quad T_r^{rNLED} = \frac{(D-2)Mke^{-\frac{k}{r^{D-3}}}}{r^{2D-4}}. \tag{2.10}$$

Using EMT from Eq. (2.10), we can obtain the D- dimensional regular black hole. We obtain the EMT for a CoS is

$$T^{\mu\nu} = \frac{m\Sigma^{\mu\rho}\Sigma_{\rho}{}^{\nu}}{\sqrt{-\gamma}} = \frac{\rho\Sigma^{\mu\rho}\Sigma_{\rho}{}^{\nu}}{\sqrt{-\gamma}}. \tag{2.11}$$

where ρ is the density of a string cloud and $\rho\ (\gamma)^{-1/2}$ is the density that doesn't depend on the gauge. The only part of the bivector Σ that is still there is $\Sigma^{tr} = -\Sigma^{rt}$. And since $T_t^t = T_r^r = -\rho\Sigma^{tr}$, and $\partial_r(\sqrt{r^n T_t^t}) = 0$, this means

$$T_t^t = T_r^r = \frac{a}{r^{D-2}}, \tag{2.12}$$

for some real constant a, which is related to the global monopole [10].

3 Einsten-Gauss-Bonnet regular black hole with Cloud of String

Now, we want to obtain D-dimensional static spherically symmetric solutions in the presence of EGB gravity with a CoS as source and NLED . We assume that the metric has the form

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\Omega_{D-2}^2, \tag{3.1}$$

where $d\Omega_{D-2}^2$ is the metric of the (D-2) sphere. Using this metric *ansatz*, the rr component of equation of motion (2.3) is

$$\left(r^5 - 2\tilde{\alpha}r^3\left(f\left(r\right)-1\right)\right)f'\left(r\right) + \left(D-3\right)r^4\left(f\left(r\right)-1\right) = \frac{D-2}{r^{D-4}}\left(\frac{2a}{(D-2)^2} + \frac{Mke^{-\frac{k}{r^{D-3}}}}{r^D}\right), \tag{3.2}$$

in which a prime denotes a derivative with respect to r, $\tilde{\alpha} = (D-3)\,(D-4)\,\alpha$, In general, Eq. (3.2) has one real and two complex solutions. Eq. (3.2) admits the following solution

$$f(r) = 1 + \frac{r^2}{2\tilde{\alpha}}\left(1 \pm \sqrt{1 + \frac{8\tilde{\alpha}M}{(D-3)r^{D-1}}e^{-\frac{k}{r^{D-3}}} + \frac{8\tilde{\alpha}a}{(D-2)r^{D-2}}}\right). \tag{3.3}$$

This is the black hole solution in the presence of CoS, GB gravity and NLED. The Eq. (3.3) reduces to black hole solution in the presence of CoS, when $k = 0, \alpha \to 0$ [10]

$$f(r) = 1 + \frac{r^2}{2\tilde{\alpha}}\left(1 \pm \sqrt{1 + \frac{8\tilde{\alpha}M}{(D-3)r^{D-1}} + \frac{8\tilde{\alpha}a}{(D-2)r^{D-2}}}\right), \tag{3.4}$$

and EGB regular black hole when CS parameter swith off [74] and also EGB black hole for $k = 0$ [5].

The numerical value of Cauchy and event horizon tabulated in the Table (1) and depicted in the Fig. 2. The size of the horizon decreases with increasing the dimensions and increases with Gauss-Bonnet the coupling constant α corresponding to the deviation parameter k. The horizon also decreases with CS parameter a. To study the general structure of solution (3.3), we take the limit $r \to \infty$ or $m = a = 0$ in solution (3.3) to obtain

$$\lim_{r\to\infty} f_+(r) = 1 + \frac{r^2}{\alpha}, \qquad \text{and} \qquad \lim_{r\to\infty} f_-(r) = 1. \tag{3.5}$$

This means that the positive branch of the solution is asymptotically de Sitter (-anti-de Sitter) depending on the sign of α ($\pm$), while the negative branch of the solution is asymptotically flat. If you choose the right values a and k, the solution's horizon will be when $f(r) = 0$. As a changes, we plot $f(r)$ vs. r in different directions, but k stays the same. When the dimension is $D < 6$, these solutions have two horizons. When the dimension is $D > 6$, they have three horizons (see Fig. 1). As the number of a CoS goes up, the horizon's radius goes up as well.

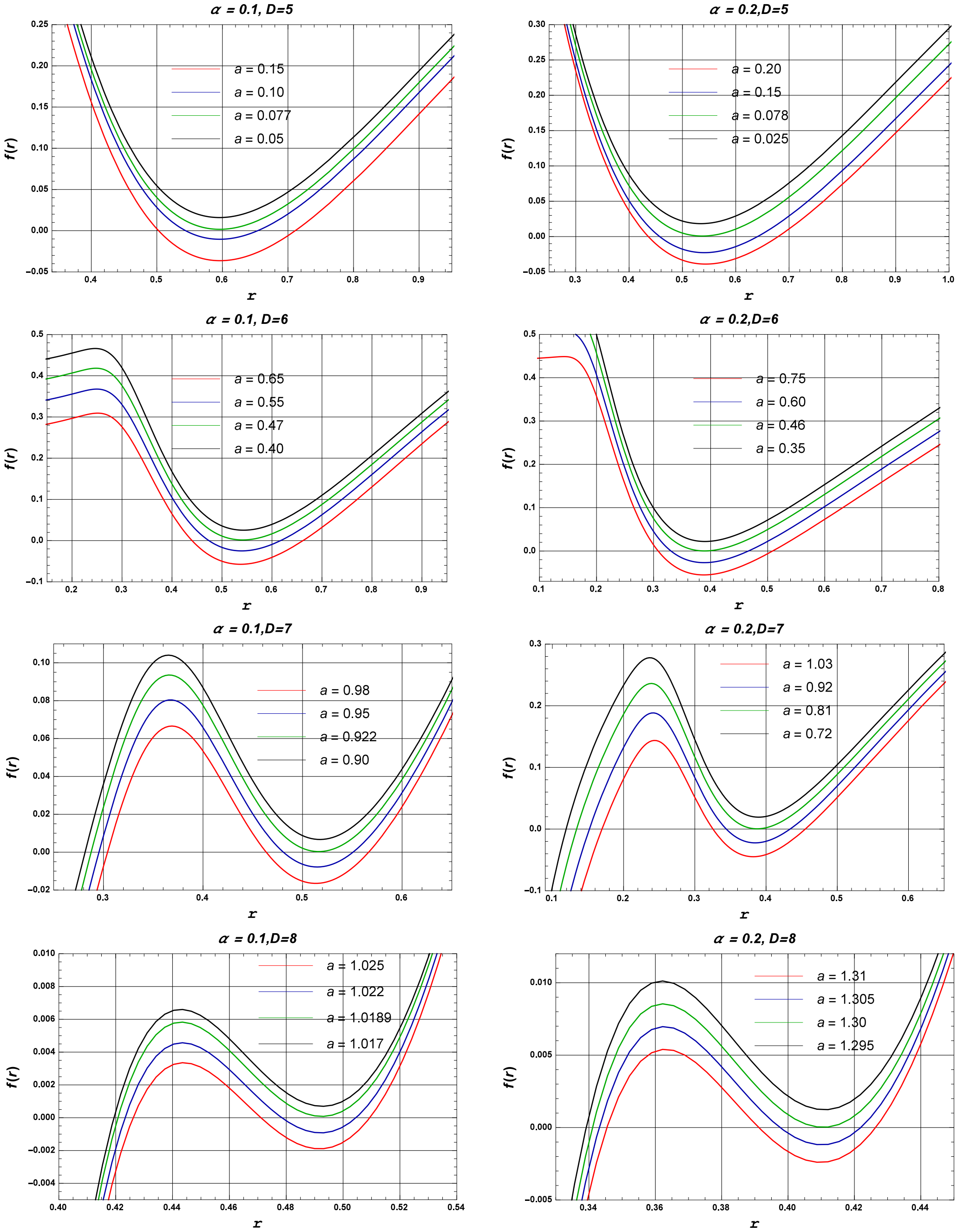

Figure 1: The plot $f(r)$ vs. horizon radius r of the various values of CoS parameter a corresponding to the deviation parameter (k) with constant values of α.

4 Thermodynamics

We examine the thermodynamic properties of regular EGB black hole with CoS, in term of the horizon radius (r_+). Our analysis will be confined to the negative branch of the EGB

Table 1: Cauchy horizon radius (r_-), the event horizon radius (r_+) and $\delta = r_+ - r_-$ for various values of CoS parameter a.

	$\alpha = 0.1$				$\alpha = 0.2$				
Dim.	a	r_-	r_+	δ	k	a	r_-	r_+	δ
$D = 5$	0.15	0.5025	0.7134	0.2109	0.12	0.20	0.4341	0.6811	0.2470
$k = 0.23$	0.10	0.5424	0.6578	0.1154		0.15	0.4585	0.6408	0.1823
	a_E=0.077	0.60	0.60	0		a_E=0.078	0.5356	0.5356	0
$D = 6$	0.65	0.4494	0.6524	0.2030	0.1	1.85	0.4381	0.6664	0.2283
$k = 0.1$	0.55	0.4763	0.6182	0.1419		1.75	0.4697	0.6207	0.1510
	a_E=0.47	0.5426	0.5426	0		a_E=1.667	0.5373	0.5373	0
$D = 7$	1.5	0.4729	0.5729	0.1002	0.04	1.92	0.3892	0.4793	0.0901
$k = 0.08$	1.125	0.4919	0.5563	0.0644		1.90	0.4036	0.4667	0.0631
	a_E=1.105	0.5261	0.5261	0		a_E= 1.875	0.4364	0.4364	0
$D = 8$	0.2	0.4156	0.5153	0.0997	0.005	0.2	0.3477	0.4141	0.0664
$k = 0.014$	0.15	0.4287	0.5052	0.0735		0.15	0.3558	0.3993	0.0400
	a_E=0.07	0.4622	0.4622	0		a_E=0.10	0.3772	0.3772	0

solution (3.3), characterized by mass (M), Gauss-Bonnet coupling (α), deviation parameter (k), and CoS parameter (a). The black hole mass is obtained in terms of horizon radius r_+ as

$$M_+ = \frac{(D-3)r_+^{D-3}}{2}\left(1 + \frac{\tilde{\alpha}}{r_+^2} - \frac{2a}{(D-2)r_+^{D-4}}\right) e^{k/r_+^{D-3}}. \tag{4.1}$$

The temperature of a regular EGB black hole with CoS is expressed as

$$\begin{aligned} T_+ =& \frac{D-3}{4\pi r_+}\Bigg[(2D-8)r_+^2 + \frac{(2D-8)(D-5)\tilde{\alpha}}{(D-2)r_+^3} - \\ & \frac{2a}{(D-2)r_+^{D-6}} + \frac{(D-3)k}{r^{D-2}}\left(\frac{2a}{r_+^{D-7}} - (r_+^2 + \tilde{\alpha})\right)\Bigg]. \end{aligned} \tag{4.2}$$

By performing the limit $a \to 0$, the temperature (4.2) reduces to regular EGB black hole

$$T_+ = \frac{D-3}{4\pi r_+}\left[(2D-8)r_+^2 + \frac{(2D-8)(D-5)\tilde{\alpha}}{(D-2)r_+^3} + \frac{(D-3)k}{r^{D-2}}\left(\frac{2a}{r_+^{D-7}} - (r_+^2 + \tilde{\alpha})\right)\right]. \tag{4.3}$$

and also EGB black hole [5] in the limit $k \to 0$. The numerical values of the temperature of the regular EGB black hole with CoS are tabulated in the table 2 and plotted in the Fig. 2. The temperature decreases with the CoS parameter a and the dimensions D.

Subsequently, we compute the other important quantity related to the black hole horizon, its entropy, which may be derived from the first law of thermodynamics [73]

$$dM_+ = T_+ \, dS_+, \tag{4.4}$$

where S_+ is the entropy of the black hole. Hence, the black hole entropy can be obtained by integrating the first law to give

$$S_+ \quad = \int T_+^{-1} \, dM_+ = \int T_+^{-1} \frac{\partial M_+}{\partial r_+} dr_+, \tag{4.5}$$

and substituting (4.1) and (4.2) into (4.5), we find the entropy of the regular EGB black hole as

$$S_+ = 4\pi \int e^{k/r_+^{D-3}} (r_+^2 + 2\tilde{\alpha}) \, dr_+. \tag{4.6}$$

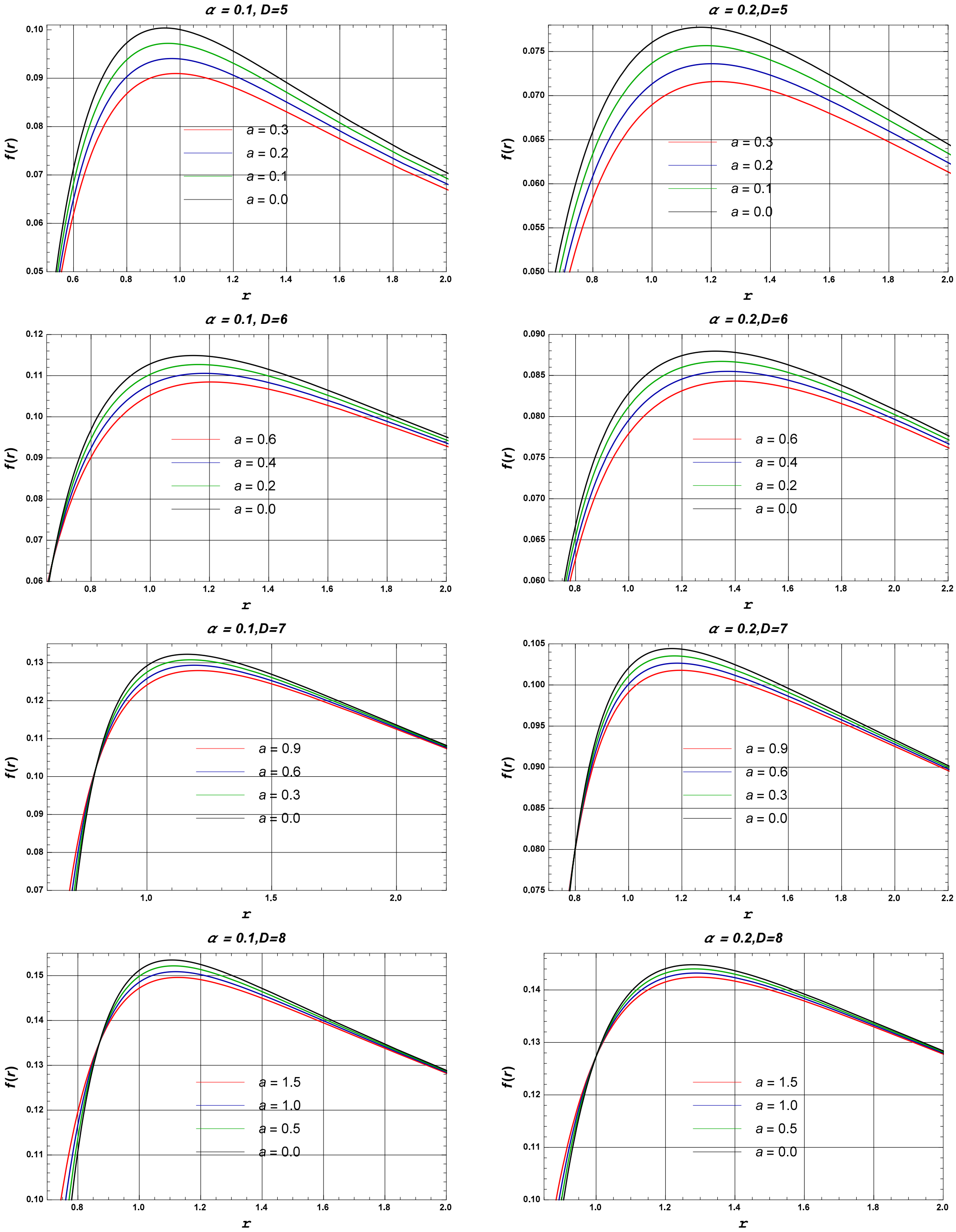

Figure 2: The plot of temperature (T_+) vs. horizon radius (r_+) with various values of the deviation parameter (k) and the CoS parameter (a)

Interestingly this can be integrated in closed form

$$S_+ = \frac{2(D-3)\pi r_+^3}{(D-2)}\left[\frac{((D-2)k + r_+^{D-3} + \frac{2(D-2)\tilde{\alpha}}{(D-4)})e^{k/r_+^{D-3}}}{r_+^2} - \frac{2\sqrt{\pi}k^{D-2/D-3}\,(k+6\alpha)}{r_+^3}\mathrm{erf}\left(\frac{\sqrt{k}}{r_+^{D-4}}\right)\right]. \tag{4.7}$$

Table 2: The maximum Hawking temperature T_+^{Max} at critical radius r_c^T for different values of CoS parameter a and different dimension $D = 5, 6, 7$ and 8 with fixed value of $k = 0.10$.

	$\alpha = 0.1$			$\alpha = 0.2$		
Dimensions	a	r_c^T	T_+^{Max}	a	r_c^T	T_+^{Max}
$D = 5$	0.0	0.9345	0.1002	0.0	1.145	0.07768
	0.1	0.9394	0.09704	0.1	1.157	0.07558
	0.2	0.9565	0.09408	0.2	1.189	0.07359
	0.3	0.9840	0.09126	0.3	1.207	0.07169
$D = 6$	0.0	1.112	0.1146	0.0	1.273	0.08785
	0.2	1.143	0.1124	0.2	1.322	0.08672
	0.4	1.156	0.1108	0.4	1.342	0.08559
	0.6	1.186	0.1107	0.6	1.371	0.08423
$D = 7$	0.0	1.148	0.1324	0.0	1.167	0.1047
	0.3	0.1306	0.1308	0.3	1.171	0.1035
	0.6	1.181	0.1294	0.6	1.182	0.1027
	0.9	1.196	0.1282	0.9	1.188	0.1018
$D = 8$	0.0	1.091	0.1536	0.0	1.269	0.1449
	0.5	1.103	0.1521	0.5	1.272	0.1440
	1.0	1.109	0.1509	1.0	1.277	0.1432
	1.5	1.118	0.1495	1.5	1.282	0.1426

Nonetheless, the entropy fails to comply with the area law. The entropy of the EGB black hole diverges from the formulation of general relativity. When $k = 0$ and $a = 0$, Equation (4.7) can be read as the entropy of the EGB black hole as follows,

$$S_+ = \frac{2(D-3)\pi r_+^3}{D-2}\left[\frac{((D-2)k + r_+^{D-3} + \frac{2(D-2)\tilde{\alpha}}{(D-4)})e^{k/r_+^{D-3}}}{r_+^2}\right], \tag{4.8}$$

is precisely equivalent to the entropy of the EGB black hole [5]. The expression $S_+ = \frac{4}{3}\pi r_+^3$ represents the entropy of the $5D$ Schwarzschild-Tangherlini black hole.

The thermodynamic stability of the black hole is assessed by examining the behavior of its heat capacity (C_+). The heat capacity of a black hole is characterized as [72,73]

$$C_+ = \frac{\partial M_+}{\partial T_+} = \Big(\frac{\partial M_+}{\partial r_+}\Big)\Big(\frac{\partial r_+}{\partial T_+}\Big), \tag{4.9}$$

substituting (4.1) and (4.2) into (4.9), the heat capacity of the regular EGB hole, determined by using Eq. (4.9) reads

$$C_+ = -4\pi r_+^3 e^{k/r^{D-3}}\left[\frac{(r_+^2+2\tilde{\alpha})^2\Big((D-2)A_1 r_+^{2D-3} - \frac{2a}{D-3}r_+^{D+3} + k\left((r_+^2+\tilde{\alpha})r_+^D - \frac{2a}{r^{D-6}}\right)\Big)}{(D-2)r_+^D B_1 - 2ar_+^6 B_2 + k(4aB_3 + (D-2)r_+^3 B_4)}\right],$$

with

$$A_1 =, r_+^2 + \frac{(D-5)}{(D-2)}\tilde{\alpha}, \qquad B_1 = 5r_+^4 + \frac{13-53}{D-3}r_+^2\tilde{\alpha} + \frac{D-5}{D-3}\tilde{\alpha}^2$$
$$B_2 = (D+1)r_+^2 + 2(D-1)\tilde{\alpha}, \qquad B_3 = \frac{(D-1)r_+^2 + 2(D-2)\tilde{\alpha}}{r^{D-9}}$$
$$B_4 = (D-2)r_+^4 + (3D+4)r_+^2\tilde{\alpha} + 2(D+2)\tilde{\alpha}^2. \tag{4.10}$$

In the limit $a = 0$ (4.10) the heat capacity reduces to regular EGB black hole and $k = 0$ and $a = 0$, EGB black hole [5]. The global stability of the black hole is calculated by the

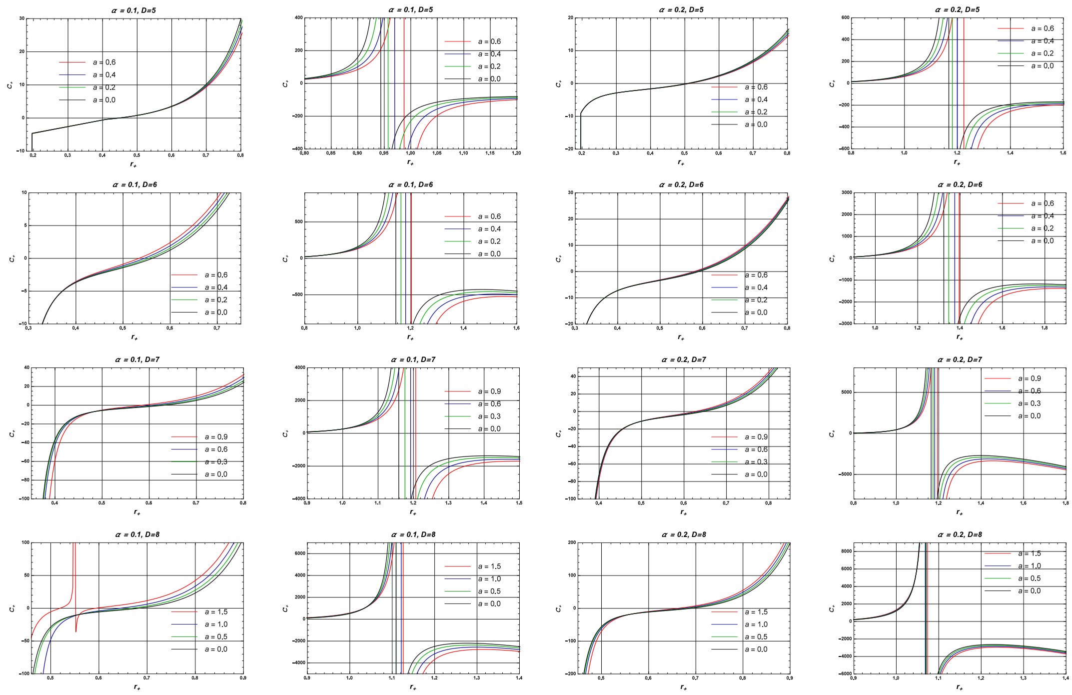

Figure 3: The plot of heat capacity (C_+) vs. horizon radius (r_+) with various values of the deviation parameter (k) and the CoS parameter (a) in various dimensions $D = 5, \& \ 6$.

free energy. The free energy is

$$F_+ = \frac{(D-3)r_+^{D-3}}{2}\left(1+\frac{\tilde{\alpha}}{r_+^2}-\frac{2a}{(D-2)r_+^{D-4}}\right)e^{k/r_+^{D-3}} - r_+^2\Big((2D-8)r_+^2+\frac{(2D-8)(D-5)\tilde{\alpha}}{(D-2)r_+^3} - \frac{2a}{(D-2)r_+^{D-6}}+\frac{(D-3)k}{r^{D-2}}\left(\frac{2a}{r_+^{D-7}}-(r_+^2+\tilde{\alpha})\right)\Big)\Big(\frac{((D-2)k+r_+^{D-2}+\frac{2(D-2)\tilde{\alpha}}{(D-4)})e^{k/r_+^{D-3}}}{r_+^2} - \frac{2\sqrt{\pi}k^{1/D-3}\,(k+6\alpha)}{r_+^3}\mathrm{erf}\left(\frac{\sqrt{k}}{r_+^{D-4}}\right)\Big). \tag{4.11}$$

The free energy is plotted in Figure 4. In the figure, we can see clearly that the black hole is stable for smaller values of the horizon radius r_+.

5 Conclusions

This study presents the higher-dimensional regular black hole solution within the framework of CoS and EGB gravity. The derived black hole solution interpolates with the Letailier black hole in the absence of NLED and EGB parameters, as well as the Schwarzschild black hole in the limit of NLED, CoS parameters, and EGB gravity. Additionally, we have analyzed the horizon structure of the derived black hole solution; the black hole possesses two horizons

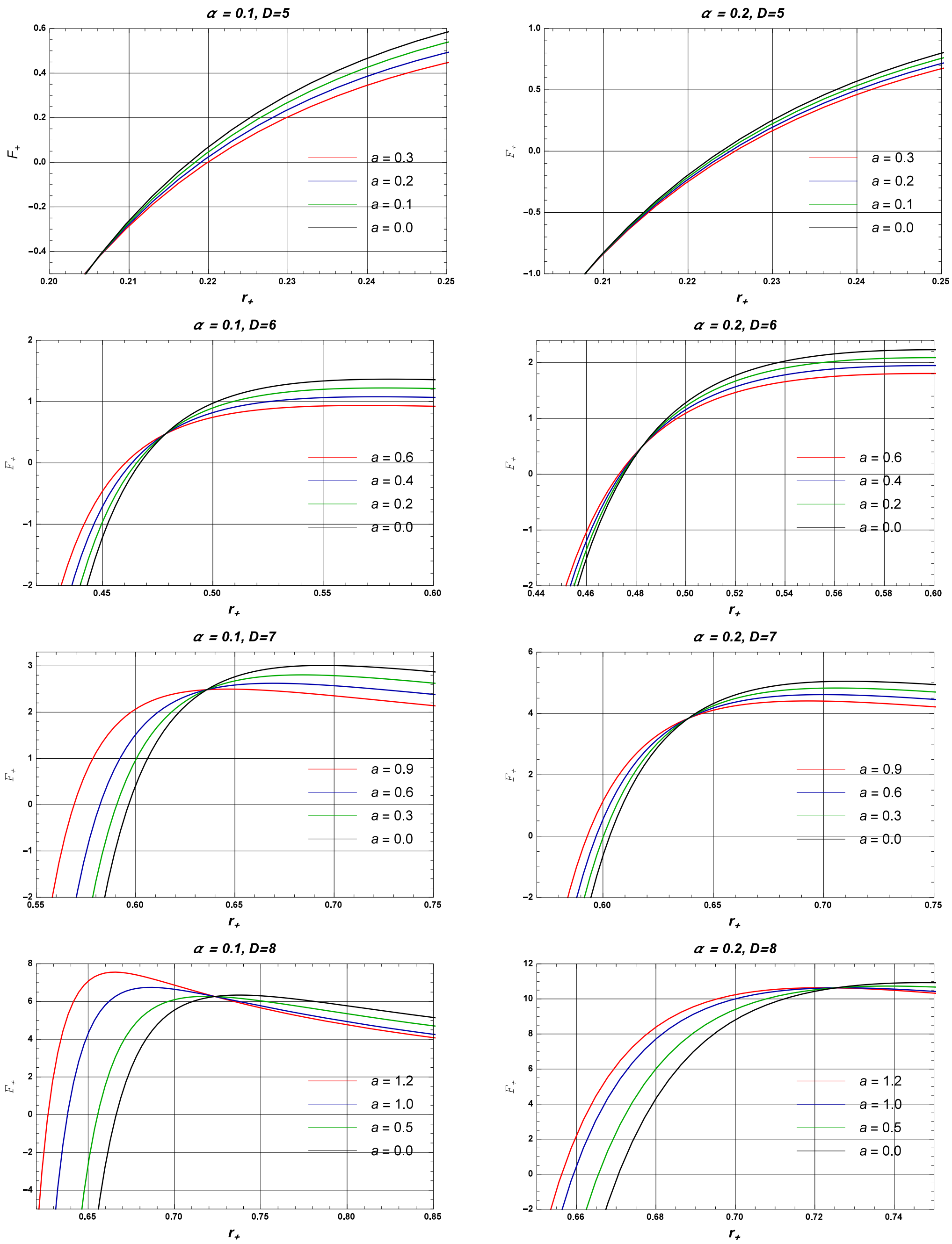

Figure 4: The plot free energy vs. horizon radius (r_+) with various values of the deviation parameter (k) and the CoS parameter (a) with fixed value of α.

for $D < 6$ and three horizons when the dimension exceeds $D > 7$. Furthermore, we have examined the thermodynamic properties related to the EGB regular black hole solution, noting the variations in thermodynamic parameters (Mass, Temperature, Entropy, Heat

Capacity, and Free Energy) in the presence of NLED, CoS, and the EGB parameter.

Authors' Contributions

All authors have the same contribution.

Data Availability

The manuscript has no associated data or the data will not be deposited.

Conflicts of Interest

The authors declare that there is no conflict of interest.

Ethical Considerations

The authors have diligently addressed ethical concerns, such as informed consent, plagiarism, data fabrication, misconduct, falsification, double publication, redundancy, submission, and other related matters.

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