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Regular article

Holographic Phase Transition of AdS Black Hole Solution Coupled with Nonlinear Electrodynamics

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Abstract. This study presents a novel AdS black hole (BH) solution coupled with NLED and a cloud of string. Our solution interpolates to the AdS Letailier BH in the absence of magnetic monopole charge (q) and deviation parameter (k), AdS Hayward Letailier BH when in the absence of deviation parameter, AdS regular Letailier BH in the absence of magnetic monopole charge, as well as Schwarzschild BH in the limit of $q = 0, k = 0, a = 0$. We have studied the horizon structure of the obtained solution; the BH has two horizons (event and Cauchy) in contrast with the Schwarzschild BH. The thermodynamic quantities associated with the BH are modified in the presence of magnetic monopole charge, cloud of string parameter, and a deviation parameter. The first law of BH thermodynamics is modified in the presence of magnetic monopole charge and deviation parameter. Additionally, we examine the thermodynamics of the AdS Letailier regular BH solution, considering the cosmological constant (Λ) as thermodynamic pressure (P), and analyze the critical points and phase structure of the BH within an extended phase space. The plot of Gibbs free energy against temperature exhibits a swallow-tail behaviour, signifying a first-order phase transition that concludes at a second-order phase transition.

Keywords: Black Hole; Thermodynamics; Holographic Phase Transition.

Contents

1 Introduction

Black holes (Bhs) are a solution of the Einstein equations and a fundamental prediction of the general theory of relativity (GTR). BH thermodynamics, fundamental components of GTR, are essential characteristics of BHs, as formulated by Bekenstein, Carter, and Hawking [1–4].In this thermodynamic system, a BH has an event horizon beyond which no particle, even light, can escape the formidable gravitational force. Because of this property, no energy or matter inside the BH can reach an outside viewer. However, when quantum effects are included, a minimal quantity of energy may be emitted beyond the BH's spacetime. Prior studies have established that the temperature of a BH can be characterised by the radiation it produces. Simultaneously, BHs may be considered thermodynamic systems characterized by the Hawking temperatures. The thermodynamic principles governing BHs as thermal systems are formulated through the concepts of temperature and entropy. The conserved quantities of the BH thermodynamic system, including charge, angular momentum, and thermodynamic potential, are also defined on the event horizon in normal phase space. Since then, a great deal of research has been done on the effects of Hawking radiation on the thermal characteristics of BHs in particular. [5–15].

We now examine the extended thermodynamics of the BH solution, in which the thermodynamic pressure (P) is considered as the cosmological constant (Λ). [16–19]. By treating cosmological constant (Λ) as a dynamic thermodynamic variable, a new thermodynamic quantity, specifically the thermodynamic pressure and volume of the BH, has been incorporated into the rules of BH thermodynamics ($dM = TdS + PdV + \Phi dQ$). This theory has led to a lot of interesting discoveries, like the van der Waals phase transition of the BH thermodynamic system. It has also led to a lot of useful results. More research needs to be done on the thermodynamic phase transition of a group of charged BHs [20–39]. It has been generally agreed upon since then that a BH is just like any other thermodynamic system.

In this paper, we extend our work to study the thermodynamics of the AdS regular BH solution. The regular BH model was first proposed by Bardeen based on Gliner and Shakarov's proposal [40,41]. Although the thermodynamics and extended phase transition of the exact BH coupled with a nonlinear electrodynamics (NLED) are discussed in [42–53]. Now there are many regular BH solutions in modified gravities based on the Bardeen Proposal and studies of the properties of BHs [54–62]. The weak cosmic supervision theory and the thermodynamic laws of BHs are still true in this expanded phase space, which is very useful in real life. This paper changes the first rule of BH thermodynamics, which says that $dM = TdS + PdV + \Phi dq + kdK$.

The rest of paper is arranged as follows. In Section 2, we briefly review the construction and structure of a BH coupled with NLED and a cloud of strings, including the horizon structure of the BH solution. Then we discuss extended thermodynamics, $P - v$ criticality, and phase transition in Section 3 and Section 4 is devoted to study for $p - v$ criticality and phase transition. Section 5 is devoted to the study of the holographic phase transition. Finally, we conclude with the results and discussion in Section 6.

2 Regular black hole solution

The action of a BH coupled with NLED in AdS space-time can be written as

$$S = \frac{1}{2}\int d^4x\sqrt{-q}[R - 2\Lambda - 2L(F)], \tag{2.1}$$

where R is ricci scalar and Λ is cosmological constant. The $L(F)$ corresponds to matter Lagrangian which depends on $F = \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$. The variation action (2.1) with respect to $q_{\mu\nu}$ and the potential A_a, we obtained the following equation of motion:

$$R_{\mu\nu} - \frac{1}{2}q_{\mu\nu}R + \Lambda q_{\mu\nu} = T_{\mu\nu} \equiv 2\left[\frac{\partial L(F)}{\partial F}F_{\mu\sigma}F^{\sigma}_{\mu} - \tilde{q}_{\mu\nu}L(F)\right], \tag{2.2}$$

$$\nabla_{\mu}\left(\frac{\partial L(F)}{\partial F}F^{\mu\nu}\right) = 0 \quad and \quad \nabla_{\mu}\left(*F^{\mu\nu}\right) = 0, \tag{2.3}$$

where $F_{\mu\nu}$ is Maxwell's field-strength tensor

$$F_{\mu\nu} = 2\delta^{\theta}_{[\mu}\delta^{\phi}_{\nu]}Y(r,\theta). \tag{2.4}$$

It is essential to emphasize that spherically symmetric solutions exist with a globally regular metric for gravity coupled to NLED characterized by the Lagrangian $L(F)$, which exhibits an accurate weak field limit just in the magnetic scenario. From Eq. (2.4), the integration of Eq. (2.2) yields

$$F_{\mu\nu} = 2\delta^{\theta}_{[\mu}\delta^{\phi}_{\nu]}Y(r,\theta) = 2\delta^{\theta}_{[\mu}\delta^{\phi}_{\nu]}q\sin\theta. \tag{2.5}$$

In this context, q is a constant (independent of r) and is validated using the exterior derivative of the differential 2-form (2.4). The field-strength tensor and, hence, the matter Lagrangian can be streamlined to

$$F_{\theta\phi} = q\sin\theta, \quad F = \frac{1}{2}\frac{q^2}{r^4}, \tag{2.6}$$

and Lagrangian density of NLED is

$$L(F) = \frac{e^{-s(2q^2F)^{1/4}}}{q^2}\left(\frac{F}{q}\right)^{1/3}\left[\frac{3(2q^2F)^{1/3}}{(1+(2q^2F)^{1/3})^2} + \frac{s(2q^2F)^{1/4}}{(1+(2q^2F)^{1/3})}\right]. \tag{2.7}$$

The components of the energy-momentum tensor that are not zero are represented by the equations

$$T^t_t = T^r_r = \frac{6q^3e^{-k/r}}{(r^3+q^3)^2} + \frac{2ke^{-k/r}}{r(r^3+q^3)}. \tag{2.8}$$

Taking into consideration the following line element, we are able to construct a static spherically symmetric BH solution in four dimensions,

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2d\Omega^2_2, \tag{2.9}$$

where $f(r)$ is the matric and $d\Omega^2_2 = d\theta^2 + \sin^2\theta d\phi^2$ is the metric of a 2-dimensional sphere. Substituting the value of the energy-momentum tensor from Eq. (2.8) into Eq. (2.2), we get

$$rf'(r) + f(r) - 1 + \frac{6l^2}{r^2} = \frac{6q^3e^{-k/r}}{(r^3+q^3)^2} + \frac{2ke^{-k/r}}{r(r^3+q^3)}. \tag{2.10}$$

Solving the above field equation (2.10), we get

$$f(r) = 1 - \frac{2Mr^2}{r^3+q^3}e^{-k/r} + \frac{r^2}{l^2}, \tag{2.11}$$

where ($k = q^2/2M$) is the deviation parameter that measures the deviation from the Hayward BH solution [54] and interpolates with the AdS regular BH in the absence of magnetic monopole charge ([55]). The BH solution (2.11) is characterised by the four parameters mass, magnetic monopole charge, deviation parameter (k) and cosmological constant ($\Lambda = -3/l^2$). The metric function $f(r) = 0$ determines the horizons of the BH solution (2.11). But this equation (2.11) is not solved analytically because it is a transcendental equation. In order to ascertain the horizons of the BH, we employ a numerical solution to the equation (2.11), and the graphs that result are presented in Figure (1)

Fig. (1) shows the horizon structure of the obtained BH solution (2.11). The derived BH solution (2.11) features two horizons (Cauchy and event) associated with the constant values of the deviation parameters (k) and deviation parameter (q), as well as the mass (M) and AdS length ($l = 10$). The Cauchy and event horizons coincide at the critical deviation parameter ($k = 0.74$) with a fixed value of $q(= 0.1)$ and the critical magnetic monopole charge ($q = 0.98$) with fixed values of $k(= 0.1)$ (see the Tab. 1). The horizon size increases as the magnetic monopole charge ((q)) and deviation parameter decrease (k).

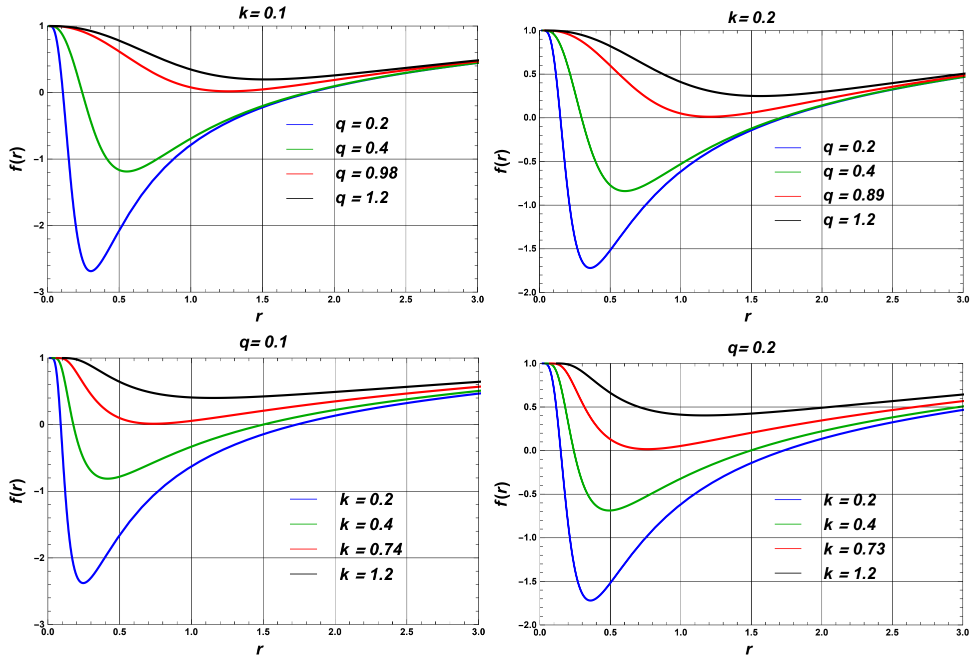

Figure 1: Plot of the metric function of ($f(r)$) vs horizon radius (r) for various values of q with a constant $k = 0.1$, and for various values of k with a fixed $q = 0.1$.

3 Extended Thermodynamics and Phase Transition

This section examines the thermal parameters, including temperature, entropy, heat capacity, and Gibbs free energy, of the AdS regular BH (2.11) as a function of the horizon radius (r_+). The mass of the AdS regular BH (2.11) is determined in relation to its horizon radius

Table 1: The table presents the horizon radius (r) for various values of q with fixed values of $k = 0.1$ and $k = 0.2$, as well as different values of k with fixed $q = 0.1$ and 0.2.

$k = 0.1$				$k = 0.2$			
q	r_+	r_-	δ	q	r_+	r_-	δ
0.2	1.84	0.1	1.74	0.2	1.73	0.15	1.58
0.4	1.84	0.25	1.59	0.4	1.72	0.30	1.42
0.98	1.30	1.30	0.00	0.98	1.21	1.21	0.00
$k = 0.1$				$k = 0.2$			
k	r_+	r_-	δ	k	r_+	r_-	δ
0.2	1.72	0.15	1.57	0.2	1.72	0.09	1.63
0.4	1.50	0.19	1.31	0.4	1.50	0.22	1.28
0.74	0.72	0.72	0.00	0.73	0.78	0.78	0.00

by solving the equation ($f(r) = 0$). It provides

$$M_+ = \frac{q^3 + r_+^3}{2}\left(\frac{1}{r_+^2} + \frac{1}{l^2}\right)e^{\frac{k}{r_+}}. \tag{3.1}$$

The mass of the derived BH solution (2.11) simplifies to the mass of the *AdS* Hayward BH ($M_+ = (q^3 + r_+^3)(r_+^2 + l^2)/2r_+^2 l^2$) [54] in the absence of the deviation parameter and to the mass of the *AdS* regular BH ($M_+ = r_+(r_+^2 + l^2)e^{k/r_+}/2l^2$) [55] when the magnetic monopole charge is deactivated. The mass (3.1) of the BH coincides with that of the *AdS* Schwarzschild BH ($M_+ = r_+(r_+^2 + l^2)/2l^2$) when there is no deviation parameter and magnetic monopole charge present.

The temperature of a BH, referred to as the Hawking temperature, is determined using the following equation,

$$T = \frac{1}{2\pi}\sqrt{-\frac{1}{2}\bigtriangledown_\mu \xi_\nu \bigtriangledown^\mu \xi^\nu} = \frac{\kappa}{2\pi}, \tag{3.2}$$

where ξ^ν is the killing vector and κ is the surface gravity related to the temperature by relation $T = \kappa/2$. The temperature of the obtained BH solution becomes

$$T_+ = \frac{f'(r)}{4\pi} = \frac{1}{4\pi}\left(\frac{3r_+^4}{l^2(q^3 + r_+^3)} - \frac{k(l^2 + r_+^2)}{l^2 r_+^2} + \frac{r_+^3 - 2q^3}{r_+(q^3 + r_+^3)}\right). \tag{3.3}$$

The temperature of the derived BH solution (2.11) converges to the temperature of the *AdS* Hayward BH ($T_+ = \frac{3r_+^5 + l^2(r_+^3 - 2q^3)}{4\pi r_+ l^2(q^3 + r_+^3)}$) [54] when the deviation parameter is absent, and the temperature of the *AdS* regular BH ($T_+ = \frac{3r_+^3 - k(r_+^2 + l^2)}{4\pi r_+^2 l^2}$) [55] when the magnetic monopole charge is deactivated. Additionally, the temperature (3.3) reduces to that of the *AdS* Schwarzschild BH ($T_+ = \frac{3r_+^2 + l^2}{4\pi r_+ l^2}$) in the absence of both the deviation parameter and magnetic monopole charge. Figure 2 illustrates the relationship between temperature and horizon radius for various values of q at a constant value of $k = 0.1$, as well as for varied values of k with a fixed $q = 0.1$. The numerical data is presented in the Table 2.

The thermodynamic properties related to the BH (2.11) must adhere to the first law of thermodynamics.

$$\mathrm{d}M = T\mathrm{d}S + \phi\mathrm{d}q + kdK. \tag{3.4}$$

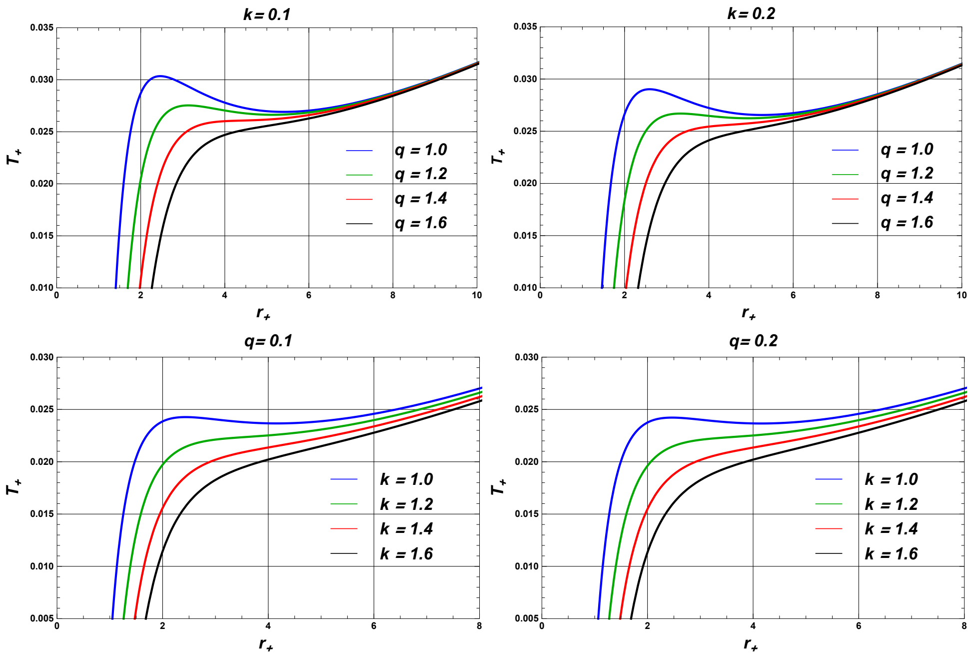

Figure 2: Plot of temperature T_+ vs horizon radius r_+ for various values of q with constant values of $k = 0.1$ and various values of k with fixed $q = 0.1$

Table 2: The table for temperature T_+ for various values of q with constant values of $k = 0.1$, & 0.2 and various values of k with fixed $q = 0.1$, & 0.2

$k = 0.1$			$k = 0.2$			$q = 0.1$			$q = 0.2$		
q	r_{1+}	r_{2+}	q	r_{1+}	r_{2+}	k	r_{1+}	r_{2+}	k	r_{1+}	r_{2+}
1.0	2.381	5.437	1.0	2.413	5.482	1.0	2.235	4.852	1.0	2.539	5.381
1.2	3.025	5.184	1.2	2.971	5.482	1.2	2.640	4.852	1.2	3.072	5.381
1.4	3.300	5.736	1.4	3.402	5.431	1.4	2.925	4.852	1.4	3.427	5.381
1.6	4.449	5.713	1.6	3.986	5.406	1.6	3.127	4.852	1.6	3.732	5.381

Given the explicit expressions for mass and Hawking temperature, the first law yields the entropy value as

$$S = \int \frac{\mathrm{d}M}{T}\mathrm{d}r = \pi(e^{k/r_+}(-2q^3k^{-1} + r_+k + r_+^2) - k^2 Exp\left[\frac{k}{r_+}\right]. \tag{3.5}$$

As we know, when heat capacity becomes negative, the system is unstable ($C_+ < 0$) and stable when the system has positive heat capacity ($C_+ > 0$). The plot illustrates the graph of heat capacity (C_+) against horizon radius (r_+) for various values of q with a constant value of $k = 0.1$, and in another plot, for varied values of k while maintaining a fixed $q = 0.1$. The formula for heat capacity at constant pressure is expressed as

$$C_+ = \frac{\partial M_+}{\partial T_+} = \left(\frac{\partial M_+}{\partial r_+}\right)\left(\frac{\partial r_+}{\partial T_+}\right). \tag{3.6}$$

By substituting the mass from Eq. (3.1) and the temperature from Eq. (3.3) into Eq. (3.6),

Table 3: The numerical values of horizon radius for various values of q with constantvalues of k and various values of k with fixed q.

$q=0.1$			$q=0.2$			$k=0.1$			$k=0.2$		
k	r_{1+}	r_{2+}	k	r_{1+}	r_{2+}	q	r_{1+}	r_{2+}	q	r_{1+}	r_{2+}
0.4	0.837	4.838	0.4	0.940	4.859	0.4	0.981	5.166	0.4	1.083	5.043
0.7	1.514	4.387	0.7	1.555	4.387	0.7	1.658	5.084	0.7	1.740	4.982
0.9	2.109	3.997	0.9	2.130	3.935	0.9	2.191	4.941	0.9	2.314	4.818
1.0	2.561	3.546	1.0	2.622	3.545	1.0	2.458	4.838	1.0	2.622	4.735

we obtain

$$C_+ = -\frac{2\pi\left(q^3+r_+^3\right)^2 e^{k/r_+}\left(q^3\left(k\left(l^2+r_+^2\right)+2l^2r_+\right)r_+^3\left(k\left(l^2+r_+^2\right)-r_+\left(l^2+3r_+^2\right)\right)\right)}{r_+\left(2q^6l^2(k+r_+)+2q^3r_+^3\left(2kl^2+5l^2r_++6r_+^3\right)+r_+^6\left(2kl^2-l^2r_++3r_+^3\right)\right)}. \tag{3.7}$$

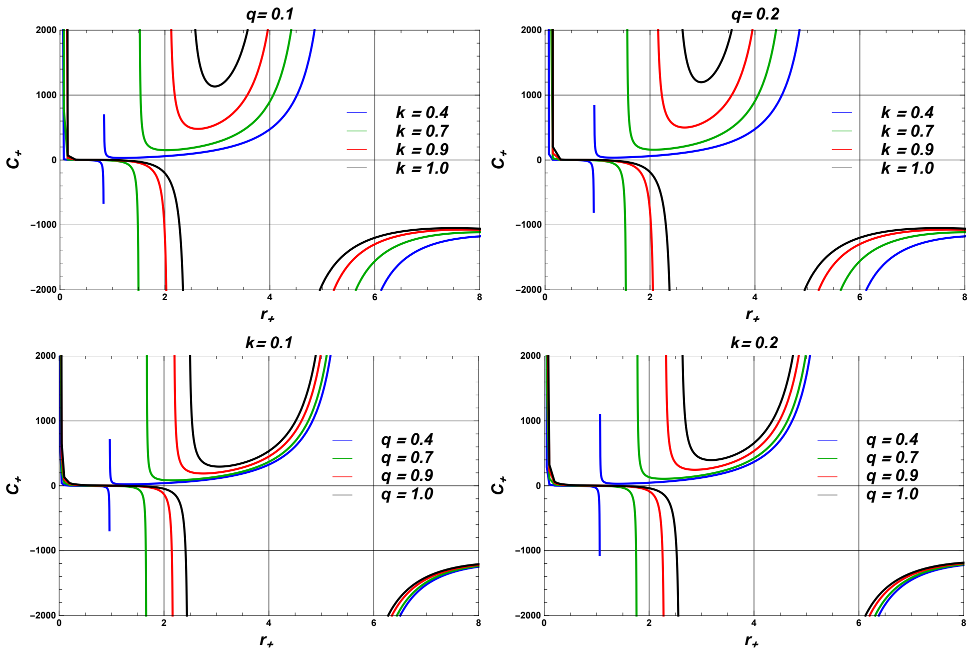

Figure 3: Plot of heat capacity (C_+) vs. horizon radius (r_+) for various values of q with constant values of $k=0.1$ and various values of k with fixed $q=0.1$

The global study of BHs is studied by using the Gibbs free energy. When the Gibbs free energy becomes negative, the system is stable ($G_+<0$), and unstable when the system has positive Gibbs free energy ($G_+>0$). It is calculated as

$$G_+ = M_+ - T_+S_+. \tag{3.8}$$

We obtain the Gibbs free energy of the AdS regular BH solution by substituting the mass from equation (3.1) and the temperature from equation (3.3) into equation (3.8).

$$G_+ = \frac{e^{k/r_+}(l^2+r_+^2)(q^3+r_+^3)}{2l^2 r_+^2} - \frac{2r_+^3(q^3+r_+^3)}{4kl^2 r_+^2(q^3+r_+^3)} - \frac{(l^2+r_+^2)((k-r_+)r_+^3+q^3(k+2r_+))(e^{k/r_+}(-2q^3+kr_+(k+r_+))-k^3Ei[\frac{k}{r_+}])}{4kl^2r_+^2(q^3+r_+^3)}. \tag{3.9}$$

The graph of free energy, G_+, as a function of horizon radius r_+ for various values of q, maintaining a constant value of $k = 0.1$. Additionally, another plot displays different values of k while keeping q fixed at 0.1.

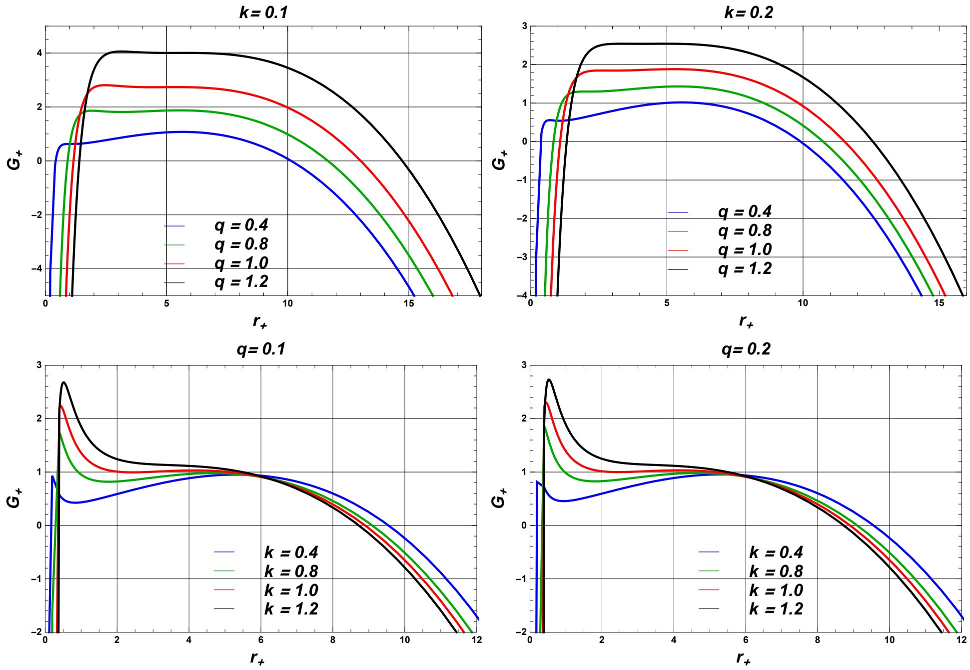

Figure 4: Plot of Gibbs energy G_+ vs horizon radius r_+ for different values of q with fixed values of $k = 0.1$ and different values of k with fixed $q = 0.1$

4 P-v Criticality and Phase Transition

This section examines the $P - v$ criticality and phase transition of the AdS regular BH solution (2.11) within the expanded phase space. In the expanded phase space, the cosmological constant functions analogously to thermodynamic pressure ($\Lambda = -3/l^2 = -8\pi P$). The pressure of the resultant solution is determined using this relation (3.3).

$$P_+ = -\frac{3\left(g^3k+4\pi g^3 r_+^2 T+2g^3 r+kr_+^3+4\pi r_+^5 T-r_+^4\right)}{8\pi r_+^2\left(g^3k+kr_+^3-3r_+^4\right)}. \tag{4.1}$$

The critical radius (r_c), critical temperature (T_c), and critical pressure (P_c) can be determined using the notion of an inflection point.

$$\left(\frac{\partial P_+}{\partial r_+}\right)_T = \left(\frac{\partial^2 P_+}{\partial r_+^2}\right)_T = 0. \tag{4.2}$$

Utilizing the aforementioned conditions, we get an equation,

$$60q^6 r + 84q^3 r^4 - 3r^7 + 9k(8q^6 + 3q^3 r^3 + r^6) = 0. \tag{4.3}$$

By solving the above equation, you can find the critical points and the horizon radius. You can't solve this problem analytically, but you can use numbers to find the critical radius r_c, critical pressure P_c, and critical temperature T_c. $T_c = 0.199333$ $P_c = 0.082022$ and now we plot a graph of pressure,P with horizon radius r_+ for different values of critical temperature

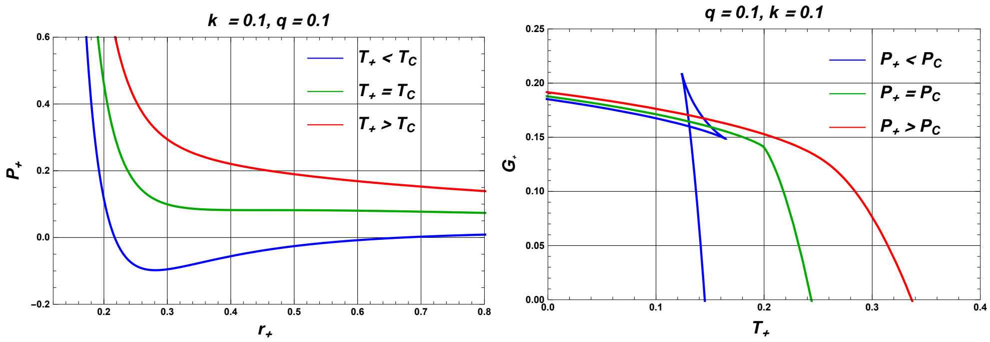

Figure 5: Plot of pressure P_+ vs horizon radius r_+ for various values of T_+ with fixed values of $k = 0.1$ and $g = 0.1$ and plot of Gibbs free energy (G_+) vs. temperature (T_+) for various values of P_+ with constant value of $q = 0.1$ and $k = 0.1$.

For each pressure number ($P < P_c, P = P_c, \text{and}, P > P_c$), we also made a graph of the Gibbs free energy versus the BH's temperature. Below the critical pressure, it behaves like a swallowtail (first-order phase transition); the first-order phase transition terminates at $P = P_c$, and there is no phase transition when $P > P_c$.

5 Holographic Phase Transition

In order to investigate the phase structure from a holographic perspective, we must first make the observation that the temperature of a specific BH is a function of the entropy of the BH, wWhich is put in writing as

$$T_+ = \frac{3S^3 - k\sqrt{\pi}(\pi l^2 + S)(\pi^{3/2}q^3\sqrt{S} + S^3) - \pi l^2(2\pi^{3/2}q^3 + S^2)}{4l^2\pi^{3/2}S(\pi^{3/2}q^3 + S^{3/2})}. \tag{5.1}$$

By applying the following relation, we are able to determine the critical radius, critical temperature, and critical entropy by making use of the attributes of the inflexion points.

$$\left(\frac{\partial T_+}{\partial S_+}\right) = \left(\frac{\partial^2 T_+}{\partial S_+^2}\right) = 0. \tag{5.2}$$

The scalar isocharges are plotted in the $T_+ - S_+$ plane in Figure 6, where the value of q_c is equal to 0.490. According to the relationship between AdS and CFT, Ryu and Takayanagi [63,64] provided a sophisticated method for calculating the holographic entanglement entropy. This method is represented by the following relation:

$$S_+ = \frac{\text{Area of horizon}}{4G}. \tag{5.3}$$

The entropy caused by holographic entanglement can be expressed in the following form: [65,66]

$$S_+ = \pi \int_0^{\phi_0} r^2 \sin^2 \phi \sqrt{r^2 + \frac{1}{f(r)} \left(\frac{dr}{d\phi}\right)^2} d\phi. \tag{5.4}$$

As the entangling surface, we chose the values of $\phi_0 = 0.15$, 0.49, and 0.60. In this case, we considered $\phi = \phi_0$ to be the encompassing surface.

With the boundary conditions $r(0) = 0$ and $r(0) = r_0$, we are able to obtain the numerical result of $r(\phi)$. We integrate S in Equation (5.4) once more up to a cut-off, which is somewhat near to ϕ_0, and then remove the pure AdS entanglement entropy (denoted by S_+) with the same entangling surface ϕ_0 at the boundary. This is done in order to regularize the entanglement entropy through the process of regularization. The regularized entanglement entropy that corresponds to this situation is represented by the equation $S_+ = S_+ - S'_+$. The plot of the $T_+ \delta S_+$ plane under the condition of $\phi_0 = 0.15$ is shown in Figure 7. The dotted curve represents the plane.

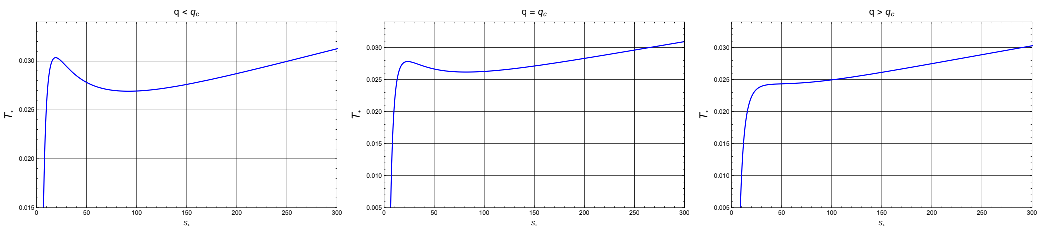

Figure 6: The plot of T_+ vs. S_+. Here we set the parameter $k = 0.1$, $l = 10$ and the critical value of magnetic monopole charge $(q_c) = 0.0490$

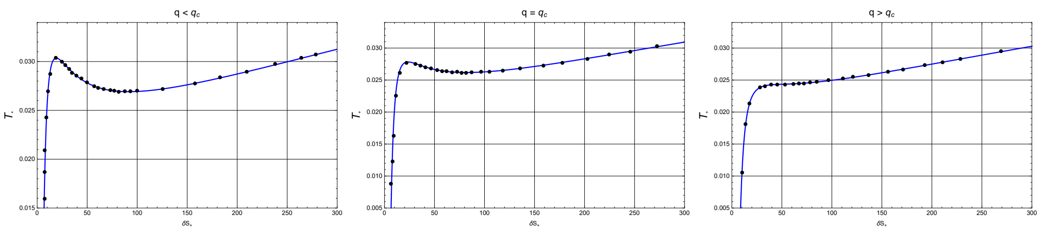

Figure 7: This is the plot of T_+ versus S_+. The parameters $k = 0.1$, $l = 10$, $\phi_0 = 0.15$, and $\phi_c = 0.0490$ are all set to their respective values below.

Furthermore, via the utilization of comparison analysis, as illustrated in Figure 7, we have discovered that the entanglement entropy also exhibits a phase transition that is similar to that of Van der Waals.

6 Results and conclusion

We have examined the regular BH solution in AdS spacetime. Our approach interpolates to the AdS Hayward BH when $k = 0$, the AdS regular BH when $q = 0$, and the AdS Schwarzschild BH in the limit as $q = 0$, $k = 0$, and $a = 0$. The derived BH solution features a dual-horizon distinction compared to the Schwarzschild BH. The curvature scalars do not diverge as $r \to 0$, indicating that the derived solution is non-singular. Additionally, we have analyzed the thermodynamics of the AdS regular BHs by examining thermodynamic parameters such as temperature, entropy, heat capacity, and Gibbs free energy. The local and global stability are examined by analyzing the heat capacity and Gibbs free energy diagrams. The heat capacity of the derived BH solution becomes infinite at the maximum temperature.

Furthermore, we examined the phase transition of the AdS regular black hole solution, considering the Cosmological constant (Λ) as the thermodynamic pressure ($P = -\Lambda/8\pi$). The BH has been categorized into three phases based on the pressure in relation to the critical pressure. Small stable BHs have been detected at pressures below the critical pressure ($P < P_c$). For pressures exceeding the critical pressure ($P > P_c$), substantial stable BHs were present, however in the critical pressure, intermediate unstable BHs developed. This classification of phases demonstrated the intricacy of the BH phase structure under differing pressure conditions.

Authors' Contributions

All authors have the same contribution.

Data Availability

The manuscript has no associated data or the data will not be deposited.

Conflicts of Interest

The authors declare that there is no conflict of interest.

Ethical Considerations

The authors have diligently addressed ethical concerns, such as informed consent, plagiarism, data fabrication, misconduct, falsification, double publication, redundancy, submission, and other related matters.

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