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Letter

Journey Through Cosmic: Tsallis Holographic Dark Energy and the Deformed Starobinsky Model

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Abstract. This study investigates the intricate dynamics of Tsallis holographic dark energy within the framework of a modified Starobinsky gravity theory. The distinctiveness of the model lies in its formulation, which involves both the Ricci scalar R and an additional positive term. The primary objective is to derive the equation of state parameter, a crucial element in understanding the behavior and properties of dark energy throughout the universe. To simplify and strengthen the analysis, we adopt an exponential form for the scale factor, commonly used in models featuring constant expansion rates due to its analytical tractability. A comprehensive stability evaluation is also carried out, with particular attention given to the squared sound speed—a critical factor in examining how fluctuations evolve within the dark energy sector. Graphical representations are employed to highlight stable regimes and visually interpret the viability of this holographic model under modified gravitational dynamics. The findings are presented in a detailed manner, including rigorous derivations and explicit formulations that underline the theoretical consistency and potential cosmological relevance of the model.

Keywords: Modified Starobinsky Gravity; Tsallis Holographic Dark Energy Model.

Contents

1 Introduction

One of the fundamental physical challenges is understanding the accelerated expansion of the universe, which is attributed to a hidden form of energy known as dark energy (DE) [1–3]. Evidence supporting this phenomenon includes observational constraints, the presence of soft dark matter, challenges to the ΛCDM cosmology [3,4], constraints on early dark energy from large-scale structure observations [5], and developments in the theory of causal fermion systems [4]. Numerous researchers have explored various aspects of DE, including black hole entropy, holographic unified inflation, the hyperbolic fracton model, subsystem symmetry, holography, K-essence, and phantom energy [6–8]. To address this problem, different modified gravity theories, which introduce new algorithmic parameters, have been proposed [9,10]. In attempts to describe the nature of DE, Einstein's General Relativity has been extended to include an additional dimension that supports gravitational interactions among particles. One such approach was suggested by Kuang, Tang, Wang, and Wang [11]. This theory proposes a possible "property of space" that creates negative pressure, thereby stretching the fabric of spacetime.

A new approach to DE research involves the application of the cosmological constant [12,13]. The cosmological constant term represents a constant energy density that fills space homogeneously, often interpreted as a "scalar field" with dynamic quantities that vary in time and space and consist of energy density [5]. The upper bound of DE contained in the universe can be evaluated using this dynamic energy density [3]. Indeed, it has been suggested that two main types of dark energy exist: the cosmological constant and scalar fields. In other words, the cosmological constant implies a constant energy density filling space uniformly and unchanging over time, while scalar fields represent dynamic quantities with energy densities that vary across time and space—examples include "quintessence" and "moduli" [5]. Specifically, quintessence involves scalar fields that depend only on time, whereas moduli arise from extra dimensions in string theory [1–4]. Several studies have utilized the holographic principle to examine DE. Building on this concept, [14] proposed using black hole entropy instead of metaphysical assumptions. The holographic nature of dark energy explains DE through the holographic principle, which states that the information content of a region of space can be encoded on its lower-dimensional boundary [5]. This principle has been suggested as a useful tool for understanding the DE problem, particularly in relation to horizon areas. Supporting this, Bousso [15] described a holographic principle concerning the entropy of DE in the universe.

Moreover, this framework incorporates the holographic principle for dark energy alongside quantum fluctuations of matter and radiation in the universe [16]. The energy density of dark matter is considered relatively small and decreases as the universe expands. Conversely, the energy density of dark energy remains constant. This inverse relationship is explained by the fact that a smaller horizon corresponds to lower energy density and vice versa [5,6]. Vogelsberger, Marinacci, Torrey, and Puchwein [17] highlighted the distinctions between dark energy and the evolution of cosmic structures. They investigated the formation of galaxies and galaxy clusters, examining the types and properties of dark energy and its varied influence on structure growth and matter distribution in the universe [18]. In the case of dark energy described by the cosmological constant, they studied structure growth until late times when dark energy dominates the energy density, causing accelerated expansion [19]. To characterize different dark energy structures, they utilized supernova observations. As standard candles, supernovae provide measurements at various epochs, and their observed brightness and redshift trace the universe's expansion history [19]. During this period, observations of the Cosmic Microwave Background (CMB) radiation—relic radiation from the early universe filling the sky—were also conducted [20,21]. Polarization fluctuations in the CMB have been analyzed to better understand the universe's geometry [22]. By measuring galaxy clustering and correlations, scientists can infer how dark energy affects the growth and evolution of cosmic structures [22].

Initial observations addressing these differences include gravitational lensing—the bending of light by gravity [12,13]. One modified gravity theory, $f(R)$ gravity, replaces the Ricci scalar R in the Einstein-Hilbert action with a more general function $f(R)$ [17,18]. Ivanov et al. introduced the Hu-Sawicki model and the Palatini formalism, interpreted within scalar-tensor gravity frameworks [21,22]. To describe gravitational dynamics, the Brans-Dicke theory is often employed. Other theories applied to dark energy include the Einstein-Aether theory, Khronometric theory, and bimetric theories. Certain gravitational models involving two interacting metrics can give rise to a massive graviton and an effective cosmological constant, depending on the coupling between metrics. Notable models include bigravity theories such as the de Rham–Gabadadze–Tolley (dRGT) model, the Hassan-Rosen construction, and related variants [23–32]. A particularly interesting extension is the modified deformed Starobinsky model, which exhibits several compelling cosmological behaviors. Its notable predictions include a de Sitter phase characterized by exponential cosmic expansion in the far future; a phantom regime where the dark energy equation of state falls below –1, potentially leading to a catastrophic event such as a Big Rip; and cyclic evolution where the universe undergoes repeated expansion and contraction cycles, each lasting longer than the previous one. Furthermore, this model supports a smooth transition out of the inflationary epoch, allowing the universe to naturally evolve into a radiation-dominated state without requiring additional mechanisms such as reheating or entropy injection [18–20].

These theoretical features form the foundation of our investigation, which is organized as follows: Section 2 provides an overview of the fundamentals of $f(R)$ gravity. Section 3 outlines the principles of Tsallis holographic dark energy. Section 4 presents the general form of the deformed Starobinsky model. Section 5 analyzes Tsallis holographic dark energy within this deformed gravity setting, considering both exponential and hybrid scale factors, and compares our results with previous studies. Finally, Section 6 summarizes our conclusions and key insights.

2 f(R) Gravity

In this section, we explore an extended theory of general relativity. We begin by considering the Einstein-Hilbert Lagrangian, modified to include the Ricci scalar R, as follows [18–22]:

$$S = \frac{1}{2\kappa} \int f(R)\sqrt{-g}, d^4x + S_M. \tag{2.1}$$

In this equation, g represents the metric determinant, S_M denotes the matter action, and $\kappa = 8\pi G$. To derive the field equations for $f(R)$ gravity, we assume a flat Friedmann-Robertson-Walker (FRW) metric:

$$ds^2 = -dt^2 + a^2(t)\left[dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta, d\phi^2\right], \tag{2.2}$$

where $a(t)$ is the scale factor. Considering a perfect fluid as the matter content, the energy-momentum tensor is given by,

$$T^{\mu\nu} = (\rho + p)u^\mu u^\nu + p g^{\mu\nu}, \tag{2.3}$$

where ρ is the energy density and p is the pressure of the fluid. The field equations can be derived as follows:

$$H^2 = \frac{1}{3f'(R)}\left(\kappa\rho + \frac{Rf'(R) - f(R)}{2} - 3H\dot{R}f''(R)\right), \tag{2.4}$$

$$2\dot{H} + 3H^2 = -\frac{\kappa}{f'(R)}\left(p - \frac{Rf'(R) - f(R)}{2} - 2H\dot{R}f''(R) + \dot{R}^2 f'''(R) + \ddot{R}f''(R)\right). \tag{2.5}$$

From these, we can define the effective energy density and pressure:

$$\rho_{eff} = \frac{1}{\kappa f'(R)}\left(\frac{Rf'(R) - f(R)}{2} - 3H\dot{R}f''(R)\right), \tag{2.6}$$

$$p_{eff} = \frac{1}{\kappa f'(R)}\left(\dot{R}^2 f'''(R) + \ddot{R}f''(R) - \frac{Rf'(R) - f(R)}{2} + 2H\dot{R}f''(R)\right). \tag{2.7}$$

Finally, the equation of state parameter ω_{tot} is given by:

$$\omega_{tot} = \frac{p + \dot{R}^2 f'''(R) + \ddot{R}f''(R) - \frac{Rf'(R)-f(R)}{2} + 2H\dot{R}f''(R)}{\rho + \frac{Rf'(R)-f(R)}{2} - 3H\dot{R}f''(R)}. \tag{2.8}$$

The parameter ω_{tot} characterizes the behavior of the $f(R)$ system and the matter content, enabling us to evaluate the evolution of the universe using a specific $f(R)$ model and its derived field equations.

3 Tsallis Holographic Dark Energy

To address the complexities of dark energy, we begin with an overview of the Holographic Dark Energy (HDE) model. This model is grounded in the holographic principle, which posits that the number of degrees of freedom associated with entropy scales with the area enclosing a system [21,31,32]. The energy density in holography is expressed as:

$$\rho = \frac{3c^2 M_p^2}{L^2}, \tag{3.1}$$

where L represents a characteristic length scale. This relationship is derived from the entropy–area relation for a black hole, $S \propto A$, where A is the area of the event horizon. Considering ultraviolet (UV) and infrared (IR) cutoffs [22], we redefine the entropy as

$$L^3\Lambda^3 \leq S^{3/4}, \tag{3.2}$$

where L and Λ are the UV and IR cutoffs, respectively. By extending the standard Boltzmann-Gibbs entropy to a non-extensive form, we apply Tsallis entropy, which is given by:

$$S = \gamma A^{\delta}, \tag{3.3}$$

where γ is an unknown constant and δ is the non-additivity parameter. When $\gamma = \frac{1}{4G}$) and $\delta = 1$, we recover the Bekenstein-Hawking entropy. The vacuum energy density is then:

$$\rho = BL^{2\delta-4}, \tag{3.4}$$

with B being an unknown parameter. Various IR cutoffs, such as the Hubble horizon, event horizon, and particle horizon, can be used. For simplicity, we choose the Hubble horizon $L = H^{-1}$, leading to the energy density:

$$\rho = BH^{4-2\delta}. \tag{3.5}$$

The parameter δ is related to the system's dimensions, defined as $\delta = \frac{d}{d-1}$. For $d = 1$, we retrieve the standard holographic dark energy. When $\delta = 2$, it corresponds to the cosmological constant. Considering the Friedmann equation and the equation of state, the conservation of energy is expressed as

$$\dot{\rho} + 3H(\rho + p) = 0, \tag{3.6}$$

where the pressure p is calculated as:

$$p = \frac{2\delta - 4}{3} B\dot{H}H^{-2\delta+2} - BH^{-2\delta+4}. \tag{3.7}$$

4 Deformed Starobinsky Model

The characteristics of the $f(R)$ gravity model can be significantly altered under various conditions and constraints. In this section, we delve into the deformed Starobinsky model, a modification of the original Starobinsky model of inflation. This deformed model is expressed as[37–39]:

$$f(R) \approx R^{2(1-n)}, \tag{4.1}$$

Here R denotes the Ricci scalar and n is a positive parameter that introduces the deformation. The original Starobinsky model, characterized by $f(R) = R + nR^2$, has been instrumental in explaining early-universe inflation. However, the deformed version, $f(R) \approx R^{2(1-n)}$, introduces a new dimension to the model by modifying the quadratic term. This adjustment allows for a more nuanced exploration of the universe's expansion dynamics.

The deformed Starobinsky model retains the desirable features of the original model, such as its ability to drive inflation in the early universe. The parameter n fine-tunes the inflationary dynamics, potentially leading to different predictions for the inflationary epoch. By modifying the R^2 term, the deformed model can also address the late-time acceleration of the universe. This dual capability makes it a versatile tool for cosmological studies, bridging the gap between early and late-time cosmic evolution.

The stability of the deformed Starobinsky model is crucial for its viability. Stability analysis involves examining the behavior of perturbations within the modified gravity framework. Ensuring that the model remains stable under small perturbations is essential for its acceptance as a realistic cosmological model.

The deformed model provides a rich landscape of cosmological solutions. By adjusting the parameter n, one can explore various scenarios of cosmic evolution, ranging from inflationary phases to dark energy-dominated epochs. This flexibility allows for a comprehensive study of the universe's expansion history.

5 Study and Discussion

This section focuses on examining a dark energy framework that merges two fundamental concepts: the holographic principle and Tsallis entropy. The holographic principle proposes that the total information contained within a region is proportional to the surface area enclosing it, rather than its volume. Tsallis entropy generalizes the conventional concept of entropy to accommodate complex systems exhibiting features such as long-range correlations and fractal structures.

In our formulation, the dark energy density depends on both the Hubble horizon scale and the Tsallis parameter, which quantifies the deviation of entropy from the classical form. This model offers an alternative mechanism to explain the universe's accelerated expansion, avoiding the need for a cosmological constant or additional scalar fields.

Moreover, we incorporate a modified $f(R)$ gravity framework that generalizes Einstein's theory by introducing a non-standard functional dependence on the Ricci scalar R within the gravitational action. This framework includes elements from the deformed Starobinsky model, enabling the description of both the inflationary phase of the early universe and the acceleration observed at late times. To analyze cosmic evolution, two types of expansion profiles are considered: (1) exponential expansion, characterized by a constant growth rate through an exponential scale factor, and (2) hybrid expansion, which combines exponential and power-law behaviors to capture the transition from a decelerated to an accelerated cosmic regime.

5.1 Exponential Case

For the exponential case, where the scale factor $a(t) = t^a$, we calculate the energy density and pressure for the modified deformed starobinsky model. Using the equations derived in previous sections, we can determine key quantities such as the equation of state,

$$\rho_{exp} = \frac{(2n-1)\left(-36a^3 + 12(4n-1)a^2 + 3(8n(4n-5)+5)a - 2(n-1)(24n+t)\right)}{6(2a-1)(n-1)t^2}, \quad (5.1)$$

and

$$p_{exp} = \Big[(36-72n)a^3 + 12(8n^2-6n+1)a^2 + 3a((2n-1)(8n(4n-5)+5) + 4(n-1)pt^2) - 2(n-1)(48n^2 + t(3pt-1) + 2n(t-12))\Big] \Big/ \Big(3(2a-1)(3a(2n-1)(2a-4n+3) + 2(n-1)\rho t^2)\Big). \quad (5.2)$$

One method to assess the stability of dark energy models is by examining the sound speed, c_s^2, which indicates how perturbations in dark energy propagate. The sound speed parameter influences both the cosmic microwave background (CMB) and the matter power spectrum. If c_s^2 differs from the speed of light, dark energy perturbations can either grow or decay, thereby affecting CMB anisotropies and large-scale matter clustering.

To measure c_s^2, we utilize a combination of CMB data and large-scale structure observations, such as galaxy surveys and weak lensing measurements. By comparing theoretical predictions with observed data, we can constrain c_s^2 and test the stability of various dark energy models. Techniques such as Bayesian analysis or frequentist approaches can be employed to derive likelihood functions and confidence intervals for c_s^2.

The stability of the dark energy model is illustrated through various plots. Figure 1 shows the equation of state as a function of time for different values of n, the power parameter of the modified deformed Starobinsky model term. The equation of state, which relates the pressure and density of the cosmic fluid, is crucial for understanding the universe's evolution.

Figure 1 depicts how the equation of state varies with n, highlighting its impact on the expansion rate. The stability regions of the modified deformed Starobinsky model for the exponential case are also shown. Positive values indicate stability, while negative values suggest instability. These regions help determine whether the model can produce consistent and realistic cosmological solutions.

In summary, our study demonstrates that the deformed Starobinsky model, combined with the holographic principle and Tsallis entropy, offers a robust framework for explaining the accelerated expansion of the universe. The stability analysis further supports the viability of this model in describing cosmic evolution.

Additionally, we present the equation of state as a function of n for various values of the Tsallis parameter. This clearly illustrates how different values of the Tsallis parameter impact the equation of state, thereby influencing the universe's expansion history.

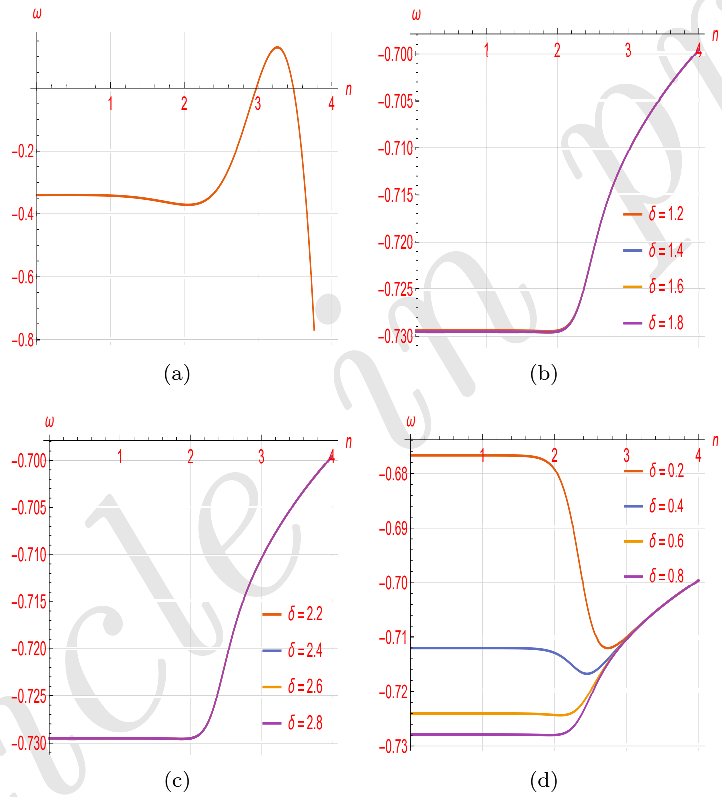

Figure 1: The graph depicts how the parameter ω varies with n across different values of δ, considering the influence of the free parameters and for various choices of the parameter n.

5.2 Hybrid Case

We extend our analysis to the modified deformed Starobinsky model using a hybrid scale factor $a = e^{ct}t^{b}$. As discussed in the previous section, we perform similar calculations for this hybrid case. Figure 2 depicts the equation of state as a function of time for different values of n, the power parameter of the modified deformed Starobinsky model term. The equation of state, which relates the pressure and density of the cosmic fluid, is crucial for describing the universe's evolution. The figure further illustrates how the equation of state varies with respect to n, highlighting its impact on the expansion rate.

Additionally, figures plot the equation of state for different values of the dimensionless parameter δ. These figures show how variations in δ influence the equation of state and, consequently, the acceleration or deceleration of the universe.

The stability analysis for the hybrid case indicates regions of stability and instability for different values of n. Positive values imply stability, while negative values suggest instability. These regions help determine whether the model can produce consistent and realistic cosmological solutions.

5.2.1 Discussion

The motivation behind investigating the modified deformed Starobinsky model lies in addressing cosmic acceleration. Researchers aim to determine whether this modification to gravity can provide an alternative explanation for the observed late-time acceleration of the universe. Additionally, understanding the behavior of $f(R)$ gravity models in various cosmological contexts is crucial for refining our understanding of fundamental physics.

The speed of sound associated with perturbations in the modified deformed Starobinsky model, c_s^2, is an important quantity. It characterizes the stability of perturbations and influences the growth of cosmic structures. In our investigation, we calculate c_s^2 to assess the model's viability. The modified deformed Starobinsky model can naturally lead to cosmic acceleration without introducing a cosmological constant. By modifying gravity, it offers an alternative explanation for the observed expansion history.

Ensuring the stability of perturbations in $f(R)$ gravity models can be challenging. Our analysis extends beyond specific cases, exploring a range of parameter values to identify viable regions that maintain consistency with both early- and late-time cosmological observations. By systematically varying the model parameters, we aim to understand its behavior across different scenarios.

In summary, the modified deformed Starobinsky model offers intriguing possibilities but also faces challenges. Researchers continue to refine the model, seeking to reconcile theoretical predictions with observational data.

To compare this work with previous research, we note that in [40], the Tsallis holographic dark energy model is investigated within a modified logarithmic $f(R)$ gravity framework characterized by both polynomial and logarithmic terms. The primary objective is to determine the equation of state parameter to better understand the dynamics of dark energy. Two scale factor models are analyzed: the exponential scale factor, representing constant expansion, and the hybrid scale factor, which captures both early deceleration and late acceleration phases of cosmic expansion. Stability is examined through sound speed analysis to assess perturbation behavior within the dark energy field. The study includes graphical representations to illustrate stability conditions and evaluates the overall viability of the model. Detailed calculations and results are thoroughly presented. The results presented in these two papers are closely connected, primarily due to the explicit elimination or setting to zero of the additional parameters involved in the models. This relationship is clearly evident in the comparative analysis, highlighting how the simplification of the parameter space directly influences the consistency and convergence of the outcomes. By constraining these extra parameters, the models effectively reduce to similar forms, which accounts for the observed alignment in their results. It is important to acknowledge that extensive research has been conducted in the areas of non-extensive entropies in black hole thermodynamics, and dark energy. These studies have significantly advanced our understanding of the fundamental connections between gravitational systems, quantum information, and cosmological phenomena. A comprehensive body of literature has explored various theoretical

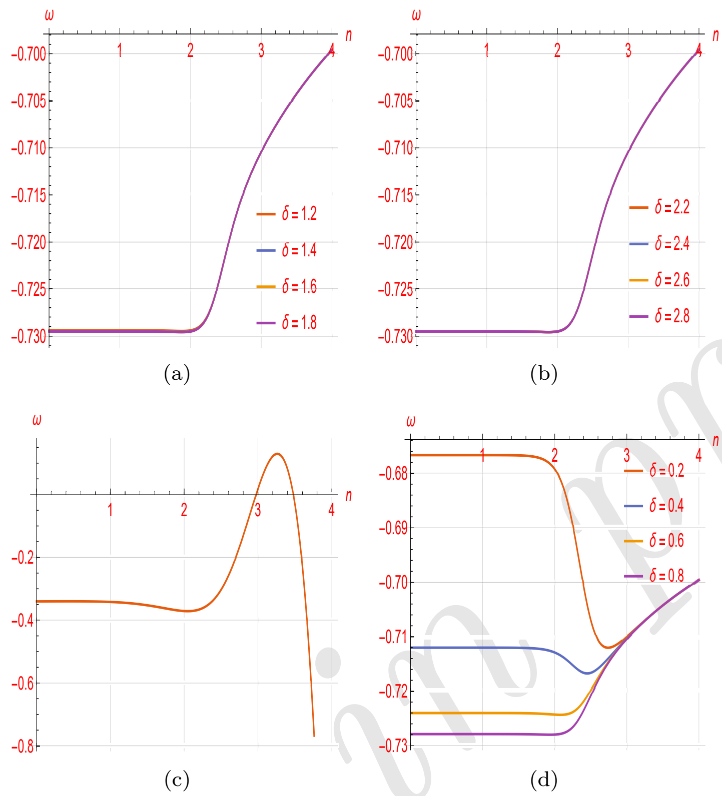

Figure 2: The graph depicts how the parameter ω varies with n across different values of δ, considering the influence of the free parameters and for various choices of the parameter n.

frameworks and models addressing these topics, including the thermodynamic properties of black holes, the role of generalized entropy measures in complex systems, and the implications of dark energy on the evolution of the universe. For a detailed overview and further reading, the references provided offer valuable insights and foundational contributions to this interdisciplinary field [41–51].

6 Conclusion

In this study, we have explored the intricacies of the Tsallis holographic dark energy model within the framework of modified Starobinsky gravity. This model is distinguished by its incorporation of the Ricci scalar (R) alongside an additional positive component. Our primary objective was to determine the equation of state parameter, which is essential for understanding the nature of dark energy in the universe.

To provide a comprehensive analysis, we employed an exponential scale factor, noted for its simplicity and relevance in cosmological models exhibiting a constant expansion rate. Furthermore, we conducted an extensive stability analysis of the model, focusing on the sound speed to evaluate how perturbations propagate within the dark energy field.

Through various plots, we illustrated the conditions under which the model remains stable and assessed the overall viability of the Tsallis holographic dark energy model in the context of modified gravity. Our findings are presented with precision, supported by detailed expressions and calculations, offering valuable insights into the behavior of dark energy in the cosmos.

Data Availability

The manuscript has no associated data or the data will not be deposited.

Conflicts of Interest

The author declares that there is no conflict of interest.

Ethical Considerations

The author has diligently addressed ethical concerns, such as informed consent, plagiarism, data fabrication, misconduct, falsification, double publication, redundancy, submission, and other related matters.

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