Journal of Holography Applications in Physics
Volume 5, Issue 4, Autumn 2025, 95–115
©Available online at http://jhap.du.ac.ir
DOI: 10.22128/jhap.2025.3054.1134
Online ISSN: 2783–3518

Regular article

Holographic Dark Energy Models in FRW Universe from Parametrization of q with $f(Q,T)$ Gravity

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Received: September 09, 2025; **Accepted:** October 2, 2025

Abstract. In this paper, we study the Holographic Dark Energy (HDE) cosmological models within the $f(Q,T)$ gravity framework. Here, Q and T represent the non-metricity scalar and energy-momentum tensor trace, respectively. In order to find the solutions in the Friedman-Robertson-Walker model, we use the deceleration parameter $q(z)$ and describe the transiting universe evolution and the Hubble parameter. We obtain the constraints on model parameters using Markov Chain Monte Carlo (MCMC) analysis with the supernovae type Ia observations from the Pantheon sample. We further investigate the cosmological parameters like the energy density, equation of state parameter, and classical stability parameter in terms of redshift with the physically plausible $f(Q,T) = \mu Q + \nu T$ form. We investigate three HDE models in this framework with different IR cutoffs. The distinct cosmological evolution scenarios have been studied with the cosmographic parameters.

Keywords: Universe; Observations; FRW; Holographic Dark Energy

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Contents

1 Introduction

The observational data from distant Ia supernovae, large scale structure, and cosmic microwave background [1–4] point to an accelerated expansion of the present universe. It suggests that there exist a new component in the universe named as 'dark energy' with negative pressure. The dark energy constitutes nearly 69% of the total energy density of the universe today. The simplest candidate to explain the dark energy in the General Relativity model is the cosmological constant (Λ) having equation of state $\omega = -1$. Although the cosmological constant offers a straightforward explanation for dark energy, it encounters two major difficulties: "the cosmological constant problem" [5] and "the cosmic coincidence problem" [6,7]. As alternatives to this constant dark energy model, the dynamical models of dark energy such as the 'quintessence' (with $-\frac{1}{3} > \omega > -1$) and the more exotic 'Phantom' scenario (with $\omega < -1$) have been proposed [8]. Furthermore, to overcome the challenges linked with the cosmological constant and Quintessence, particularly those arising from negative pressure, the Chaplygin gas may be useful for describing dark energy.

The holographic principle, initially proposed by 't Hooft [9] in the context of black hole physics, has led to one of the prominent dark energy models known as holographic dark energy (HDE) [10–14]. The holographic principle emphasizes that black hole entropy is determined by the surface area of the horizon, rather than by the enclosed volume. Granda and Oliveros [15,16] proposed a new infrared cutoff for the HDE density, showing that it can account for the accelerated expansion of the universe and remains consistent with current observational data. The HDE model has been extensively studied to describe late time accelerating stage of the universe [17–21]. By employing an extended horizon entropy relation, Barrow [22] recently formulated a new HDE model. Alternatively, Tsallis entropy [23–25] provides a generalized framework of the conventional Bekenstein entropy.

The HDE models in the modified gravity frameowrk may describe the universe evolution dominated by dark energy during the late-times [18,20]. The varying dark energy in these models may be compatible with the observations. In the present work, we investigate HDE models within the framework of $f(Q,T)$ gravity using classical stability criteria, considering different infrared (IR) cutoffs such as the Granda–Oliveros (GO) cutoff and the Modified Ricci Radius cutoff [26–31]. We aim to obtain the varying dark energy scenario in the HDE models and test its compatibility with the observational data in the present study. In order to do the study on varying dark energy scenario, we investigate the Friedmann– Robertson–Walker (FRW) model within the framework of $f(Q,T)$ gravity by considering a specific form of the deceleration parameter $q(z)$. The deceleration parameter plays a crucial role in characterizing the universe's evolution, and the validity of its chosen form can be assessed through observational data. In particular, constraints on the model parameters are derived using Type Ia supernova observations. Furthermore, we determine the best-fit values of the parameters by applying the MCMC algorithm to the data sample [32].

The manuscript is divided into 5 different sections: In the Section 2, we summarize the field equations of the $f(Q,T)$ gravity. The Section 3 deals with the basic information on the parametrized form of deceleration parameter q in terms of the redshift z and analyze the Hubble parameter using observational data. We study three distinct models of HDE in $f(Q,T)$ gravity framework in the Section 4. We also examine the cosmological parameters, namely the energy density ρ, the equation of state (EoS) parameter, and the classical stability parameter, to assess the compatibility of these models with the standard cosmological model. The Section 5 presents the summary of results.

2 The field equations in $f(Q,T)$ gravity

The $f(Q,T)$ gravity [27] may be visualized as an extended version of the symmetric teleparallel gravity [33,34]. This theory is composed of the gravitational action defined in terms of the energy-momentum tensor trace (T) and the non-metricity scalar (Q). This theory belongs to the non-conservative class of modified theories. Introducing a coupling between Q and T in HDE scenarios may lead to interesting results.
The modified Einstein-Hilbert action in the $f(Q,T)$ gravity theory equipped with the matter Lagrangian $\mathcal{L}_m$ may take the form [27]

$$S=\int\left(\frac{1}{16\pi}f(Q,T)+\mathcal{L}_m\right)\sqrt{-g}d^4x, \tag{2.1}$$

where g represents the determinant of metric tensor g_{ij} and $f(Q,T)$ denotes a general function of T and Q. In this theory, the non-metricity tensor and non-metricity scalar may respectively be given by

$$Q_{\mu ij}\equiv\nabla_\mu g_{ij},\quad Q\equiv-g^{ij}\left(L^\mu_{\nu i}L^\nu_{j\mu}-L^\mu_{\nu\mu}L^\nu_{ij}\right). \tag{2.2}$$

In this theory, the universe evolution will be governed by the field equation derived from the action (2.1) as [27]

$$L^\mu_{\nu\gamma}=-\frac{1}{2}g^{\mu\lambda}\left(\nabla_\gamma g_{\nu\lambda}+\nabla_\nu g_{\lambda\gamma}-\nabla_\lambda g_{\nu\gamma}\right). \tag{2.3}$$

$$-\frac{2}{\sqrt{-g}}\nabla_\mu\left(f_Q\sqrt{-g}\mathcal{P}^\mu_{ij}\right)-\frac{1}{2}g_{ij}f+f_T\left(T_{ij}+\Theta_{ij}\right)-f_Q\left(\mathcal{P}_{i\mu n}Q_j^{\mu n}-2Q_i^{\mu n}\mathcal{P}_{\mu nj}\right)=8\pi T_{ij}, \tag{2.4}$$

where $f_Q\equiv\frac{\partial f}{\partial Q}, f_T\equiv\frac{\partial f}{\partial T}$, the energy-momentum tensor is $T_{ij}=-\frac{2}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}\mathcal{L}_m\right)}{\delta g^{ij}}$ and $\Theta_{ij}=g^{\mu\nu}\frac{\delta T_{\mu\nu}}{\delta g^{ij}}$. In $f(Q,T)$ theory, the super-potential is given by [27]

$$\mathcal{P}^\mu_{ij}=\frac{1}{4}\left(-Q^\mu_{ij}+2Q^\mu_{ij}+Q^\mu g_{ij}-\tilde{Q}^\mu g_{ij}-\delta^\mu_{(i}Q_{j)}\right). \tag{2.5}$$

In the above equation, the trace of the non-metricity tensor may be expressed as

$$Q_\mu=Q^i_{\mu i},\quad \tilde{Q}_\mu=Q^i_{\mu i}. \tag{2.6}$$

By varying the gravitational action (2.1) with respect to the connection, the field equations can be written as [27]

$$\nabla_i\nabla_\mu\left(\sqrt{-g}f_Q\mathcal{P}_j^{i\mu}+4\pi H_j^{i\mu}\right)=0, \tag{2.7}$$

where the hyper-momentum tensor density given by $H^{ij}_\lambda=\frac{\sqrt{-g}}{16\pi}f_T\frac{\delta T}{\delta\tilde{\Gamma}^\lambda_{ij}}+\frac{\delta\sqrt{-g}\mathcal{L}_m}{\delta\tilde{\Gamma}^\lambda_{ij}}$.

In order to obtain solutions of the field equations within the $f(Q,T)$ gravity, appropriate simplifications should be made. In this study, we take the homogeneous and isotropic FRW metric having a flat-spatial section as

$$ds^2=-dt^2+a^2(t)\delta_{ij}dx^idx^j, \tag{2.8}$$

where the scale factor is denoted by $a(t)$. By using this quantity, one may derive the expansion rate of the universe as $H(t)\equiv\frac{\dot{a}}{a}$. The time derivative is denoted by the overhead dot. We may also have the relation between non-metricity scalar and Hubble parameter as $Q = 6H^2$. Within the FRW background, the field equations are given by [27]

$$(f/2) - 6FH^2 - \frac{2\tilde{G}}{1+\tilde{G}}\left(F\dot{H} + \dot{F}H\right) = 8\pi\rho, \tag{2.9}$$

$$-(f/2) + 6FH^2 + 2\left(F\dot{H} + \dot{F}H\right) = 8\pi p, \tag{2.10}$$

where $F = f_Q$ and $\tilde{G} = \frac{f_T}{8\pi}$. By adding (2.9) and (2.10), one can obtain

$$\dot{H} + \frac{\dot{F}}{F}H = \frac{4\pi}{F}\left(1+\tilde{G}\right)(\rho + p). \tag{2.11}$$

We consider the EoS, $p = (\gamma - 1)\rho$, where ρ represents the energy density, and whereas p designate cosmic fluid pressure. By solving equations (2.9) and (2.11), we get an equation of matter density as

$$\rho = \frac{f - 12FH^2}{16\pi(1+\gamma\tilde{G})}. \tag{2.12}$$

By defining the EoS parameter $\omega \equiv \gamma - 1$, the phantom kind of dark energy evolution may be visualized by $\omega < -1$, whereas, for $-1 < \omega < -\frac{1}{3}$, one may have the quintessence kind of dark energy.

3 The evolution of deceleration parameter (q) and the observational constraints

In the present work, we proceed with the functional form $f(Q,T) = \mu Q + \nu T$ [27], where μ and ν are the constants. Here we take $\mu = F = f_Q$, $\nu = f_T$, which we use as the free-constant parameters in the model as per the requirements. The negative and positive of these parameters will indicate decreasing and increasing behavior of f with Q and T respectively. We also get

$$Q = 6H^2, \quad T = \rho - 3p. \tag{3.1}$$

The equation (3.1) and $p = (\gamma - 1)\rho$ would yield

$$f = 6\mu H^2 + (4 - 3\gamma)\nu\rho. \tag{3.2}$$

By using equation (3.2) in (2.12), we get

$$\rho = \frac{-6\mu H^2}{[2(8\pi + \gamma\nu) + \nu(4-3\gamma)]}, \tag{3.3}$$

$$p = \frac{-6\mu H^2(\gamma - 1)}{[2(8\pi + \gamma\nu) + \nu(4-3\gamma)]}. \tag{3.4}$$

The deceleration parameter plays a vital role to describe the expanding behavior of the universe. Initially, the expansion of the universe was slowing down due to the strong gravitational attraction between matter and radiation. However, after the expansion of the universe matter become more dispersed and the gravitational force weakened, altering the cosmic dynamics. The universe has now entered an accelerating phase, characterized by a negative q. Analyzing this transition is significant for understanding the basic mechanism

driving the universe's expansion in a cosmological model. The deceleration parameter, defined as $q = -\frac{\ddot{a}}{aH^2}$, can be used to describe the rate of expansion of the universe and can equivalently be expressed as $q = -1 + \frac{d}{dt}\frac{1}{H}$. The cosmological models can be constructed based on the deceleration parameter, as it directly depends on the derivatives of the scale factor and the Hubble parameter.
The parameterization of deceleration parameter q may have a significant impact on universe's expanding rate in a model. A parametric approach provides a practical framework for analyzing the transition from deceleration to acceleration in the universe, while improving the effectiveness of upcoming cosmological observations. Inspired by these facts, in the present work, we have chosen a special form of $q(z)$ which may provide a signature flip describing the deceleration to accelerated expansion phase. We proceed with the form [35,36]

$$q(z) = q_0 + q_1 \left(\frac{\ln(z+2)}{z+1} - \ln 2 \right), \tag{3.5}$$

where a is the scale factor, and q_0 and q_1 are dimensionless quantities that can be constrained through observational data. The relation between redshift z and the scale factor is given by $\frac{a_0}{a} = z + 1$, where in agreement with observational value $a_0 = 1$ is for the present-day universe, $0 < a < 1$ for the past, and $a > 1$in the late-time universe [37]. Related to redshift scale, $0 < z < \infty$ corresponds to the past, whereas $-1 < z < 0$ for the future universe and the present day universe marked by red shift z almost zero [37]. The logarithmic form of $q(z)$ corresponds to the divergence-free parametrization of the dark energy equation of state parameter. The simplest calculations of $q(z)$ form may provide limits on the following cases:

$$q(z) = \begin{cases} q_0 - q_1 \ln 2, & z \to \infty \text{ (in the early universe)} \\ q_0, & z = 0, \text{ (at the present)}. \end{cases}$$

The $H(z)$ and $q(z)$ may be represented in terms of z as

$$H(z) = H_0 \exp\left(\int_0^z \frac{q(z\prime) + 1}{z\prime + 1} dz\prime \right), \tag{3.6}$$

where H_0 indicates the current value of the Hubble parameter. On solving equations (3.5)-(3.6), $H(z)$ can be evaluated as

$$H(z) = H_0 2^{2q_1} (z+1)^m (z+2)^{\frac{-(z+2)q_1}{(z+1)}}, \tag{3.7}$$

here $m \equiv 1 + q_0 + q_1(1 - \ln 2)$. The equation $a = \frac{a_0}{z+1}$, for $a_0 = 1$ implies that

$$\frac{d}{dt} = \frac{dz}{dt}\frac{d}{dz} = -(1+z)H(z)\frac{d}{dz},$$

and it can be used to obtain the following equations

$$\dot{H} = -(z+1)H(z)\frac{dH}{dz}, \tag{3.8}$$

$$H'(z) = H(z) \left(\frac{\mu_1}{1+z} - q_1 \left[\frac{\ln(2+z)}{(1+z)} + \frac{1}{(z+1)} - \frac{(z+2)\ln(z+2)}{(1+z)^2} \right] \right), \tag{3.9}$$

where $'$ denotes the derivative with respect to z. Equations (3.3) and (3.4) in form of z can be written as

$$\rho = \frac{-6\mu H^2(z)}{[2(8\pi + \gamma\nu) + \nu(4 - 3\gamma)]}, \tag{3.10}$$

$$p = \frac{-6\mu H^2(z)(\gamma-1)}{[2\,(8\pi+\gamma\beta)+\nu\,(4-3\gamma)]}. \tag{3.11}$$

On differentiating equation (3.11) with respect to z, we get

$$\frac{dp}{dz} = \frac{-(\gamma-1)12\mu H'(z)H(z)}{[2\,(8\pi+\gamma\nu)+\nu\,(4-3\gamma)]}. \tag{3.12}$$

3.1 Observational constraints

In this section, we use the Cosmic chronometer [38] and Pantheon [39] dataset to test the compatibility of the Hubble parameter (3.7) with the observations.

Cosmic chronometer data: The cosmic chronometer (CC) data describes the model independent estimates of the present day Hubble parameter (H_0) based on the differential ages of the slowly evolving galaxies [38]. It consists of the 31 data points from the redshift range $0.07 < z < 1.965$. In the present analysis, we use the data compiled in [38] with the corresponding χ^2 function denoted by χ_c^2 as

$$\chi_c^2(\theta) = \sum_{i=1}^{31} \frac{(H_t(\theta,z_i) - H_o(z_i))^2}{\sigma_i^2}. \tag{3.13}$$

Here, H_t and H_o represent the theoretical and observational values of H with σ_i is denoting the observed value of errors in the corresponding H_o values. For the present parametric model (3.7), the parameter space is $\theta = \{H_0, q_0, q_1\}$. Pantheon data: The supernovae type Ia observations suggest that the universe expansion is accelerating [1,2]. We use the observations from supernovae type Ia composed of redshift (z), apparent magnitude ($m_{b_{obs}}$), and the corresponding error from the Pantheon sample [39]. We use the emcee package [32] to conduct the MCMC analysis. In order to constrain the model parameter, we use the theoretical apparent magnitude (m_b) in terms of the Hubble free luminosity distance $D_L(z)$. As a result, we define $\mathcal{M} \equiv M - 5\log_{10}\left(\frac{c/H_0}{\text{Mpc}}\right) + 25$ [36,40–43]. This parameter is a combination of the absolute magnitude (M) of the supernovae and present day Hubble parameter (H_0). We also constrain this parameter in the present analysis. We use the parameter $\mathcal{M}$ because H_0 and M cannot be estimated simultaneously. We define the χ^2 for the Pantheon data denoted by χ_p^2 as [40–43]

$$\chi_p^2 = \delta V_i {C_{ij}}^{-1} \delta V_j, \tag{3.14}$$

where ${C_{ij}}^{-1}$ is inverse of total covariance matrix and $\delta V_i = m_{b_{obs}} - m_b(z_i)$.
Corresponding to the Pantheon dataset, the parameter space is $\{q_0, q_1, \mathcal{M}\}$.
In this study, we use the Hubble parameter (3.7) to extract constraints on model parameters $\{q_0, q_1, H_0, \mathcal{M}\}$. For the CC dataset, we use following priors $65 < H_0 < 85$, $-4.0 < q_0 < 2.0$ and $-15.0 < q_1 < 3.0$. In the MCMC analysis with the CC dataset, we use 36 walkers and 30000 iterations. The corresponding χ^2_{min} value is 14.79. In this analysis, we find

$$H_0 = 67.66 \pm 0.83 \;\; km/s/Mpc, \quad q_0 = -0.461 \pm 0.058, \quad q_1 = -4.41 \pm 0.52.$$

We use the getdist python package [44] to plot Fig. (1). It describes the posterior phase space of parameters subjected to the CC data.

In the standard cosmological model, the Hubble parameter for the late-time universe is

$$H(z) = H_0\sqrt{\Omega_{m0}(1+z)^3 + \Omega_{\Lambda 0}},$$

where Ω_{m0} and $\Omega_{\Lambda 0}$ are matter and dark energy densities during the present time and it follows $\Omega_{m0} + \Omega_{\Lambda 0} = 1$. For this Λ cold dark matter (ΛCDM) model, we get χ^2_{min} value as 14.49 from the same dataset. We derive following constraints of ΛCDM model by CC dataset based on MCMC analysis as $H_0 = 67.8 \pm 3.0\ km/s/Mpc$ and $\Omega_{m0} = 0.318^{+0.066}_{-0.057}$. It describes the correctness of the derived results of the parametric model.

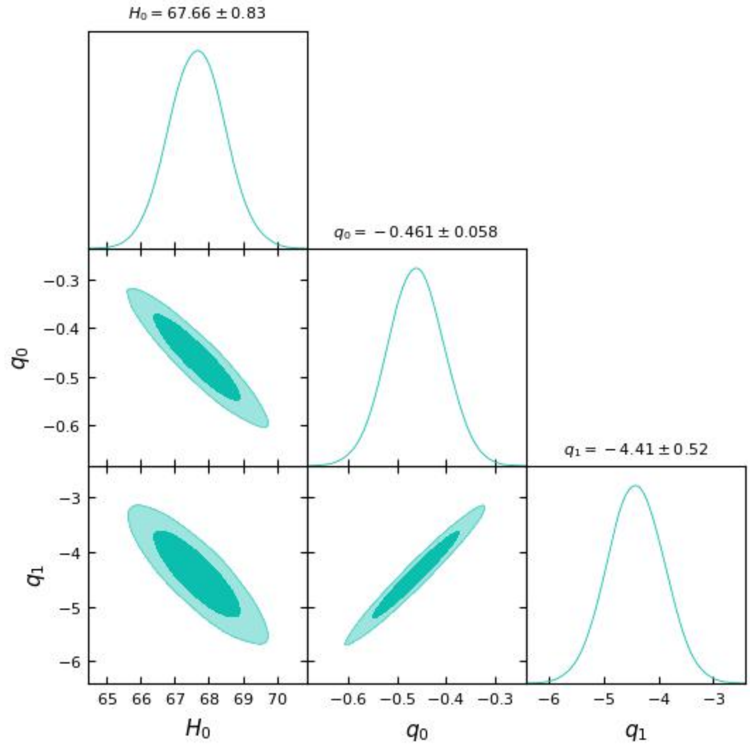

Figure 1: $1\sigma,\ 2\sigma$ contour plot for q_0, q_1 and H_0 by using CC data

In the analysis with Pantheon data, we use the following priors $-4 < q_0 < 2$ and $-15 < q_1 < 3$ and $23.0 < \mathcal{M} < 24.0$. In this analysis, we use 36 walkers and 12000 iterations. In this case, the χ^2_{min} value is 1026.32. We obtain

$$q_0 = -0.578 \pm 0.095, \quad q_1 = -5.9 \pm 2.1, \quad \mathcal{M} = 23.807 \pm 0.014.$$

The behavior of the posterior phase space of parameters subjected to the supernovae type Ia observations from the Pantheon sample has been given in Fig. 2. The posterior phase space of parameters subjected to the Pantheon data clearly demonstrates the convergence of chains in the MCMC analysis. We may use these constrained values to describe the universe's evolution in the further discussion.

For this Λ cold dark matter (ΛCDM) model, we get χ^2_{min} value as 1026.67 from the same dataset. We derive following constraints of ΛCDM model by CC dataset based MCMC analysis as $\Omega_{m0} = 0.301 \pm 0.022$ and $\mathcal{M} = 23.810 \pm 0.011$.

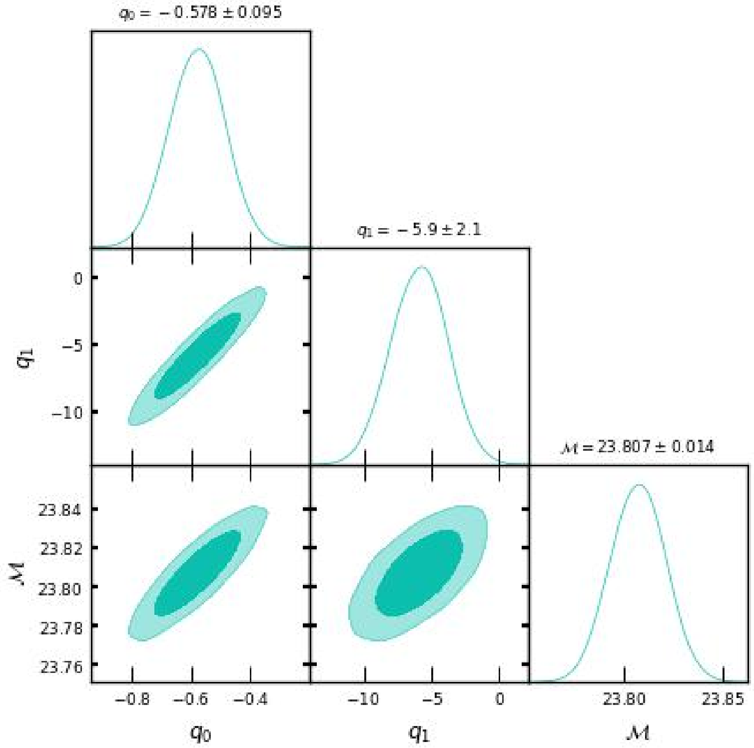

Figure 2: 1σ, 2σ contour plot for q_0, q_1 and $\mathcal{M}$ by using Pantheon data

We further derive the joint constraints on the model parameters using $\chi_t^2 = \chi_c^2 + \chi_p^2$. For the parameter space $\{H_0, q_0, q_1, \mathcal{M}\}$ using CC+Pantheon dataset, we obtain

$$H_0 = 68.8 \pm 1.9\ km/s/Mpc, q_0 = -0.557^{+0.072}_{-0.066}, \quad q_1 = -5.29 \pm 1.1, \quad \mathcal{M} = 23.806 \pm 0.015.$$

In this analysis, we use the same priors as those of Pantheon data set analysis and run 36 walkers with 12000 iterations.

4 HDE models with $f(Q,T)$ gravity

We study the evolution of HDE with different IR cut-offs in the $f(Q,T) = \mu Q + \nu T$ gravity model for the deceleration parameter q given by equation (3.5).

4.1 HDE model with Granda-Oliveros (GO) cut-off

The UV cut off is related to vacuum energy, and IR cut off is related to Large scale of Universe viz. Hubble Horizon, future event horizon or particle horizon.
The event horizon is a global concept of space-time. An event horizon is determined by the universe's future events and is present only if the universe continues to expand at an

accelerated rate indefinitely [45]. Influenced by this, Granda and Oliveros [15] put forward a new infrared cut-off for the HDE, by using the term proportional to the square of the Hubble scale and the time-dependent derivative of the Hubble scale. IR cut-off was introduced by Granda and Oliveros in terms of $\dot{H}$ and H^2. This IR cut-off is given by [15]

$$L_G = \left(\mu_2 \dot{H} + \nu_2 H^2(t)\right)^{\frac{-1}{2}}, \tag{4.1}$$

where μ_2 and ν_2 both are constant variables. ρ_G of HDE in the GO cut off model expressed as

$$\rho_G(t) = 3\left(\mu_2 \dot{H}(t) + \nu_2 H^2(t)\right). \tag{4.2}$$

Eq. (4.2), in form of z becomes

$$\rho_G(z) = 3\left[-\mu_2(z+1)H'(z)H(z) + \nu_2 H^2(z)\right]. \tag{4.3}$$

By using equations (3.11) and (4.3), the DE EoS parameter is defined as

$$\omega_d = \frac{-2\mu H^2(z)(\gamma-1)}{\left[2\left(8\pi+\gamma\nu\right)+\nu(4-3\gamma)\right]\left[-\mu_2 \quad (z+1)H(z)H'(z)+\nu_2 \quad H^2(z)\right]}. \tag{4.4}$$

On differentiating equation (4.3), we get

$$\frac{d\rho_G}{dz} = 3[-\mu_2(z+1)H''(z)H(z) - \mu_2(z+1)(H'(z))^2 + (2\nu-\mu_2)H'(z)H(z)]. \tag{4.5}$$

The classical stability parameter $C^2_{sG}(z)$ in this scenario may be calculated with the help of equations (3.12) and (4.5) as

$$C^2_{sG}(z) = \frac{(1-\gamma)4\mu H'(z)H(z)}{[-\mu_2(1+z)H''(z)H(z) - \mu_2(1+z)(H'(z))^2 + (2\nu-\mu_2)H'(z)H(z)]\,[2 \quad (8\pi+\gamma\nu)+\nu(2-3\gamma)]}. \tag{4.6}$$

The evolution of $\rho_G(z)$ and $C^2_{sG}(z)$ have been shown in Fig. (3) and (4). For this, we take the values $H_0 = 67.4, q_1 = -6.0, q_0 = -0.585, \mu_2 = -0.0001, \nu_2 = 10.0$. We have $\rho_G \geq 0$ which is due to $\frac{\dot{H}}{H^2} \leq -\frac{\nu_2}{\mu_2}$. The present choice of μ_2 and ν_2 gives $\frac{\dot{H}}{H^2} \geq 0$ for late time expansion it yields $q < 0$. The energy density $\rho_{RG}(z)$ remains positive, decreasing from relatively high values in the early universe ($z \geq 1$) to lower values at later times, as z approaches -1.

The model exhibits classical stability at the present epoch, and was stable in the past. The behavior of adiabatic sound speed with z is given in Fig. (4).

4.2 Modified holographic Ricci dark energy (MHRDE) model

The MHRDE model is a modified version of the original holographic Ricci dark energy model. This section focuses on the analysis of the HDE model in which the IR cut-off is defined by the modified Ricci radius. For the MHRDE model, energy density is expressed as

$$\rho_m(t) = \frac{2}{\mu_3 - \nu_3}\left(\dot{H}(t) + \frac{3\mu_3}{2}H^2(t)\right), \tag{4.7}$$

where μ_3 and ν_3 are constants. In this study, the holographic principle [46] is applied by relating the infrared cutoff L to the modified Ricci radius. We take L^{-2} as a linear combination of $\dot{H}$ and H^2, with this MHRDE $\rho_m = 3C^2M_p^2L^{-2}$ [16,47,48] and it lead to the form

$$\rho_m(z) = \frac{2}{\mu_3 - \nu_3}\left(\frac{3\mu_3}{2}H^2(z) - (z+1)H'(z)H(z)\right), \tag{4.8}$$

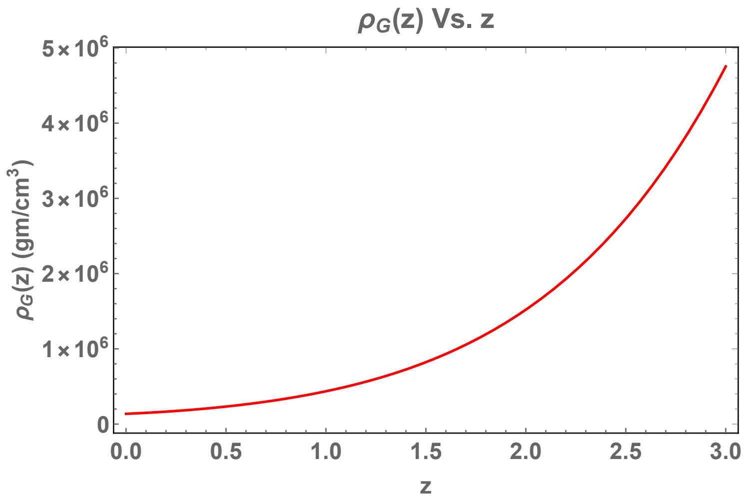

Figure 3: Variation of $\rho_G(z)(gm/cm^3)$ Vs. z for HDE GO cut-off model for the values $H_0 = 67.4, q_1 = -6.0, q_0 = -0.585, \mu_2 = -0.0001, \nu_2 = 10.0$

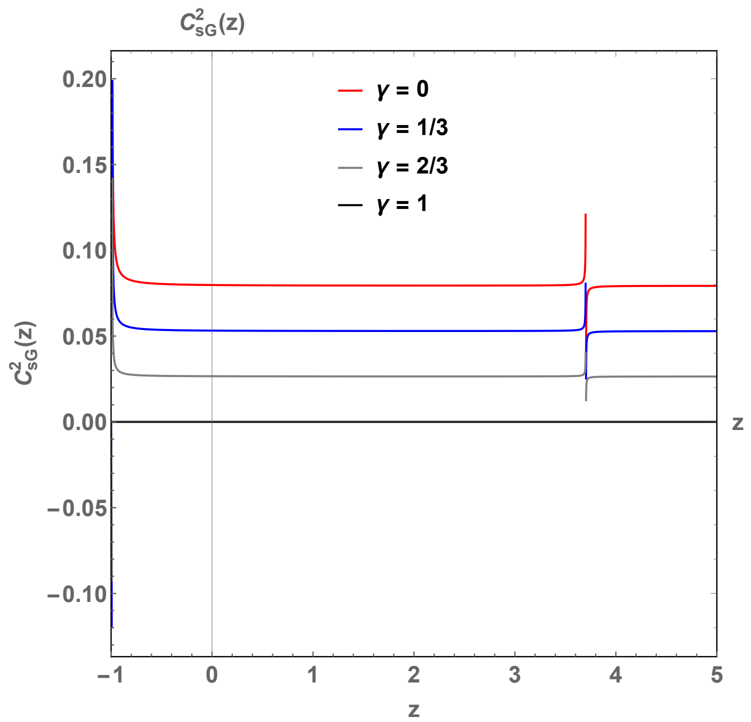

Figure 4: The evolution of $C^2_{sG}(z)$ for HDE of GO cut off with the values $H_0 = 67.4, q_1 = -6.0, q_0 = -0.585, \mu_2 = -0.0001, \nu_2 = 10.0$.

where $H = \frac{\dot{a}}{a}$ is the Hubble parameter, a is scale factor and μ_3, ν_3 are constants. The EoS parameter for the MHRDE model is given by

$$\omega_m(z) = \frac{-6\mu H^2(z)(\gamma-1)(\mu_3-\nu_3)}{[2(8\pi+\gamma\mu)+\nu(4-3\gamma)] \quad [3\mu_3 H^2(z) - 2 \quad (z+1)H'(z) \quad H(z)]}. \tag{4.9}$$

On differentiating equation (4.8) with respect to z, we get

$$\frac{d\rho_m}{dz} = \frac{2}{\mu_3 - \nu_3}\left((3\mu_3 - 1)H'(z)H(z) - (z+1)H''(z)H(z)(z) - (z+1)(H'(z)^2)\right). \quad (4.10)$$

For MHRDE model, for this case C_s^2 can be expressed by using the equations (3.12) and (4.10)

$$C_s^2 = \frac{(1-\gamma) \quad (\mu_3 - \beta_3) \quad 6\mu H'(z)H(z)}{[2(8\pi + \gamma\nu) + \nu(4 - 3\gamma)] \quad \{(3\mu_3 - 1)H'(z)H(z) - (1+z)H''(z) \quad H(z) - (1+z) \quad (H'(z))^2\}}. \quad (4.11)$$

Graphs of $\rho_m(z)$ and C_s^2 have been given in Fig (5) and (6) for the values of $H_0 = 67.4$, $q_0 = -0.585$, $q_1 = -6$, $\mu = 0.1$, $\nu = 0.05$, $\mu_3 = -0.01$, and $\nu_3 = 2.00$. In this case, for the accelerating evolution of the universe we need $\rho_m > 0$. Under these conditions, the $\rho_m(z)$ is always positive and approaches to a constant value (almost zero) in the remote future.

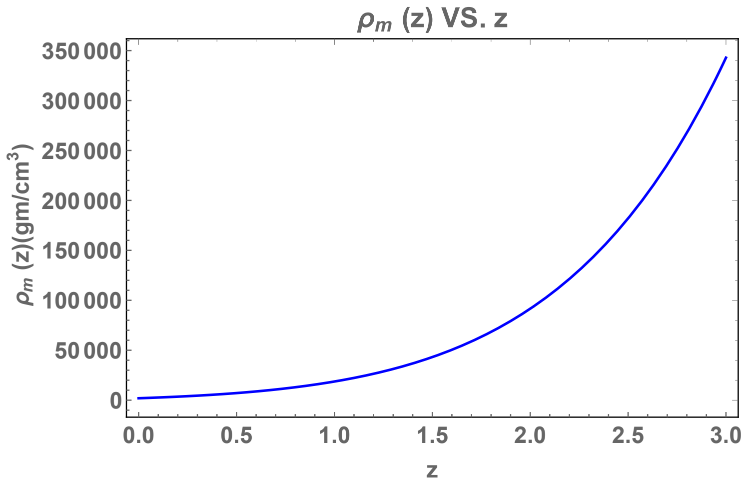

Figure 5: Variation of $\rho_m(z)(gm/cm^3)$ versus z for MHRDE model for the values $H_0 = 67.4, q_0 = -0.585, q_1 = -6, \mu = 0.1, \nu = 0.05, \mu_3 = -0.01, \nu_3 = 2.00$

.

4.3 Rényi holographic dark energy (RHDE) model

In recent years, a few new HDE models have been evolved, such as the RHDE model [49], Sharma-Mittal [50], and Tsallis HDE, RHDE is a more stable model among these models. By employing a generalized form of the entropy–area relationship, the holographic dark energy and gravity model equations can be extended. This approach motivates the use of Rényi entropy to construct a model describing the universe's accelerating phase.

Assume an m-state system with probability distribution P_k, where the normalization condition $\sum_{k=1}^{m} P_k = 1$ holds. Within this context, the Rényi and Tsallis entropies are regarded as prominent formulations of generalized entropy [51].

$$S_R = \frac{1}{\delta} ln \sum_{k=1}^{m} P_k^{1-\delta}, \quad S_T = \frac{1}{\delta}\sum_{k=1}^{m}(P_k^{(1-\delta)} - P_k), \quad (4.12)$$

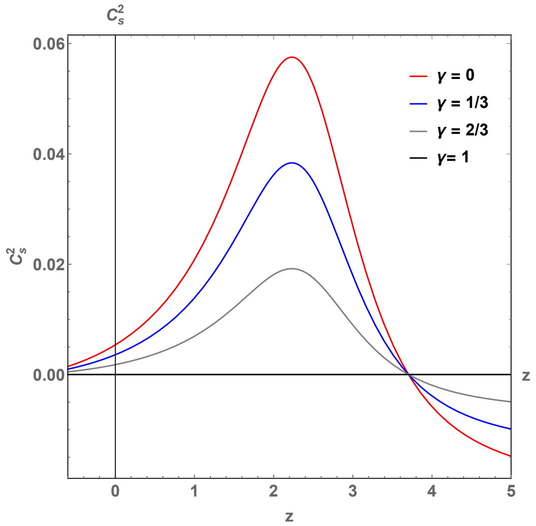

Figure 6: The evolution of $C_s^2(z)$ Vs. z is shown for MHRDE model for the values $H_0 = 67.4, q_0 = -0.585, q_1 = -6, \mu = 0.1, \nu = 0.05, \mu_3 = -0.01, \nu_3 = 2.00$.

here $\delta = 1 - U$, U is a real variable. Combination of eq. (4.12), yields

$$S_R = \frac{1}{\delta} ln(1 + \delta S_T), \tag{4.13}$$

where δ is constant and S_T is Tasllis entropy.For homogeneous systems, S_R represents one of the most general forms of entropy. It has been argued that the Bekenstein entropy ($S = \frac{A}{4}$) can be viewed as a special case of Tsallis entropy, leading to [52,53]

$$S_R = \frac{1}{\delta} ln\left(1 + \delta\frac{A}{4}\right). \tag{4.14}$$

If δ approaches to 0, then the Rényi entropy becomes

$$\rho_{de} = \frac{3}{8}\frac{d^2}{\pi L^2}\left(1 + \pi\delta L^2\right)^{-1}, \tag{4.15}$$

where d^2 is constant.To evaluate this equation, we employ the relations $T = \frac{1}{2\pi L}$, $V = \frac{4\pi}{3}L^3$, and $A = 4\pi L^2$, which are consistent with the FRW space-time. Subsequently, the RHDE model with the GO cutoff is analyzed to study its different characteristics. Using equations (3.14) and (4.15), the corresponding energy density for this model is obtained as

$$\rho_{RG}(t) = \frac{3}{8}\frac{d^2}{\pi}\left\{\frac{\left(\nu_2 H^2(t) + \mu_2 \dot{H}(t)\right)^2}{\left(\mu_2 \dot{H}(t) + \nu_2 H^2(t)\right) + \pi\delta}\right\}. \tag{4.16}$$

The above equation in the form of z is given by

$$\rho_{RG}(z) = \frac{3}{8}\frac{d^2}{\pi}\left\{\frac{\left[\nu_2 H^2(z) - \mu_2(1+z)H'(z)H(z)\right]^2}{\left[\pi\delta + \nu_2 H^2(z) - \mu_2(1+z)H'(z)H(z)\right]}\right\}. \tag{4.17}$$

The EoS parameter for this model with z is expressed as

$$\omega_d = \frac{16\pi\mu H^2(z)(\gamma-1)\left[\pi\delta + \nu_2 H^2(z) - \mu_2(z+1)H'(z)H(z)\right]}{d^2\left[\nu_2 H^2(z) - \mu_2(z+1)H'(z)H(z)\right]^2}. \tag{4.18}$$

On differentiating equation (4.17), we get

$$\frac{d\rho_{RG}}{dz} = \frac{3}{8}\frac{d^2}{\pi}\left\{\frac{(\pi\delta + M)2MN - M^2N}{[\pi\delta + M]^2}\right\}, \tag{4.19}$$

where

$$M = \nu_2 H^2(z) - \mu_2(z+1)H'(z)H(z),$$

and

$$N = (2\nu_2 - \mu_2)H'(z)H(z) - \mu_2(z+1)(H'(z))^2 - \mu_2(z+1)H''(z)H(z).$$

The stability parameter C_s^2 in this case, in the form of z is obtained as

$$C_S^2 = \left\{\frac{-32\pi\alpha(\gamma-1)H'(z)H(z)(\pi\delta + M)^2}{[2(8\pi + \gamma\beta) + \beta(4-3\gamma)]\,MNd^2[2(\pi\delta + M) - M]}\right\}. \tag{4.20}$$

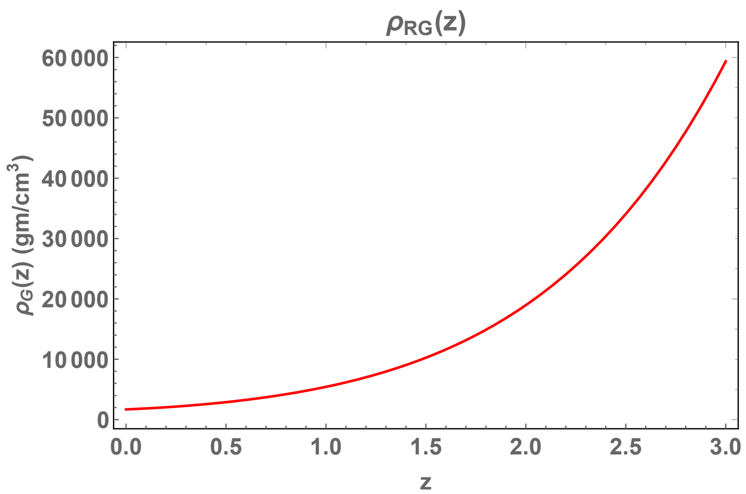

Figure 7: The evolution of $\rho_{RG}(z)(gm/cm^3)$ Vs. z for RHDE model is shown. The values are $H_0 = 67.4, q_1 = -6.0, q_0 = -0.585, \mu_2 = -0.0001, \nu_2 = 10.0$.

4.4 Discussions

Cosmographic parameters provide model-independent information regarding the evolving universe. The Hubble parameter characterizes the expansion history and is examined ot derive the constraints on the model parameters. In particular, the deceleration parameter describe the universe expansion history and relates the universe journey through different era of thermal evolution. For example, during the radiation era, $q = 1$ while in the matter dominated era, $q = 0.5$. The universe journey from the decelerated to accelerated expansion era may be visualized by the signature change of q. The Fig. (10) depicts the nature of q

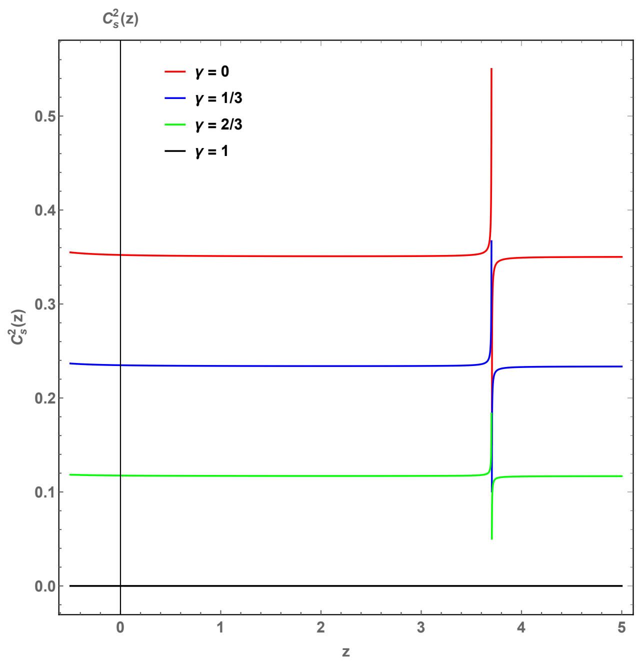

Figure 8: Depict the evolution of $C_s^2(z)$ Vs. z for RHDE model, for the values $H_0 = 67.4, q_1 = -6.0, q_0 = -0.585, \mu_2 = -0.0001, \nu_2 = 10.0$.

versus z. It has been noted that the deceleration parameter $q(z)$ is positive for high redshift z and negative for lower z. From the plot, it can be seen that $q > 0$ for $z > 0.5$, indicating a decelerating phase that allows the formation of cosmic structures, whereas negative values of q correspond to the current accelerating universe. The information regarding rate of change of deceleration parameter may be visualized by the jerk parameter $q(z)$. The jerk parameter in form of z is given by [54]

$$j(z) = (z+1)\frac{dq}{dz} + q(z) + 2q^2(z), \tag{4.21}$$

for this model it can be expressed as

$$j(z) = (z+1)q_1 \left[\frac{(z+1) - (z+2)\ln(z+2)}{(z+1)^2(z+2)} \right] + X + 2X^2, \tag{4.22}$$

where $X \equiv q_0 + q_1 \left(\frac{\ln(z+2)}{(z+1)} \right) - q_1 \ln 2$. The positive value of $j(z)$ signifies the transiting universe's evolution.
We use the equation of state (EoS) parameter as $\omega = \frac{1}{3}(2q-1)$ [55,56]. The variation of ω with z has been displayed in Fig. (10). The models may be interpolate the matter-dominated phase with the dark energy dominated phase.

The ΛCDM model possesses the matter dominated past with EoS parameter $\omega = 0$ and $q = 0.5$. In the accelerating era, one may have $\omega_0 \approx -0.69$ and $q_0 \approx -0.55$ [8]. In this model, $j = 1$. The ΛCDM model approaches to $\omega \to -1, q \to -1$ and $j = 1$ in the asymptotic limit $z \to -1$. In contrast, the model (3.7) possess $q_0 = -0.585$, $j_0 = 1.258$ and $\omega_0 = -0.7233$, during present times. The deceleration parameter suggests that the universe is accelerating and the universe may have the phantom evolution (with varying energy density) in the future, since $q < -1$ and $\omega < -1$. However, the model suggests that the dark energy in these models is of the quintessence kind during the present era.

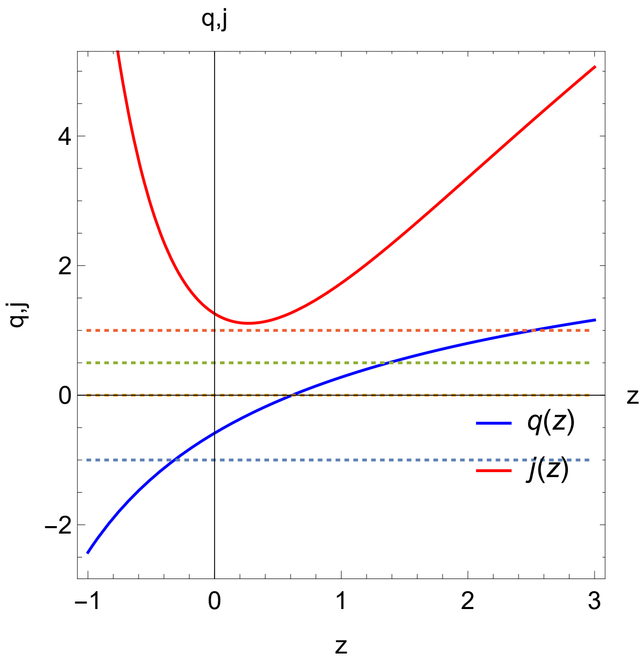

Figure 9: The plot of q, j Vs. z.

5 Conclusions

We study a flat FRW universe model within the $f(Q,T) = \mu Q + \nu T$ gravity with three holographic dark energy (HDE) models. These models have following distinct IR cutoffs: (1) HDE with the GO cutoff, (2) MHRDE, and (3) RHDE. To analyze the cosmic evolution in these models, we adopt an ansatz based approach for the deceleration parameter and derive the exact solutions for the Hubble parameter, energy density and the EoS parameter in these HDE models.

We derived form of the Hubble parameter has been scrutinized with the observational data of CC and Pantheon datasets. We constrained the model parameter using MCMC analysis with the supernovae type Ia data along with the Joint dataset. To illustrate the evolution of cosmological parameters, we plotted graphs of quantities such as energy density and the squared sound speed (C_s^2) to analyze the behavior of all three HDE models.
The key-take away about these three models has been summed up as follows:

- The Hubble parameter and deceleration parameter of the model illustrates the transiting universe evolution under the influence of varying dark energy.
- The model possesses $q_0 = -0.585$, $j_0 = 1.258$ and $\omega_0 = -0.7233$, during present times.
- The HDE model with GO cut off has been classically stable in the past, present, as well as in the far future. The universe for this model posses positive energy density

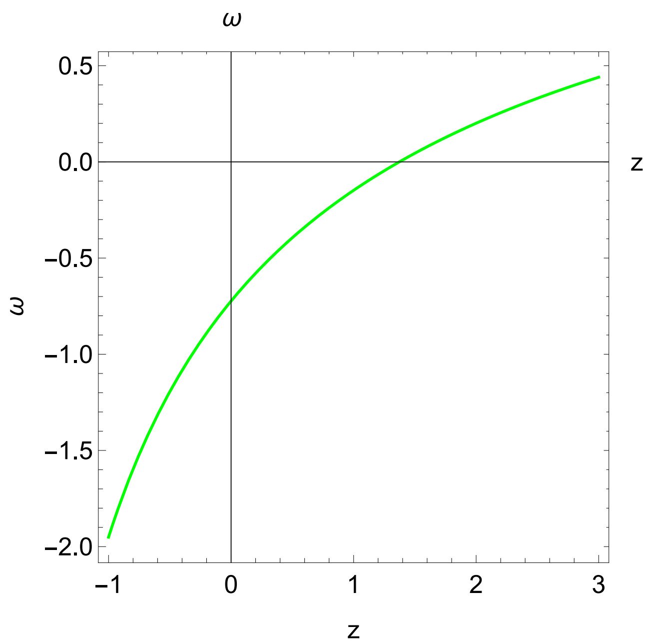

Figure 10: The plot of ω Vs. z.

during it's evolution corresponding to various red shift values.

- The MHRDE model has been classically stable at present as well as in the future and unstable in the past. In this model too, the energy density retains positive values during its evolution for various redshifts.

- The RHDE model with the GO cut off is classically stable at present, in the future, but unstable in the past. Energy density is positive for the RHDE model with the GO cut-off.

The model illustrates the universe evolution under influence of phantom energy in future although the universe is dominated by quintessence energy in the present times. These aspects are clearly deviated from the standard cosmological model, where the dark energy density follows $\omega = -1$. In the present work, rather than constraining the model parameters μ and ν of $f(Q,T)$ gravity, we treat them as free parameters according to the requirements of the model. It is observed that the behavior of physical parameters is highly sensitive to the values of these parameters.

Authors' Contributions

All authors have the same contribution.

Data Availability

The data supporting this work are presented in this paper.

Conflicts of Interest

The authors declare that there is no conflict of interest.

Ethical Considerations

The authors have diligently addressed ethical concerns, such as informed consent, plagiarism, data fabrication, misconduct, falsification, double publication, redundancy, submission, and other related matters.

Funding

This research did not receive any grant from funding agencies in the public, commercial, or non-profit sectors.

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